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AUTHOR Heidema, Clare; Schweitzer, Janis
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ABSTRACT

This guide represents the final experimental version of a pilot project conducted in the United States between 1973 and 1976. The ideas and manner of presentation are based on the works of Georges and Frederique Papy. They are recognized as having introduced colored arrow drawings ("papygrams") and models of our numeration system (the Papy "minicomputer") into the teaching of mathematics at the elementary and secondary level in Belgium. This program follows the "spiral approach." Suggestions are given for acquiring materials for lessons. This guide is divided into four sections which include: Notes to the Teacher, Suggested Schedule of Lessons, Questions and Answers About the Comprehensive School Mathematics Program, and The World of Numbers. Worksheets for the students accompany the guide. (Author/SA)

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Comprehensive School Mathematics Program
CEMREL, Inc.
Final Experimental Version
2-33401

CSMP Mathematics for the Upper Primary Grades
Part II

Teacher's Guide

U.S. DEPARTMENT OF HEALTH,
EDUCATION & WELFARE
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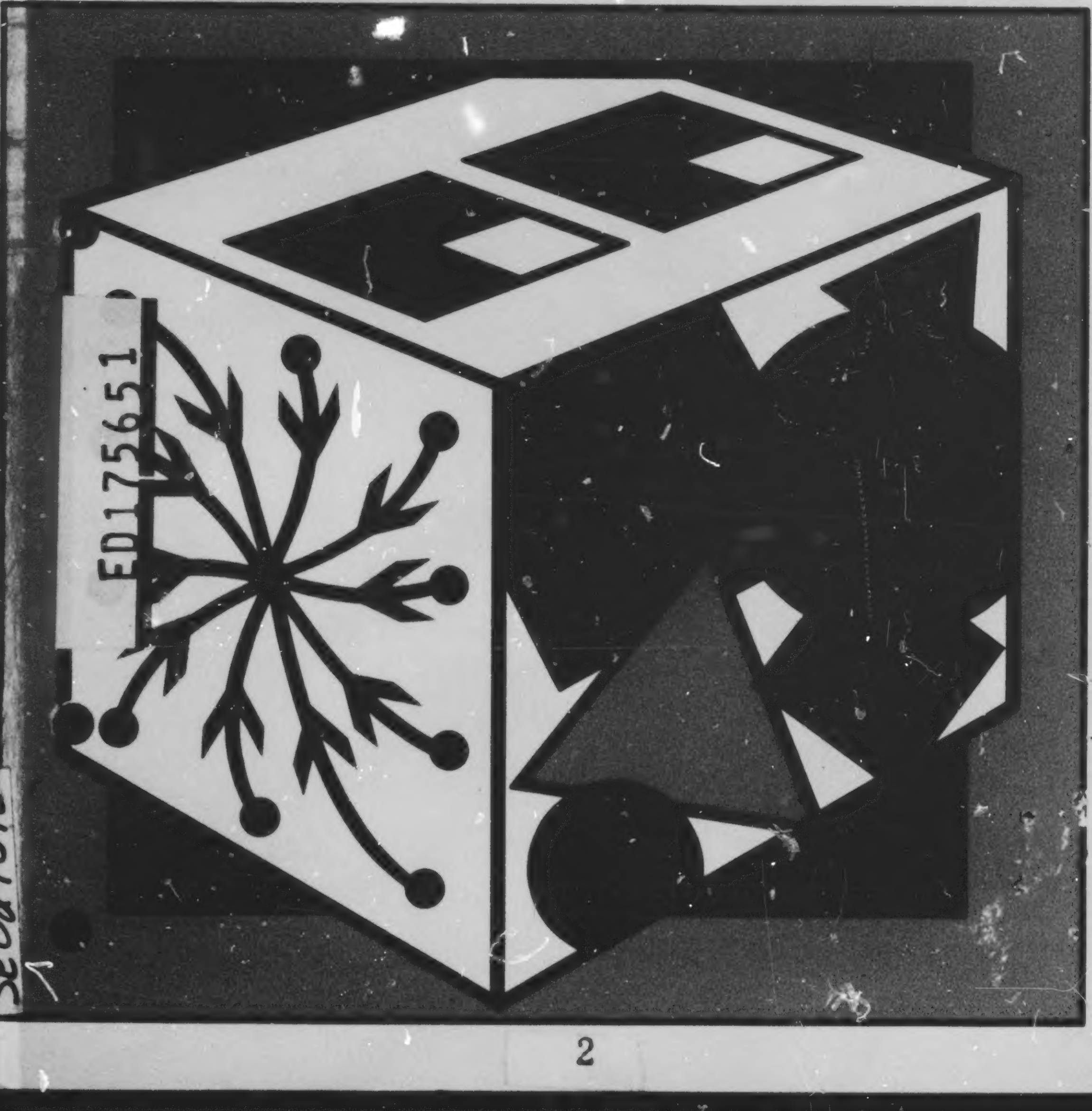
General Introduction
The World of Numbers



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Comprehensive School Mathematics Program
3120 59th. Street, St. Louis, Mo. 63139

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Comprehensive School Mathematics Program

CSMP Mathematics for the Upper Primary Grades

Part II

Teacher's Guide

CEMREL, Inc.

PREFACE

CSMP Mathematics for the Upper Primary Grades (UPG) owes its existence to the coming together under the leadership of Burt Kaufman (CSMP Director) of a special group of people who have been concerned not only about the kind of mathematics that is being taught to children but also about the manner in which it is taught. This is even more important in the crucial early years of a child's life than in later years.

The CSMP elementary school curriculum is in large part the outgrowth of a great deal of active involvement with children in ordinary classroom settings, and its present form was strongly influenced by information gathered from classes participating in an Extended Pilot Test conducted nationwide during 1974-77. But its manner of presentation and its main pedagogical ideas and tools, which stand out from all other curricula, are based on the pioneering works of Georges and Frédérique Papy acknowledged here with deep gratitude.

Use of colored arrow diagrams[*] -

Georges Papy, Die Erste Elemente der Moderne Mathematik; Shriftenreihe zur Mathematik, Hefte 10-11 (Frankfurt and Hamburg: Otto Salle Verlag, 1962-63)
Mathématique Moderne, Vols. 1, 2, 3, 5, 6 (Brussels and Paris: Didier, 1964-1967)
Modern Mathematics 1, 2 (London: Collier-Macmillan Ltd., 1968)

Frédérique and Georges Papy, L'Enfant et les Graphes (Brussels, Montreal and Paris: Didier, 1968)
Graphs and the Child (Montreal: Algonquin, 1970)

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[*] In 1959, Georges Papy introduced colored arrow diagrams into the teaching of mathematics at the elementary and the secondary school level. For this reason, they are sometimes referred to as "papygrams" in his honor.

Les Enfants de 4 Ans et le Langage des Graphes (Paris : Classiques Hachette, 1973)

Origin of the Papy Minicomputer-

Georges Papy, Minicomputer (Brussels : IVAC, 1968) in collaboration with Frédérique Papy

Use of graphs and the Papy Minicomputer with young children-

Frédérique Papy, Les Enfants et la Mathématique[*], Vols. 1, 2, 3, 4 (Brussels, Montreal and Paris : Didier, 1970-1976)

Mathematics and the Child 1, (Montreal : Algonquin, 1971)

By making this wealth of experience available to CSMP, Frédérique and Georges Papy have made an invaluable contribution to the production of this program. It must also be acknowledged that the fundamental idea of presenting mathematics to children via "pedagogical situations" is due to the mathematician and psychologist Caleb Gattegno.

Various prototype versions of the UPG Program dating back to 1973 were due to the efforts of former CSMP staff members José Agusti, Phyllis Klein, Edward Martin, and Christiane Vandeputte, and staff members Burt Kaufman and Frédérique Papy.

The large amount of information gleaned, thanks to the interest and concern of many teachers, in the Extended Pilot Tests since 1974 led to the revision of the program that culminated in this final version. During the 1976-77 school year, this final version of Part II was tried out by CSMP staff members Clare Heidema and Janis Schweitzer in Jackson Park Elementary School, University City, Missouri. In this effort they received valued collaboration and support of second grade teacher Marylou Long and the motivating responses of her students.

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[*] A series of books reporting on an experiment in teaching mathematics in grades 1 to 6 in Brussels, using the languages of strings, arrows, and the Papy Minicomputer. The reports of the first four years have been published by Didier; the reports of the last two years exist only in manuscript.

NOTE ON AUTHORSHIP

This printed copy of the final experimental version of CSMP Mathematics for the Upper Primary Grades, Part II, is the result of the work of the following people :

Development Team Leader :	Frédérique Papy (CSMP Associate Director for Research and Development)
Writers :	Clare Heidema Janis Schweitzer
Advisor on Content :	Burt Kaufman (CSMP Director)
Consultants :	Vince Haag (CSMP Staff Associate) Georges Papy (CSMP Staff Associate)
Editors :	Clare Heidema Pamela Pedersen
Production Coordinator :	Pinky Kaufman
Graphics Specialists :	John Brutcher Elliot Green Cyndi Heimburger Steven Sims
Typists :	Kim Davidson Gail Salas Linda Schneider

Comprehensive School Mathematics Program

CSMP Mathematics for the Upper Primary Grades

Part II

General Introduction

CEMREL, Inc.

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SECTION ONE

NOTES TO THE TEACHER

WELCOME

You are about to teach the second semester (Part II) of CSMP Mathematics for the Upper Primary Grades (grades 2 and 3), perhaps for the first time. We wish you success in your teaching and hope that both you and your students will enjoy the activities suggested in this teacher's guide. It is the purpose of these Notes to the Teacher to help you use this book effectively; to describe the materials and to give suggestions for organizing the program; and to answer some frequently asked questions which may also be your questions about CSMP.

You will find, as you and your students take part in the activities during the year, that there is a definite CSMP viewpoint about elementary school mathematics, a viewpoint that we hope you will come to understand and to share. We believe that learning takes place when children react to interesting situations (sometimes called a "pedagogy of situations"[*]), whether real life or fantasy, such as in stories or games. These situations involve children personally and allow arithmetic to take the form of "adventures in the world of numbers".

We believe that every child can and does learn something from each situation encountered in the program, sometimes suddenly and dramatically, sometimes latently. For this reason, CSMP teachers do not stick to one topic until it is "mastered". Indeed our experience indicates that true learning often stops when the purpose of a lesson is merely the mastery of a skill. Instead, we believe that learning is more a "spiral" process than a "straight line" process, that intuitive leaps play as big a role as acquiring small successive pieces of information, and that a variety of situations can provide the interrelated experiences through which we learn.

The Question and Answer section of these Notes to the Teacher gives more explanation of what we mean by a "pedagogy of situations" and a "spiral development", but you will probably learn their meanings only by experiencing them yourself as you teach the curriculum. Meanwhile we assure you that a large number of CSMP teachers across the country have come to understand and embrace the CSMP style of teaching and learning and are greatly pleased with the results.*

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[*] This phrase is due to the mathematician and psychologist, Caleb Gattegno.

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* See evaluation Report 2-A-1 in the CSMP Evaluation Report Series, CEMREL, Inc., St. Louis, Missouri.

PROGRAM DESCRIPTION

CSMP Mathematics for the Upper Primary Grades (UPG) is designed as a program for students in second and third grades. There are four parts to the curriculum, each corresponding to one semester's work. This is Part II of the four.

Since the learning process occurs through many interrelated experiences, we have developed this curriculum so that no single lesson is an end in itself. It is neither intended nor expected that every student will meet the full challenge of a particular situation in any given lesson. Furthermore, we have found that it is most effective to vary the situations from day to day rather than to continue one type of situation or topic until so-called mastery has occurred. This spiral development (see also Question 1 of Section Two) is reflected in the materials and scheduling of various topics in the curriculum. It is consistent with CSMP's pedagogy of situations since with such an approach, a child experiences each of several ideas a little at a time and then proceeds through increasing levels of sophistication as the situations become more challenging. We hope you will keep in mind the spiral approach as you teach the CSMP curriculum.

1. Description of Materials

a) Teacher's Guide

This set of two volumes (five books) which is directed specifically to you, the teacher, is known as the "Teacher's Guide to CSMP Mathematics for the Upper Primary Grades, Part II". (By way of abbreviation, in what follows we shall refer to this collection simply as the "Teacher's Guide".) With the exception of this book, "General Introduction", each book of the Teacher's Guide addresses itself to a single area of the curriculum, and each of these areas is known as a "strand". The four strands with abbreviations given in parentheses are:

- World of Numbers - (N)
- Languages of Strings and Arrows - (L)

- Geometry and Measurement - (G)
- Workbooks - (W)

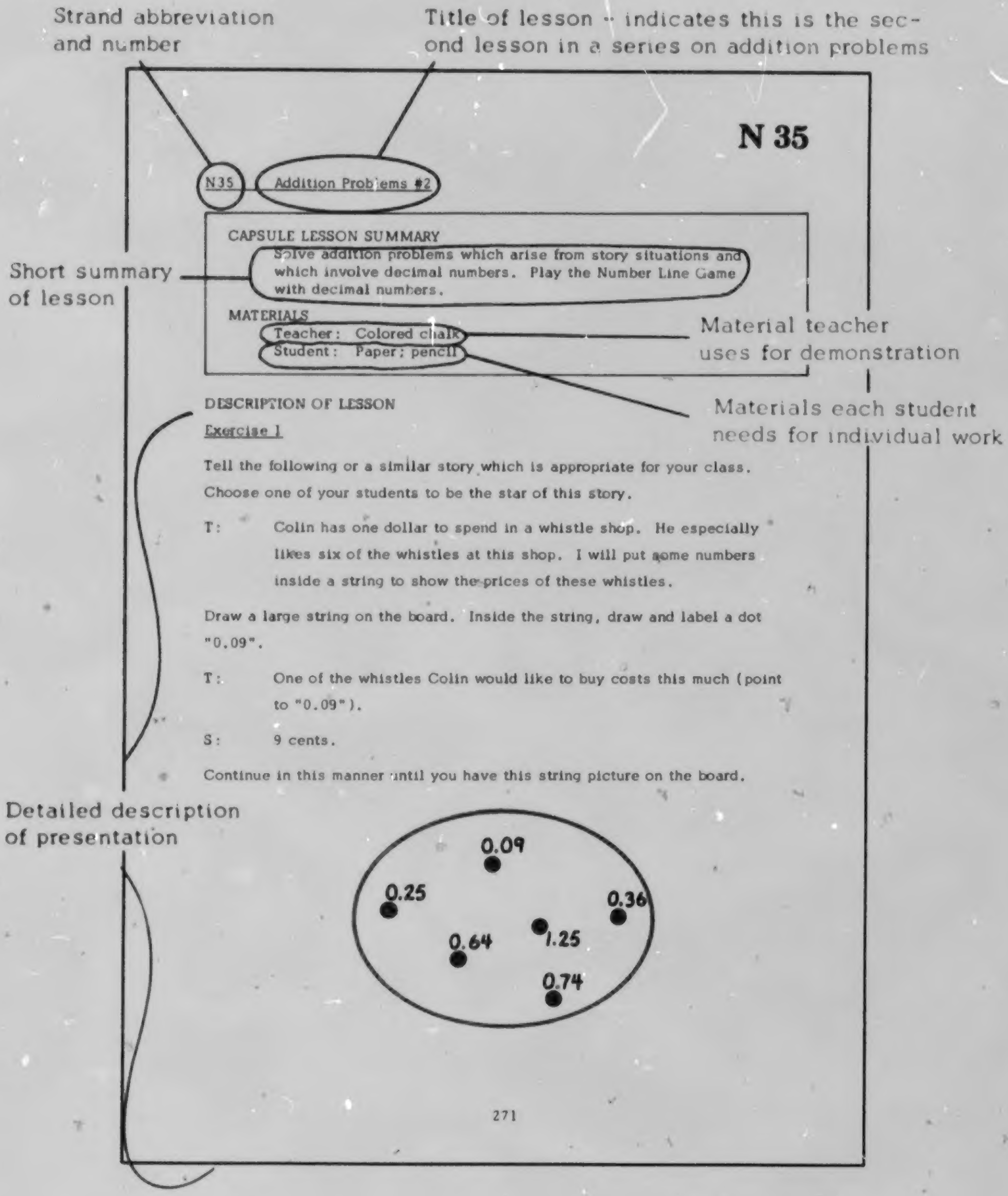
The five books comprising the Teacher's Guide are bound into two volumes, with a separation page inserted between the various books of a given volume. This volume contains both the General Introduction and the World of Numbers strand. The other volume contains the Languages of Strings and Arrows strand, the Geometry and Measurement strand, and the Workbooks strand. Each of these five books has its own table of contents and is paginated individually.

The strands have the following arrangement of material.

- A table of contents. This enables you to locate whatever you need in that particular strand.
- An introduction to the strand. This provides you with information explaining why certain material finds its place in that strand, what are the overall goals of study in that strand, and how this material fits in with that which is described in other strands.
- A series of lesson plans. These plans include Capsule Lesson Summaries, lists of needed materials for both teacher and student, and lesson descriptions which suggest possible classroom dialogue and indicate the use of worksheets and manipulative materials. The lesson descriptions also provide illustrations of diagrams which the teacher puts on the board or otherwise displays for the class and answer keys for worksheets or other independent work during the lessons.

In each of the strand books, N, L, G, and W, the lessons are consecutively numbered. Each lesson is designated to be used during one full class period. For example, Lesson N8 is the eighth lesson in the N (World of Numbers) strand. Since this lesson is interspersed among lessons from other strands, it might be taught on, say, Wednesday of the fourth week of the term and the next N lesson (N9) might be taught on Monday of the fifth week.

Each lesson is presented in a form that includes its strand and number, title, summary, materials, and description. For example, here is the first page from the thirty-fifth lesson in the N (World of Numbers) strand.



Strand abbreviation and number

Title of lesson -- indicates this is the second lesson in a series on addition problems

Short summary of lesson

Material teacher uses for demonstration

Materials each student needs for individual work

Detailed description of presentation

N 35

N35 Addition Problems #2

CAPSULE LESSON SUMMARY

Solve addition problems which arise from story situations and which involve decimal numbers. Play the Number Line Game with decimal numbers.

MATERIALS

Teacher: Colored chalk

Student: Paper; pencil

DESCRIPTION OF LESSON

Exercise 1

Tell the following or a similar story which is appropriate for your class. Choose one of your students to be the star of this story.

T: Colin has one dollar to spend in a whistle shop. He especially likes six of the whistles at this shop. I will put some numbers inside a string to show the prices of these whistles.

Draw a large string on the board. Inside the string, draw and label a dot "0.09".

T: One of the whistles Colin would like to buy costs this much (point to "0.09").

S: 9 cents.

Continue in this manner until you have this string picture on the board.

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Unlike the day-to-day presentation of lessons in the teacher's guide for the CSMP First Grade Program, the organization of lessons in the teacher's guide for UPG-II is a strand-to-strand presentation. By this we mean that on one day of the week a lesson is taught from a certain strand; and then on the next day of the week a lesson is taught from another strand; and so on. In this way we achieve the same spiraling effect as in the first grade program, but we also gain an extra advantage, namely, the benefit of organization of topics according to strands. For example, suppose you are interested in the way a topic in geometry is developed in the second grade. With a day-to-day format you would need to search through the entire guide for geometry lessons, but with a strand-to-strand format all these lessons are found, in their sequential order, in one part of the guide, the G-strand. You can then read the lessons in their natural order to gain an overview of the Geometry strand in second grade (second semester). At the same time, the strand-to-strand arrangement allows you to anticipate what is coming on the next "spoke" of the spiral development of any topic simply by reading ahead in the strand book in which that topic is developed.

b) Teacher's Materials

In addition to the teacher's guide, there are some demonstration manipulative materials you will need. The following materials are available from CSMP and should be in your classroom set of materials.

- Demonstration Minicomputer Set
  (4 magnetic Minicomputer boards; 1 set of magnetic checkers)
- A-Block String Game Kit
  (2 sets of A-Blocks; 2 attribute posters; 2 sets of string game cards; magnetic material)
- 0-109 Numeral Chart
- Number Line
- UPG-II Poster Set (six Geometry posters and two Language posters)
- Robot Walk Grid
- Two Dice

Other materials needed for individual lessons are listed under MATERIALS at the beginning of each lesson. Usually such materials (those not in your classroom set) are readily available or can be easily prepared from usual classroom supplies.

c) Student's Materials

There are some individual materials that each student (each pair of students in the case of storybooks) will need. The following materials are available from CSMP and should be in your classroom set of materials.

- Worksheet Booklet for UPG-II (with perforated pages which can be torn off sheet by sheet as needed)
- Workbooks
  (five titles: "A Caravan of Problems #1", "Length and Area", "A Caravan of Problems #2", "Nora's Neighborhood", "Fishing for Numbers (Part II)")
- Tangram Booklet and Tangram Sheet (used also in Part I)
- "Addition Problems" Booklet (used also in Part I)
- Storybooks from the STORIES BY FREDERIQUE Collection and an accompanying story-workbook from THE CSMP LIBRARY ("The Magic Box"; "Summer School in the Old Days" and "Summer School: 0's Discovery")
- Individual Minicomputer Set (used also in Part I)
  (2 sheets of Minicomputer boards; 1 sheet of punch-out checkers)
- Temperature Data and Charts
- Centimeter Tape Measure (used also in Part I)

Other materials needed for a lesson are listed under MATERIALS at the beginning of the lesson. Materials not available from CSMP are readily available or can be easily prepared from usual classroom supplies.

Some teachers find it convenient to provide each student with a sturdy cardboard box for storing manipulative materials. Others find it more convenient to store all the materials of one kind together in a container of appropriate size and to distribute these materials to the students as they are needed. In any case, you should probably keep the worksheet booklets, workbooks, and storybooks in your storage area and distribute them as called for in the lessons.

Some storybooks in the STORIES BY FREDERIQUE collection are provided in your classroom set of materials although they are not used in any lessons of the UPG-II Program. There are two copies of each such storybook; we suggest that these be available in a reading corner for students to read whenever they wish.

2. Role and Use of Worksheets, Workbooks, and Storybooks

a) Worksheets

With some lessons there are worksheets for independent work. Worksheets are designated with numbers corresponding to the lessons with which they are used and with stars indicating the order in which they are to be done by the students. The stars also suggest an increasing level of difficulty and, thereby, provide for various abilities of students. IMPORTANT: It is imperative that you do not insist on every student finishing every worksheet before going on to the next lesson. The worksheets are designed to accommodate differences in students' interests and abilities. Some students will finish more worksheets than others during the allotted time. But the same ideas will emerge in new settings in later lessons, so that students will have other opportunities to work independently on the same topics.

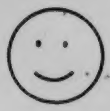
b) Workbooks

Part II of the CSMP Upper Primary Grades Program (UPG-II) includes five workbooks titled: "A Caravan of Problems #1", "Length and Area", "A Caravan of Problems #2", "Nora's Neighborhood", and "Fishing for Numbers (Part II)". Each is a thirty-two page booklet that reviews problems which have been encountered up to that point in the curriculum. Two lessons are scheduled for each workbook, and we suggest that all students be directed to start at the beginning of the booklet and to work independently through as many pages as they can during the two scheduled periods.

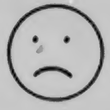
Ordinarily the first ten pages of a workbook are easy enough to guarantee some success for all the students; the next ten to twelve pages are average level difficulty; and the remaining pages are more difficult, often challenging even your best students. We would estimate that about two-thirds of your students should correctly finish the first ten pages during the two scheduled lessons, one-third should finish the first twenty pages, and a few might finish all or most of the pages. Of course, these proportions will vary from class to class.

Two full class periods are devoted to each workbook. At the end of the first period, you should collect the workbooks and check some or all the pages completed. Then when you return the workbooks at the beginning of the next workbook period, the students can correct their mistakes before going on with new problems.

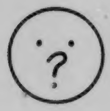
Notice that the cover page of each workbook includes spaces for marking your evaluation of the work on each page. One way to do this is to use these symbols:



Meaning "well done" or "correct"



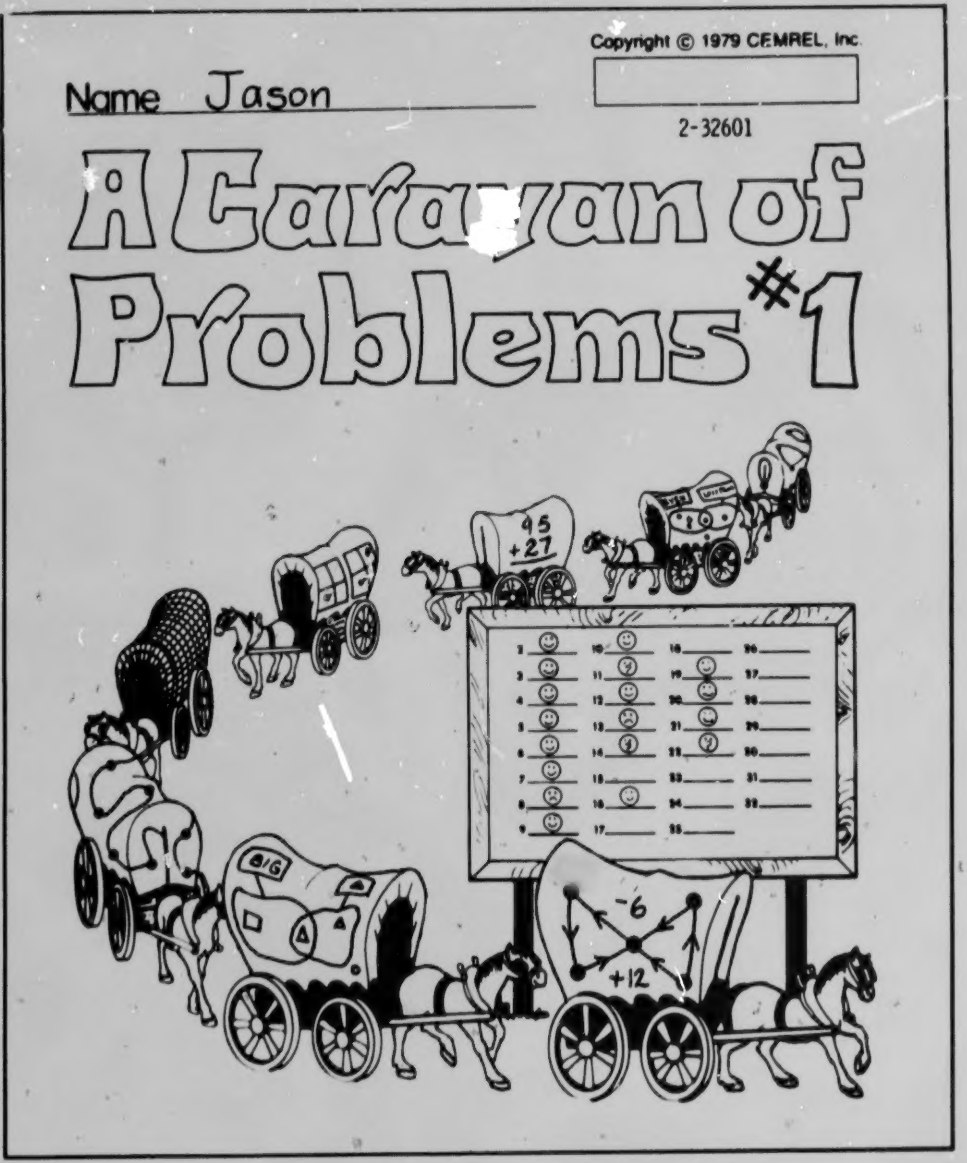
Meaning "something wrong, try again"



Meaning "did you forget this problem?" or "do you need a hint?"

The next page shows the front cover of one student's A Caravan of Problems #1 workbook after it was checked following the first lesson devoted to this workbook.

NOTE: This is the way Jason's workbook was returned to him at the beginning of the second lesson devoted to this workbook. The blanks on lines 15, 17, 18, and 23-32 indicate that he has not yet worked on those pages. Before Jason begins on those pages during the second lesson, he should correct his mistakes on pages 8 and 13 and finish his work on pages 11, 14, and 22.



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Name Jason

2-32601

A Caravan of Problems #1

IMPORTANT: Again we ask that you not require or even encourage every student to finish every problem in a workbook. If, at the end of the two lessons allotted for a workbook, some students have not completed as much of the workbook as you expect that they are capable of, you might suggest that they spend some extra time on it during the next adjustment day class. It would not be useful to demand completion or "mastery" before permitting work to begin on the next workbook series.

After the second workbook period, and after you have checked the results, you might want to send the workbooks home with students. This could be one way to keep parents informed of their children's progress.

c) Storybooks

Presently, there are twenty-four storybooks for young readers in the collection STORIES BY FREDERIQUE. Each story is presented in a colorful newsprint booklet and can be used for individual or collective reading at school or at home. The stories are independent of each other and for the most part need no teacher explanations. The titles of the books in the collection and the intended age range for each story are :

Ages 5 to 8

THE PLAYFUL NUMBERS
81 ROSES
I AM A VERY HAPPY BOY
ONE OUT OF SEVEN
THE HAPPY PUPPET
TWO BY TWO
THE LITTLE DREAMER
THE LITTLE DONKEY
THE MAGIC BOX
THE BABY IS BORN
THE WEIRD STORY OF 24
THE OLD SHOEMAKER

Ages 8 to 12

WHERE'S MY NOSE?

SINGING FRIENDS

DANCING FRIENDS

THE LIVING LINES

I AM NOT MY NAME

NABU WINS AN AWARD

THE SQUARE TRAP

SUMMER SCHOOL IN THE OLD DAYS

Ages 10 to 14

THE HIDDEN TREASURE

A VERY STRANGE NEIGHBORHOOD

ELECTION IN THE NUMBER WORLD

A VALENTINE MYSTERY

Two of these storybooks are used as the basis of lessons in the UPG-II Program. Young people can become personally involved in these stories; there are questions to answer, pictures to relate to the story, information to be found in the pictures, and new adventures to invent for the characters in the stories. They find that in the world of numbers there are emotions to be expressed and aesthetic senses to be developed just as in the world of people. Your students might enjoy rereading (or just looking at) a storybook many times after they have used it for some lessons. Other storybooks (additional titles from the STORIES BY FREDERIQUE collection have been included in your classroom set of materials although they are not used with lessons) could be made available for individual reading during free time.

3. Role and Use of Hand-Calculators

The electronic hand-calculator, if used creatively, can have a remarkably beneficial effect on the students' interest in and understanding of arithmetic. Its use can be a contributing factor in the improvement of their capabilities in that area. The reasons for this are many and varied, but among them are certainly the following:

- By removing the burden of mechanical computation from the children's shoulders, the calculator frees them to concentrate on the thinking that is a necessary part of the solution to any meaningful problem.
- The instant feedback provided by the calculator enables the children to indulge in making and testing conjectures, an area into which they would not otherwise venture because the quantity of computation involved would be overwhelming.

This pedagogical tool is particularly useful in the introduction of new numerical notions such as negative integers, decimal numbers, multiples of a given whole number, and so on, because through suitably chosen, rich situations these concepts can be made to arise in a natural way.

Two battery powered hand-calculators are recommended for UPG-II* and many different kinds are available from CSMP as a distributor for National Semi-Conductor, Inc. and Texas Instruments, Inc. Any of the products of these two companies can be obtained at discount prices.

While two hand-calculators are adequate for the lessons in UPG, the ideal situation would be for each student to have one. Encourage the students to bring calculators from home. The hand-calculators that are available from CSMP have algebraic logic and function keys which are permanently in a constant mode. You should check to be sure that any calculators brought in by students also have these characteristics.

- Algebraic logic. This means that the calculator responds to instructions given in the order in which they are usually written. To check that this is the case, press $\boxed{2}\ \boxed{+}\ \boxed{3}\ \boxed{=}$. If you have "5" on the display, then all is well.

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* Several UPG-II lessons require two hand-calculators in the class. If you already have calculators, be sure to check them for algebraic logic and constant mode (as explained on this and the following page).

- Constant mode. Some calculators have a switch so that the calculator can be set either in a constant mode or in a chain mode. If your calculator lacks such a switch, it may well be that the calculator has an automatic constant. For calculators with an automatic constant, the first number entered during multiplication is usually the constant, while the second number entered during the other three operations (addition, subtraction, division) is usually the constant.

  To check your calculator for the constant mode operation of $\boxed{+}$ is a simple procedure. Press $\boxed{0}$ $\boxed{+}$ $\boxed{2}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ and so on. If the number on the display is increasing by two at each depression of $\boxed{=}$, then $\boxed{+}$ is operating in a constant mode. Check the constant mode operation of $\boxed{-}$ in a similar fashion.

  To check the constant mode operation of $\boxed{\times}$ (where the first number entered will be the constant), press $\boxed{2}$ $\boxed{\times}$ $\boxed{1}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ and so on. If the number on the display doubles at each depression of $\boxed{=}$, then $\boxed{\times}$ is operating in a constant mode.

While it would be possible to use calculators without these characteristics in the lessons, the adjustments that would be required in the instructions make the calculators unsuitable.

In several lessons, the hand-calculator is used to explore patterns in a sequence of numbers which appear on the display. For example, put 3 on the display and press $\boxed{+}$ $\boxed{5}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ and so on. The sequence 3, 8, 13, 18, 23, 28, ... should appear. Another example in which the positive multiples of some number are generated is to put 4 on the display and press $\boxed{+}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ and so on. The sequence of positive multiples of 4 should appear; that is, 4, 8, 12, 16, 20, 24, 28, ... Even if your calculators have algebraic logic, it may be necessary for you to check that they operate in the way described in the lessons. If you find that your calculators do not operate in this manner, then try replacing (for example)

$\boxed{+}\ \boxed{5}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \ldots$ with $\boxed{+}\ \boxed{5}\ \boxed{=}\ \boxed{+}\ \boxed{5}\ \boxed{=}\ \boxed{+}\ \boxed{5}\ \boxed{=}\ \ldots$

and

$\boxed{4}\ \boxed{+}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \ldots$ with $\boxed{4}\ \boxed{+}\ \boxed{4}\ \boxed{+}\ \boxed{4}\ \boxed{+}\ \boxed{4}\ \boxed{+}\ \boxed{4}\ \ldots$

or

$\boxed{4}\ \boxed{+}\ \boxed{4}\ \boxed{=}\ \boxed{+}\ \boxed{4}\ \boxed{=}\ \boxed{+}\ \boxed{4}\ \boxed{=}\ \ldots$

SUGGESTIONS FOR ORGANIZING THE PROGRAM

1. Schedule of Lessons

The design and schedule of the lessons in this guide should be regarded as suggestions; they reflect the experiences of the CSMP staff in developing the curriculum. It is difficult for curriculum developers to predict which topics will be easy for your students and which lessons will require more or less time for your particular class. Only you will be able to judge that as you teach the lessons. Thus the program is not meant to be a rigid sequence of lessons to be followed exactly, but only to be a suggested guide.

The master schedule shown on pages 22 and 23, was developed as a one semester course for a hypothetical second grade class of average ability. It illustrates some important ideas about organizing the program. You will notice that several different strands are scheduled each week and that even within a strand the topics are interspersed. This was done to carry out the idea of a spiral curriculum. If you organized your own schedule, it is best to follow a similar pattern rather than to spend two or three full weeks, for example, "covering" one particular topic such as composition of functions or negative numbers. In a usual five day week, two days should be devoted to the World of Numbers strand, one day to the Languages of Strings and Arrows strand, one day to the Geometry and Measurement strand, and one day to the Workbook strand.

In the master schedule, a block represents one full period of approximately 45-50 minutes duration. It may very well be, however, that you and your class will spend more than one period or, possibly, less than one period on some lessons. Exercise your own professional judgement in deciding when to stop one particular lesson and when or whether to take it up again. Just be sure you do not insist that every single child understands every single aspect of a lesson before you go on. It is better to stop a lesson at a point when student interest is high and a problem remains unsolved than to teach so methodically that everyone in the class understands everything, but is completely bored.

Inside each block of the master schedule, the lesson of the day is designated in several ways. First, there is listed a number which indicates its suggested chronological order among the lessons in that strand. Hence, "N5" indicates that the lesson in question is suggested as the fifth lesson in the N (World of Numbers) strand. This identifying number also appears in the upper outside corner of each page in the description of that lesson, thus enabling a lesson to be located simply by flipping through the pages of the strand book. Second, you are told on which page of the strand book the description of the lesson in question begins. Third, you are given the title of the lesson.

Please bear in mind that the model schedule was designed for a hypothetical class with the usual range of abilities. In order to be successful with the program, you should adjust the schedule to the ability, maturity, and interests of your class. For example, you may wish to extend a lesson or a workbook for an additional period beyond those scheduled.

2. Adjustment Days

From time to time an adjustment day appears in the schedule. Since we have intended that UPG-II would be the mathematics curriculum for the second semester of the second grade and a usual school semester is eighteen weeks, the schedule has been designed to indicate eighteen weeks of lessons. However, it is also usual for there to be days when no mathematics lesson is taught; e.g., parent-teacher conference days, field trip days, snow days, and so on. These adjustment days should account for such non-class days, and should also give you opportunities to do many kinds of things. You could use an adjustment day to review a lesson you feel needs review, or to complete an unfinished lesson, or to extend some interesting ideas from completed lessons. You could also use an adjustment day to give your students time to become more familiar with the manipulative materials in the program. The students may especially enjoy playing games such as the Number Line Game or the String Game, and an adjustment day could give them another opportunity to play these games. You may wish to use an adjustment day to do some mental arithmetic exercises, or to teach a topic in mathematics you think would be especially interesting or useful to your class. For suggestions on ways to engage in mental arithmetic with your class, see the next section and Appendix B: MENTAL ARITHMETIC in the World of Numbers strand.

3. Supplementary Activities and Mental Arithmetic

Throughout the lessons there are many activities that children enjoy doing over and over in a variety of forms. For example, the String Game and the Number Line Game (to mention only two such activities) can be played whenever an extra time period is available. Of particular importance and enjoyment are the activities dealing with mental arithmetic; fully one-third of the lessons involve such activities. Other general suggestions about mental arithmetic are given in Appendix B to the World of Numbers strand. Teachers can invent clever variations of these mental arithmetic activities to be used often during a school day as adjuncts to the natural school routines. Some of the most obvious are: checking class attendance (How many present? How many should be here? How many absent?); the opening exercises (What day of the month is it? How many days in this month? How many days left?); distributing materials (How many students? Two pencils for each. How many pencils?).

Some teachers might want to do some supplementary drill work on writing numerals or on basic number facts. We would suggest that the drill be kept to a minimum so as not to suggest to children that arithmetic is a chore.

SUGGESTED SCHEDULE OF LESSONS

Master Schedule for the CSMP Upper Primary Grades, Part II

Week	World of Numbers	Languages of Strings and Arrows	World of Numbers	Workbooks	Geometry and Measurement
1	N1 Subtraction Problems #1 N p. 1	L1 Trees #1 L p. 1	N2 The Functions 2 x and 1/2 x N p. 9	W1 A Caravan of Problems #1 (Lesson 1) W p. 1	ADJUSTMENT DAY
2	N3 Doubling and Halving #1 N p. 16	L2 The Empty Set L p. 6	N4 Consecutive Jumps on the Number Line N p. 25	W2 A Caravan of Problems #1 (Lesson 2) W p. 4	G1 Estimation of Area G p. 1
3	N5 Doubling and Halving #2 N p. 34	L3 A Valentine Story L p. 13	N6 Roads #1 N p. 46	W3 Detective Story #1 W p. 17	G2 Length #1 G p. 7
4	N7 Addition Algorithm #1 N p. 53	L4 The String Game #1 L p. 23	N8 The Functions +10 and -1 N p. 61	W4 Length and Area (Lesson 1) W p. 21	G3 Taxi-Geometry #1 G p. 12
5	N9 Addition Algorithm #2 N p. 67	L5 Trees #2 L p. 26	N10 Midpoints on the Number Line N p. 77	W5 Length and Area (Lesson 2) W p. 26	ADJUSTMENT DAY
6	N11 Doubling and Halving #3 N p. 85	L6 Temperature Bar Graphs L p. 32	N12 Roads #2 N p. 93	W6 Detective Story #2 W p. 35	G4 Millions of Dots / Tangram Puzzles G p. 19
7	N13 Doubling and Halving #4 N p. 99	L7 Composition Games L p. 36	N14 The Function 3 x N p. 106	W7 A Caravan of Problems #2 (Lesson 1) W p. 43	G5 Length #2 G p. 25
8	N15 Subtraction Problems #2 N p. 112	L8 Telling Stories with Arrows L p. 44	N16 Composition of Functions #1 N p. 123	W8 A Caravan of Problems #2 (Lesson 2) W p. 46	G6 Taxi-Geometry #2 G p. 32
9	N17 Subtraction Problems #3 N p. 135	L9 Multiples #1 L p. 50	N18 Return Arrows N p. 143	W9 Detective Story #3 W p. 57	ADJUSTMENT DAY

10	N19 Introduction to Decimal Numbers #1 N p. 148	L10 Trees #3 L p. 57	N20 Frames of Differing Shapes N p. 158	W10 Nora's Neighborhood W p. 62	G7 Fractional Parts of Shapes #1 G p. 40
11	N21 Introduction to Decimal Numbers #2 N p. 166	L11 The String Game #2 L p. 63	N22 Composition of Functions #2 N p. 174	W11 Detective Story #4 W p. 71	G8 Area Puzzle G p. 50
12	N23 Multiplication Problems N p. 179	L12 Two Dice #1 L p. 69	N24 The Function 10 x N p. 186	W12 Fishing for Numbers (Lesson 1) W p. 79	G9 Length #3 G p. 52
13	N25 Addition Algorithm #3 N p. 196	L13 Two Lists L p. 77	N26 Number Friends N p. 199	W13 Fishing for Numbers (Lesson 2) W p. 82	ADJUSTMENT DAY
14	N27 The Functions 3 x and 1/3 x N p. 207	L14 Two Dice #2 L p. 85	N28 Subtraction with -10 Arrows N p. 215	W14 Detective Story #5 W p. 95	G10 Area Problems with Tangram Pieces G p. 58
15	N29 Sharing Three Ways N p. 218	ADJUSTMENT DAY	N30 Common Multiples #1 N p. 227	W15 Summer School in the Old Days W p. 107	G11 Fractional Parts of Shapes #2 G p. 65
16	N31 Addition Problems #1 N p. 238	L15 The String Game #3 L p. 93	N32 Roads #3 N p. 245	W16 Summer School: 0's Discovery W p. 110	G12 Length #4 G p. 73
17	N33 Introduction to Decimal Numbers #3 N p. 252	L16 Multiples #2 L p. 95	N34 Composition of Functions #3 N p. 262	W17 Detective Story #6 W p. 114	ADJUSTMENT DAY
18	N35 Addition Problems #2 N p. 271	L17 Telling Stories with Strings L p. 104	N36 Common Multiples #2 N p. 278	ADJUSTMENT DAY	G13 Mazes G p. 77

SECTION TWO

QUESTIONS AND ANSWERS ABOUT CSMP

As you teach the CSMP curriculum or even as you prepare to teach it, there will be some basic questions that come to your mind. Very likely they are the same kinds of questions that parents and supervisors will ask about a curriculum that appears to be different from the traditional program. Perhaps some of the questions that concern you are answered here.

1. WHAT IS THE SPIRAL APPROACH? HOW DOES IT WORK?

A: We learn through many interrelated experiences. But no experience, particularly a second grade math lesson, is an end in itself. With this in mind, CSMP introduces an idea in one lesson, then another in the next lesson, then another. Later the first idea reappears in a slightly new setting or in a slightly expanded version; then a second topic reappears; and so on. Each of the many times that a particular idea or area of study reappears, it is treated in a different manner and at a slightly higher level than the last time. This is called a spiral development.

Children learn at different times and at different rates. Only part of the learning of a given topic actually takes place during a lesson on the topic. In between the times when the topic occurs on the learning "spiral", the child is mentally digesting the idea, sometimes consciously, usually unconsciously. In this way the spiral development gives each student a new chance to "catch on" at each stage.

We find it highly effective to follow this kind of development rather than continuing on and on with a topic until so called "mastery" has occurred.

Sticking with the spiral idea requires an act of faith on your part, particularly if you have been used to teaching a given topic until you are satisfied that all (or at least most) of the class knows it "cold". Belief in the spiral approach implies that some lessons may be stopped before some (or even a majority of) children appear to have caught on, or in a lesson involving worksheets or workbooks, before everyone has successfully completed all the pages. It involves knowing that the topic will reappear again and again, and that varying degrees of understanding will come at varying rates and times. It entails a belief that an idea planted now may not sprout until much later, and that it is best not to force its development. All our experience to date (including five years of extended pilot testing of the curriculum) indicates that this approach really works.

2. Q: I UNDERSTAND HOW THE SPIRAL DEVELOPMENT WORKS, BUT HOW CAN I ASSESS MY STUDENTS' PROGRESS UNDER SUCH AN APPROACH?

A: We don't believe it is desirable or reasonable to try to evaluate student mastery of skills from day to day or even from week to week. On the other hand, teachers, parents, and the children themselves benefit from some monitoring of their progress and their involvement in the activities. Most teachers have their own methods for doing this with which they are comfortable, and we suggest that you continue to use your own monitoring methods. You will find that group interaction in the lessons and the individual worksheets and workbooks in the materials provide some valuable information. The spiral development in fact may help rather than hinder your long term assessment of students' progress.

3. Q: FOR WHAT SORT OF STUDENT IS THE CSMP ELEMENTARY CURRICULUM DESIGNED?

A: It is designed for all children in the schools. Each learning situation is designed to involve all students at their own levels in such a way that even the slowest can succeed in dealing with some aspects of the situation that challenge them. In fact, it has been found that such a "pedagogy of situations" (see Question 4) is particularly effective with the so-called "slow" learners, and recent work suggests it might be effective also with educable mentally handicapped children.

4. Q: BRIEFLY, HOW DOES CSMP DIFFER FROM A TRADITIONAL MATHEMATICS CURRICULUM?

A: The core content of CSMP is not revolutionary. The program aims to inculcate ideas and facts that have always been considered important in mathematics. The major differences are in method. Traditional teaching takes pride in disciplining students to be submitted to the usual paper and pencil algorithms. CSMP students are involved as well in many other real mathematical activities, with understanding at their level. For that purpose, CSMP employs new teaching aids such as non-verbal languages (see Question 7), which are themselves part of mathematics.

A pedagogy of situations is used to permit children to participate in the learning process. The students are faced with situations and are led by their reactions to important ideas in mathematics. Thereby, students begin to mathematize situations, a process which is of fundamental importance whenever mathematics is applied.

CSMP also makes use of a spiral approach rather than the mastery approach found in most mathematics curricula. (See Question 1)

5. Q: ISN'T THIS THE "NEW MATH"?

A: There is a current popular belief that all school mathematics today is either "new math" or "old math". Neither of these labels is well defined. "New math" usually denotes the programs that heavily emphasize the teaching of sets; introduce technical mathematical language for concepts such as "commutativity", "associativity", "distributivity", and so on; and that teach "concepts" before "skills". The "old math" refers to programs of the "good old days": memorization of the multiplication tables and complete emphasis on paper and pencil drill in calculations.

The CSMP curriculum is definitely not the "new math" nor the "old math" by any definition. It uses no technical language, yet achieves the understandings sought by the "new math". It uses no repetitive drills, yet elicits the ability to calculate stressed by the "old math". But these achievements are just by-products of a more pervasive goal: to help children use their curiosity, their intuition, their powers of reasoning to formulate and think about interesting problems and situations.

6. Q: ARE ALL THE TOPICS IN THE CSMP CURRICULUM REALLY MATHEMATICS?

A: Yes. Mathematics is much more than calculation with numbers, and its methods include more than paper and pencil algorithms. The CSMP curriculum is filled with experiences that directly and indirectly relate to the wide world of numbers, their interrelationships and the kinds of logical reasoning about these relationships that make mathematics an activity of interest and concern.

7. Q: WHY USE THE LANGUAGES OF STRINGS, ARROWS, AND THE PAPY MINICOMPUTER?

A: It is surprising that the language of strings was not used earlier and more extensively in the teaching of mathematics. Mathematicians have used strings for a very long time as they support classification notions so fundamental and natural that they go back to Aristotle. The scope of the string diagrams is broader than mathematics, the string diagram being a marvelous instrument for logical thinking. String diagrams permit the understanding of a situation at a glance; they generate math concepts and ideas through child-related discussion.

Arrow diagrams are probably the most important teaching aid in CSMP. They are used everywhere in our daily lives; signals, advertisements, directions, and so on. Why not use them in teaching mathematics? Arrows are a pictorial way to represent relations. Relations have always been important in mathematics, but in the past seemed to be something rather abstract, elusive, and mysterious. With arrows they become vivid, concrete, and easy to understand.

The Minicomputer allows very young children to become familiar with numbers and operations on them (especially large numbers and the more difficult calculations) before they are able to work with them on paper. Later, and more importantly, the Minicomputer is used as a vehicle for posing situations involving mental arithmetic and for examining the workings of the arithmetic operations. It provides the experience necessary so that, rather than memorizing an algorithm and applying it to situations, the situations give rise to the algorithm.

8. Q: IF THE STUDENTS USE THE MINICOMPUTER, WILL THEY BE ABLE TO CALCULATE WITHOUT IT?

A: This question suggests that the only purpose of the Minicomputer in the CSMP curriculum is to perform calculations. To be sure, the Minicomputer does enable very young children to deal with "large" numbers and perform relatively difficult calculations long before they know the paper and pencil algorithms for the calculations. But very soon these experiences with the Minicomputer suggest how the paper and pencil algorithms come about, and from that moment the Minicomputer is no longer regarded as a calculating device. Later, for the remainder of the curriculum, the Minicomputer assumes its primary roles: as a source of very interesting problems and games, as an effective vehicle for stimulating mental arithmetic, as a teaching device for introducing more advanced topics such as the negative numbers and the decimal numbers, and, in general, as an alternative to the usual representation of numbers.

9. Q: WHAT IS THE CSMP POSITION ON HAND-CALCULATORS AND WHAT ROLE DO THEY PLAY IN THE PROGRAM?

A: Hand-calculators should be a part of any mathematics curriculum. As more and more children have hand-calculators, these tools will become incorporated in the elementary and the secondary school curricula in a natural way with uses very similar to those of the Minicomputer. In some cases, they may be used as a tool for mechanical computation where the difficulty of the calculation might turn an otherwise interesting problem into an exercise in drudgery. In other cases, they may be used as the context for posing situations leading to the discovery of important mathematical properties or providing opportunities for interesting exercises in mental arithmetic.

Hand-calculators are of particular benefit to "slow" learners. Many slow learners have mechanical difficulties rather than problems in thinking about the situation. The hand-calculator alleviates this problem.

Some lessons in the CSMP curriculum require only two calculators, while some lessons suggest that each student have one. Most often, however, lessons of this latter sort also include suggestions on how they may be modified since not every student may have a hand-calculator. In either situation, students with access to hand-calculators should always be encouraged to bring them to class.

10.Q: ARE THERE GOALS FOR EACH LESSON?

A: At the beginning of each lesson plan there is a CAPSULE LESSON SUMMARY, which describes briefly the activities and experiences included in the lesson. Indirectly these summaries suggest goals but, of course, the goals of the curriculum spread over whole series of lessons and cannot be broken into specific goals for individual lessons. If you read the introduction to the strand in which a lesson appears as well as the Capsule Lesson Summary for that lesson, we expect you will easily see the direction and intent of the lesson. We decline to predict for any one lesson what will happen in terms of student learning.

11.Q: SHOULD I SUPPLEMENT THE PROGRAM WITH MORE PRACTICE IN NUMBER FACTS?

A: There is a surprising amount of such practice throughout the program. It is implicit in the activities and is not in the form of practice for its own sake. If you still feel it is necessary to give supplementary practice, we suggest that you keep repetitive drill to a minimum so as not to endanger the students' interest in numerical situations.

12.Q: WHY IS THERE A LACK OF DRILL AND LITTLE EMPHASIS ON "SKILLS"?

A: CSMP believes that training for rote skill should never precede understanding. The avowed purpose of drill in arithmetic is to acquire the "skills", that is, the skills of calculation. Certain numerical skills are essential but, unfortunately for school children, the usual drill techniques for acquiring them are dull, stultifying and sometimes even counterproductive. The CSMP curriculum appears to place little emphasis on "skills" because it does not resort to drill. But as a matter of fact, the essential skills of both mental and written calculation are learned almost inadvertantly in response to the many situations where the ability to calculate is needed in order to respond to the situation: to win the number game, to identify the secret number of the detective story, or to build a number road. Calculation is always considered as a tool for doing interesting things, rather than a chore done for its own sake.

13.Q: HOW DO STUDENTS LEARN TO USE MATHEMATICS IN PRACTICAL SITUATIONS?

A: For students of elementary school age, a situation that intrigues them and catches their imagination or their curiosity will certainly motivate them to learn whatever is necessary in order to deal with the situation. But situations that adults consider "practical" very often are outside the realm of interest of school children and do not provide motivation for learning. CSMP has found that students will indeed learn to use mathematics in situations they consider worthy of their attention. As they approach adulthood they will soon consider "practical" situations to be most worthy of their attention, but in the meanwhile, they are more interested in games, fantasy, puzzles, stories, and so on.

14.Q: HOW ARE TELLING TIME, MONEY, AND THE METRIC SYSTEM TREATED?

A: Telling time is not included in the CSMP program. It is our belief that telling time is more naturally a part of a language arts program than of a mathematics program, although it is frequently included in the mathematics curriculum. Since there are many excellent materials available to help you teach telling time, we urge you to teach this topic using those methods and materials you particularly like or that have proved successful to you in the past.

The topic of money is first introduced in the CSMP curriculum in situations involving combinations of coins. It is treated again in the Upper Primary Grades and the Intermediate Grades as a support for the teaching of the decimal numbers. Although money is not treated as a separate topic, these situations provide experience with our monetary system. Should the students need more exposure to this topic, teachers should feel free to develop supplementary materials in order to provide extra practice.

Metric units are used whenever there is cause to measure in standard units. Activities involving the metric system are begun in the First

Grade Program and continue throughout the Geometry and Measurement strand of the Upper Primary Grades Program. We recommend that you do not ask the students to convert from the English to the metric system or vice versa (inches to centimeters, yards to meters, Celsius to Fahrenheit, grams to ounces, and so on). It would be confusing for children to deal with two systems simultaneously, especially at this early stage. Since the United States has not yet "gone metric", your students will still need to be familiar with the English system. We would suggest that you use whatever materials and methods you have found successful in the past for teaching this topic, but we repeat our recommendation that you not ask the students to convert from one system to the other.

15.Q: WHAT IS THE ROLE OF STORYBOOKS IN THE CURRICULUM?

A: The storybooks in the collection STORIES BY FREDERIQUE include twenty-four stories for young children, presented in colorful newsprint booklets. Two stories from this collection are used for lessons in UPG-II, with fifteen copies of each storybook provided in a classroom set of materials. (If your school can afford it, we recommend that you order enough extra copies to provide one for each child.) Other storybooks, two copies of each, are provided for supplementary reading. Each of the stories is portrayed by the mathematical language of dots, arrows, and strings which reinforce the ideas of the stories. Young readers can become personally involved in the stories; there are questions to answer, pictures to relate to the story, and new adventures to invent for the characters of the stories. They find that emotions and aesthetic senses are involved in the world of mathematics just as in the world of people.

16.Q: WHAT PROVISION IS MADE FOR INDIVIDUALIZATION IN THE CSMP PROGRAM?

A: In many of the lessons there are worksheets to be done individually by the students. Sometimes the worksheets are done at the beginning of a lesson, followed by a full group discussion of them. But ordinarily a topic is first developed in a full group setting, followed by individual student work on related worksheets. If "individualization" means a procedure for allowing students to work alone at their individual rates and levels, then the worksheets (and also the workbooks, discussed below) do provide for (partial) individualization. A series of worksheets for a given lesson offers a whole spectrum of levels from very simple to quite challenging. Students all start with the first worksheet (one-star) and proceed at whatever pace is appropriate and to levels according to their capabilities. The spiral approach accommodates this kind of work in the same manner as the full group work; that is, no worksheet is a prerequisite for the content of any succeeding lesson.

Ten lessons during the second semester of the second grade are devoted to the five UPG-II workbooks. Each workbook is also designed to move through a spectrum of levels of difficulty. We believe that the worksheets and workbooks realize the best sense of "individualization".

17.Q: WHAT PROVISIONS ARE MADE FOR THE VARIETY OF ABILITIES IN A USUAL CLASS?

A: Every lesson includes something for all students; there are opportunities for the slowest students to participate in a meaningful way, and there are challenges for even the very brightest of students. This means that you should not exclude any students from a full group lesson. At the same time, you should not expect every student to understand everything during the course of the lesson. Furthermore, with the spiral development many situations are open-ended and allow students to experience them at levels as high as they are capable.

Evaluation has shown that students traditionally labeled "slow" are doing better in CSMP classes than in non-CSMP classes. We believe this is due to the spiral development, to full group work on mathematical activities, and to the presence in the CSMP curriculum of certain nonnumerical activities in which many supposedly "slow" students can excel.

Additionally, CSMP provides for a variety of ability levels in the student materials. Worksheets and workbooks offer problems in a range from very easy to quite challenging. Indeed, some of the "higher starred" worksheets and pages toward the ends of workbooks may be accessible only to your very best students.

In extreme cases, you might seriously consider the possibility of arranging for exceptionally bright (slow) students to take their math a higher (lower) grade level CSMP class.

18.Q: I PREFER TEACHING SMALL GROUPS, BUT MOST OF THE LESSONS SEEM TO BE AIMED AT THE WHOLE GROUP. WHAT DO YOU SUGGEST?

A: If your school routine allows you to teach each lesson twice, once to each half of your class, that would be an alternative way to proceed. But if you do this, it might be wise to vary the composition of the groups from day to day so that you are not tempted to group your students according to your perception of their ability. Otherwise you would guarantee that half your students would not have as rich an experience as the other half, through no fault of their own. The purpose of the full group experiences is to allow interaction among all the children in every situation posed in the lessons. Students learn quite a bit from each others' reactions. You will be surprised to find that in some situations the key responses are made by students you had labeled "slow". We feel the spiral approach has maximum potential in full group teaching.

The above response might give the impression that CSMP teaching takes place only in full group settings. This is not the case. There is much time devoted to individual student work on worksheets and workbooks, and there are various games to be played. We also encourage you to arrange small group sessions whenever you have time, particularly with your slower students. Small group activities could supplement the full group lessons, but should not replace them. Extra small group work should not be simply for drill and for rehashing ideas, but rather should provide additional experiences.

19.Q: WHY IS A "DIALOGUE" STYLE USED AS A FORMAT FOR THE LESSON PLANS? ARE TEACHERS EXPECTED TO FOLLOW THIS DIALOGUE EXACTLY?

A: Most of the lessons are written in considerable detail; step by step descriptions are frequently given, together with suggestions for the exact words a teacher might use (in the form of a possible dialogue between teacher and students). These details are given only for the teachers who feel the need for them. You should never feel obligated to follow any lesson word for word as it is written in the guide. By the same token, you should never insist that your students give the same responses as those given in the imaginary dialogues. Use your own creativity in drawing out students' responses as the situations suggest. Read the lesson plans very carefully beforehand to get an idea how the lessons are expected to proceed.

20.Q: IS IT NECESSARY TO FOLLOW THE SUGGESTED SCHEDULE EXACTLY?

A: This program is not meant to be a static sequence of exercises to be followed rigidly. The lessons in this guide are only suggestions based on our own experience teaching the materials. If this will be the first time you are teaching the program and you would like specific suggestions for organizing the lessons, we offer some guidelines and a suggested schedule of lessons. You may wish to follow it at the beginning of the school year and then deviate from it as the needs of your students may dictate. It is probably best to follow a one-day, one-strand pattern when you organize your own schedule, rather than to spend two or three full weeks "covering" one particular topic. In a usual five day week, two days should be devoted to the World of Numbers strand, one day to the Languages of Strings and Arrows strand, one day to the Geometry and Measurement strand, and one day to the Workbook strand.

Exercise your own professional judgment in deciding when to stop a particular lesson. It is better to stop a lesson when student interest is high than to teach so methodically that everyone in the class understands everything, but is completely bored. In order to be successful with this curriculum, you should adjust the schedule to the ability, maturity, and interests of your class.

21.Q: HOW DO I HANDLE STUDENTS WHO HAVE AN EXTENDED ABSENCE OR STUDENTS (NEW TO CSMP) WHO TRANSFER INTO MY CLASS DURING THE YEAR?

A: The spiral approach allows a student to miss one or two weeks of school without much "catch up" work needed. When the student returns, either you or classmates can help with any material that is deemed necessary.

If several months of classes have been missed, you might need to treat the student as you would a transfer student new to CSMP. Give the student a little extra attention during a lesson especially when the questions are review for most of the class.

In most cases, the differences between new and veteran students tend to fade rather quickly. One exception might be in lessons involving the Minicomputer. Veteran students who are unusually adept with the Minicomputer very often are good tutors for the transfer students in the use of this abacus.

It is probably best not to send worksheets and/or workbooks home to a child; without participating in the collective lessons that introduce the worksheets and workbooks, it would be very difficult for the student to proceed.

22.Q: HOW DO I HANDLE WHOLE CLASSES WHO ARE NEW TO CSMP?

A: CSMP does not assume that kindergarten and first grade students have any prior experience with CSMP. The Kindergarten and First Grade Programs will need no adaptation for classes who are new to CSMP.

Second grade classes that are new to CSMP will initially progress more slowly than veteran classes. The lessons at the beginning of the second grade program are designed to serve either as an introduction to CSMP languages for new classes or as a quick review for veteran classes.

Third grade classes which are new to CSMP will need to become fluent in the CSMP languages before beginning the lessons in UPG Part III. A special schedule has been prepared for these classes which consists of prerequisite lessons and workbooks from UPG Parts I and II followed by a modified UPG Parts III and IV program.

23.Q: WHAT SHOULD I DO WITH MY CLASS IF THEY DID NOT FINISH THE PREVIOUS YEAR'S SCHEDULE?

A: No special measures are required. Each level of the CSMP curriculum begins with introductory lessons. In addition, the spiral approach in the lessons permits students to participate at their own level of understanding, even if that level is not quite as high as it might have been if the previous year's schedule had been completed.

24.Q: HOW DO CSMP STUDENTS DO ON STANDARDIZED TESTS?

A: While CSMP disagrees with the philosophy underlying standardized testing, test results are available to us from several sources. The results indicate that CSMP classes generally do as well or better on standardized tests than do non-CSMP classes. It should be pointed out that standardized testing and traditional programs are correlated, and that CSMP students have been exposed to a great deal of mathematics that is not taught in more traditional programs or that is not considered in the construction of the tests. There are also many unmeasurable or unmeasured areas such as enjoyment of mathematics, strategic thinking, organizational ability, and depth of understanding of the methods and uses of mathematics where we feel CSMP students have a definite advantage. Evaluation Report 2-A-3 in the CSMP Evaluation Report Series gives more detailed answers to this question. Copies are available upon request.

25.Q: WHAT PROBLEMS WOULD CSMP STUDENTS HAVE IF THEY RETURN TO A MORE TRADITIONAL PROGRAM IN THE NEXT SCHOOL YEAR?

A: There should be no problems for CSMP students who return to a traditional program. Test results indicate that CSMP students are at or above the level of non-CSMP students in material covered in traditional courses. In addition, CSMP students have encountered many situations that are not introduced in traditional programs nor tested for on standardized tests (see Question 24). In particular, their reasoning ability should be much sharper than their non-CSMP counterparts.

There are, of course, differences in timing as to when different programs introduce the standard algorithms and often CSMP's introduction will occur later than in some of the other programs. The reason for this is our belief that it is essential that the creative freedom of the student not be impaired by a premature emphasis on the standard algorithms for adding, subtracting, multiplying, and dividing. We believe that many of the present computational difficulties experienced by elementary and junior high school students were caused by their spending an inordinate amount of time trying to master the algorithms too early in their mathematical education. This shuts the students off from other experiences necessary for their mathematical growth. Students need plenty of time to explore the world of numbers. Mastering an algorithm should be the last step of this exploration, a sort of synthesis of previous experience. Furthermore, as electronic hand-calculators become available to nearly everyone, algorithmic mastery as a prerequisite to other numerical experiences should become less important.

CSMP students usually can solve a problem before having learned the standard algorithm. If they enter a program that expects an algorithm they have not learned, they are still more than adequately prepared for the algorithm and should have no trouble quickly learning to use it.

CSMP introduces the usual addition algorithm in the second grade, one of the two standard subtraction algorithms in the third grade, the multiplication algorithm in the fourth grade, and the division algorithm in the fifth grade.

26.Q: WHAT ARE SOME WAYS TO ORGANIZE MATERIALS AND MANAGE MY CLASS?

A: It is very important to organize your CSMP materials before school begins. Decide where you will store the materials once they are organized. Filing drawers, shelves, boxes, or closets are all satisfactory as long as you can easily get to the materials. Take them all out of their shipping boxes and spread them on several tables and/or desk tops in your classroom so they can be organized into the order you will be using them.

Workbooks: The teacher's guide is needed to find the order in which the workbooks are used. Locate the Workbook strand column in the Master Suggested Schedule of Lessons and collect all the workbooks in the order in which they are listed. After organizing the workbooks in order, place them in an upright position in your file drawer, box, shelf, etc., so that the first workbook to be used is at the front of the stack.

Worksheets: There is a separate worksheet booklet for each student and you should decide whether you will store these booklets and distribute them when a particular lesson calls for a worksheet, or whether you will let the students keep their own worksheet booklets in their desks.

Each worksheet can be removed from the booklet or left in depending on how you wish to manage worksheets. However, once a worksheet is removed from the booklet it cannot be replaced, so make sure students are careful about tearing off the correct sheet.

Manipulatives: Storage of each manupluative in a separate container, properly marked, is one way to make everything accessible. Another way is to provide a sturdy box (for example, a shoe box) for each student and to put individual manipulative materials in the box, properly labeled with the student's name.

Additional Materials: Be sure that you have plenty of high quality colored chalk, large sheets of unlined white paper, and that your students have colored pencils, pens, or crayons. Plan well ahead to be sure you have any needed materials that are not supplied by CSMP. Keep all teacher materials where they are easily accessible. You may find the guides easier to use if they are put in looseleaf binders; the books are perforated so that pages can be removed easily. The demonstration Minicomputer boards, the number line, and the 0-109 numeral chart should always be on display in the room.

27.Q: SOME OF THE CSMP MATERIAL IS NEW TO ME. HOW DO I LEARN ENOUGH TO TEACH A CLASS?

A: Every CSMP teacher should have been trained by his or her coordinator.

Kindergarten and first grade teachers receive a minimum of eighteen hours training, teachers of the Upper Primary Grades Program receive up to thirty-two hours training, and teachers of the Intermediate Grades Program receive up to fifty-six hours training. During the training sessions the CSMP content is taught to new teachers and suggestions are given for organization of their classes.

If the suggested training schedule has been followed, many lessons taken from the CSMP curriculum, or very similar lessons, have been presented. The training schedule also makes use of the spiral approach so that training sessions aim not only to teach CSMP mathematics but to familiarize teachers with the spirit and the pedagogy of the CSMP curriculum.

28.Q: WHAT SHOULD BE DONE WHEN THERE IS A SUBSTITUTE TEACHER?

A: Some school systems have included people who frequently substitute in their CSMP training program. When this is not the case, it would be very difficult for the untrained substitute to teach from CSMP materials. One solution is to arrange for another CSMP teacher to teach math to your class while the substitute teacher teaches another subject to that teacher's class. If that cannot be arranged and if you will only miss one or two days, you may find certain CSMP material, uncompleted workbooks for example, that the substitute can manage. Otherwise, use those days to cover material not included in the CSMP curriculum, such as telling time. If your absence will be an extended one, arrangements should be made with your coordinator to train a substitute, to rearrange schedules to put a CSMP teacher in your class, or to otherwise provide for your students.

29.Q: THE PROGRAM MAKES USE OF THE STUDENT'S CREATIVE FREEDOM HOW MUCH ROOM IS THERE FOR THE TEACHER'S INGENUITY?

A: CSMP not only allows for your ingenuity but depends on it. While the lessons may appear to be very rigid by presenting dialogue for teacher and student, it should be remembered that these are suggested questions, answers, and comments based on our experience with development classes. Even if you were to follow your part of the "script," your students are likely to offer responses which differ from those we have listed. It is the student's response that will determine the content and the direction of your next question or comment.

In a traditional program your creativity is called upon the first time you develop a particular lesson, and to a lesser extent, as you revise lessons in succeeding years. There is little room for spur-of-the-moment creativity once a lesson begins. The CSMP curriculum, by creating a free and open situation emphasizing the student's creative freedom, continually presents you with unanticipated student response and calls on your ingenuity at all times.

30.Q: HOW CAN I EXPLAIN CSMP TO PARENTS?

A: The best way would be for your coordinator to organize a workshop for interested parents. The parents can then see the languages of the program in use and have some of the same experiences their children have had. Workshops can be of almost any length but anything short of two hours does not allow for much familiarity with the material. In such a workshop we would suggest that the language of the Minicomputer be explained with particular emphasis on its use in presenting situations where mental arithmetic is called upon and in developing algorithms (for example, physically carrying a checker from the ones' to the tens' board in the "8 + 2 = 10" play, paralleling "carrying" in the paper and pencil operation). A lesson using the storybook "Where's My Nose?" provides a good example of the power of the language of strings. A nonnumerical example, such as the brother-sister graph and a numerical example, such as building a road from 2 to 23 using +5 and -2 arrows, will provide an introduction to the language of arrows. The lessons that are the favorites of the children would be excellent for a presentation to parents as well.

You might also extend an invitation to parents to visit your class during the math period. Many times such an invitation leads eventually to a closer parent-teacher relationship, with the parent offering voluntary services as an aide.

You will find a letter to parents at the end of this section which may be reproduced and sent home with the student.

TO THE PARENTS OF CSMP STUDENTS

Your child is participating in the Comprehensive School Mathematics Program's (CSMP) elementary school curriculum, developed under a contract with the National Institute of Education. While the program cannot be fully explained in a brief note, we hope that this letter will give you some feeling for the substance of your child's mathematics education.

The goals of the CSMP K-6 program includes a development of the standard number systems and their operations (including an early introduction to the integers, rational numbers and decimal numbers) interwoven with a development of geometry and measurement. On an equal footing with the above, the CSMP program includes a strong component of probability, statistics, and combinatorics which extends throughout grades 2-6. Powerful pedagogical devices are used, such as several nonverbal languages. These make the unifying ideas of the program accessible to very young children.

These languages include:

1) The Language of Strings This "language" mimics the basic methods of collecting and classifying data applying to the problems at hand. That is, it deals with the fundamentally useful and important mathematical notion of set.

2) The Language of Arrows This graphical language models the process of comparing and analyzing sets and operations on them. That is, it deals with the mathematical notions of relations and functions including, but not restricted to, ordering relations and the numerical processes of adding to, subtracting from, multiplying by, and sharing equally among.

3) The Language of the Papy Minicomputer The Papy Minicomputer, an extremely effective paper abacus invented by Georges Papy, models the positional structure of our system of numeration and lends itself to a multitude of numerical algorithms, including the standard algorithms for the basic numerical operations.

Among other things, these languages allow for the development of the basic operations on numbers in a natural way based on previous experiences with the languages. They also provide a context in which situations arise that call for the repeated use of arithmetic operations without resorting to drill exercises, which most children find boring.

The content is learned in an atmosphere of constant applications utilizing a "pedagogy of situations".* These vary from simple story situations in the early primary grades to challenging applications and nontrivial simulations of real world problems in the upper grades.

One of the tenets of CSMP's development philosophy is that mathematics is a unified whole and should be learned as such. As a result, along with CSMP's concern with the variation in ability levels in the typical heterogeneous class, the curriculum is structured in a spiral format throughout the K-6 program. The content is completely sequenced in spiral form so that each student is brought into contact with each area of content continuously throughout the program. This consciously avoids atomizing the content and mastering each bit before continuing to the next. Rather, each student spirals through repeated exposures to the content, building interlocking experiences of increasing sophistication.

Another tenet of CSMP's development philosophy, based on years of experimentation in the classroom, is that no single method of classroom management suffices to meet the needs of every student. Hence, the program is constructed to allow numerous opportunities for whole class participation, small group participation, and individual independent experiences. Individualization throughout the K-6 program is achieved through the independent work on workbooks and worksheets of varying levels of difficulty. Due to the spiral nature of the curriculum, each student can engage in a workbook without losing touch with the progress of the class while at the same time working to the limits of his or her ability.

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* Inspired by the work of the mathematician and psychologist, Caleb Gattegno.

A feature unique to the CSMP program is a collection of storybooks written by Frédérique Papy (STORIES BY FREDERIQUE) and a collection of story-workbooks (THE CSMP LIBRARY). These probe the affective domain of learning and bring children into contact with areas of mathematics until now unexplored by them.

CSMP does not have its own system of student assessment and progress reporting. While standardized testing indicates that CSMP students develop the "basic skills" as early or earlier than do non-CSMP students, we hope you will not be obsessed with your child's development of these "skills". Each child is an individual and should not be judged against norms but on what he or she can and does accomplish. Many extremely important areas--development of self-confidence, ability to think and reason, etc.--are very difficult to measure and grade. If you have any questions regarding the progress of your child, his or her teacher is in the best position to answer those questions.

THE CSMP STAFF

Comprehensive School Mathematics Program

CSMP Mathematics
for the
Upper Primary Grades
Part II

The World of Numbers

CEMREL, Inc.

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THE WORLD OF NUMBERS

INTRODUCTION

Numbers are among the most important things that mathematics (at all levels) is about. Mathematicians are interested in numbers just as astronomers are interested in stars, botanists in plants, and sociologists in the organization and dynamics of human societies. Surely everyone agrees that one of the primary goals of any elementary school mathematics curriculum should be to introduce students to the world of numbers - to give them the opportunity to become familiar with numbers, their properties, and the relations between them. The ability to calculate with numbers is an important part — but not the only part — of being familiar and comfortable with numbers. The World of Numbers strand of the CSMP Upper Primary Grades Program is designed to provide the students with a wide variety of challenging experiences with numbers so that gradually they will become not only familiar, but comfortable with numbers; they will, so to speak, get to know numbers on a "first name basis".

The Minicomputer and the World of Numbers

The CSMP Upper Primary Grades program uses the Papy Minicomputer[*] as a support for the positional system of numeration; for calculations and estimation; for number patterns and mental arithmetic; and for modeling the basic operations involving whole numbers, integers, and decimal numbers. Although the Minicomputer can be seen as a tool for calculation and as a device to help students learn routine methods for calculations, its more exciting use is as a vehicle for posing interesting problems which challenge a child's intellectual curiosity about numbers and for presenting situations which dually encourage strategic thinking and reinforce numerical skills. The lessons which make use of the Minicomputer are often intended to be explorations into the world of numbers.

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[*] See also Questions 7 and 8 of Section Two, Questions and Answers About CSMP of the General Introduction and APPENDIX A: STANDARD CONFIGURATION AND STANDARD MINICOMPUTER MOVES.

Standard Algorithms of Arithmetic

CSMP seeks to develop basic numerical skills as well as an understanding of the underlying mathematical ideas. We are fully in agreement with the thesis that, along with the growth of understanding of the world of numbers, there must be a concommitant growth of familiarity and facility with numbers and operations on them. But facility should not be confused with understanding; they are partners in the growth of mathematical maturity. A balanced growth of each must be maintained, neither being sacrificed for the other.

Students must eventually learn mechanical algorithms for the basic operations (addition, subtraction, multiplication, division). However, premature presentation of these algorithms may actually stunt a student's ability to develop alternative algorithms, to do mental arithmetic, or to estimate.

Consider the problem of calculating "385 + 99". A second grader may have difficulty performing the standard (paper and pencil) addition algorithm. An easier and more efficient way to proceed is to add 100 to 385 and then to subtract 1 (385 + 100 = 485 and 485 - 1 = 484). To insist on a mechanistic response to such a problem would be to encourage inefficiency and might also inhibit the development of the flexibility necessary for problem solving. On the other hand, a rich array of situations in which students interact with numbers provides them with opportunities to gain the necessary facility with standard algorithmic procedures while retaining the openness required to respond creatively to new situations in the world of numbers.

Numerical Relations

One of the main aims of the World of Numbers strand is to familiarize students with numbers by studying explicitly and in a variety of contexts relations between numbers. (For more general comments about mathematical relations, see the introduction to the Languages of Strings and Arrows strand.) Arrow diagrams represent relations in a simple, suggestive, and pictorial way -- usually more conveniently than the same information could be given in words or other symbols.

Students are brought into contact with an assortment of challenging situations, many of which would be totally inaccessible to them were it not for the arrow diagrams. The problems and activities of this strand include many examples: solving linear equations presented in terms of arrows; studying iterated processes and patterns in sequences of numbers; tackling problems which may have many solutions or no solution; estimating or testing that a solution is reasonable; and exploring properties of operations on numbers.

In summary, what is most important in the study of numbers is to confront the students with a variety of problems and situations which capture their interests, challenge their abilities to reason, and stimulate their curiosities about numbers -- and, at the same time, provide them with tools for coping with these situations and problems.

N1 Subtraction Problems #1

CAPSULE LESSON SUMMARY

Explore the effect of moving various checkers in a configuration on the Minicomputer – in each case, does the move increase, decrease, or leave the same number on the Minicomputer? Put a number on the Minicomputer with many checkers and then alternate estimating the number and making some plays until standard configuration of the number is displayed. Estimate the result of a subtraction calculation involving three digit numbers and then do the calculation using the Minicomputer.

MATERIALS

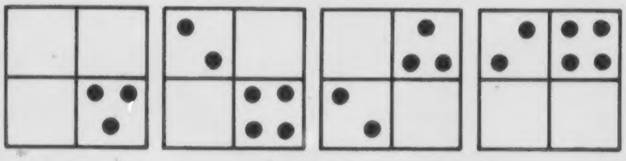
Teacher: Minicomputer set

Student: Worksheets N1*, **, ***, and ****; pencil; Minicomputer set[*] (optional)

DESCRIPTION OF LESSON

Exercise 1: Transforming a Number

Put this configuration on the Minicomputer.



T: I put a number on the Minicomputer. Is this number more than 1,000? How do you know?

S: Yes. The number is more than 3,000, because there are three checkers on the 1,000-square and more checkers on other squares.

---

[*] A students' Minicomputer set consists of two individual Minicomputer sheets (each having two boards) and a sheet of individual checkers.

T: We know this number is more than 3,000. Is it more than 4,000? How do you know?

S: Yes, because there are three checkers on the 1,000-square and there are two checkers on the 800-square. 800 + 800 is more than 1,000.

T: Is this number more than 5,000?

It is possible that a student will be able to explain and to convince the class why this number is more than 5,000. In any case, your class should estimate the number to be larger than 4,000 and possibly larger than 5,000.

T: We know that the number on the Minicomputer is more than 4,000 (5,000). We do not need to figure out exactly what number it is to compare it to other numbers.

Write these words on the board close to the Minicomputer.

More
Same
Less

T: I am going to move, remove, or add some checkers. Each time tell me if the number on the Minicomputer is more than, less than, or the same number as before.

Move a checker from the 1,000-square to the 2,000-square. Point to each of the three words in turn as you say:

T: If you think this number is more than the number we had, hold up your hand.
If you think the number is the same, hold up your hand.

If you think this number is less than the number we had, hold up your hand.

Repeat the move very obviously if many students do not know that the number now on the Minicomputer is more than the previous one.

T: How much more is this number than the number we had before? How do you know?

S: 1,000 more, because you moved the checker from the 1,000-square to the 2,000-square.

Continue this activity with the following moves or similar ones.

- Move a checker from the 4-square to the 10-square. (6 more)
- Replace any checker by a checker of another color. (Same)
- Move a checker from the 20-square to the 1-square. (19 less)
- Make a "40 + 40 = 80" play. (Same)
- Move two checkers from the 100-square to the 200-square. (200 more)
- Make an "800 = 400 + 400" play. (Same)
- Make an "8 + 4 = 10 + 2" play. (Same)

When appropriate, ask the students exactly how much more or less a number is than the previous one. After you make four or five moves yourself, invite students to transform the number on the Minicomputer.

At the end of this activity look carefully at the number on the Minicomputer. If you think the number is more than 10,000, you should "transform" it into a number which is less than 10,000. This can be accomplished by removing one or more checkers from the thousands' board or by moving checkers on the thousands' board to squares of lesser value.

At the end of this activity, leave the checkers on the Minicomputer for Exercise 2.

Exercise 2: Estimation

Guide the students in estimating whatever number is now on the Minicomputer in the same way as you did at the start of Exercise 1. Whenever the class has made a fairly good estimate or some individual members begin to disagree about an estimate, invite students to make some plays.

After several plays are made, guide the class to make a closer estimate. Then allow students to guess what the number is and record their guesses on the board.

Continue making plays until standard configuration is obtained. Invite a student to write the number below the Minicomputer, and decide which guess is the closest. Erase the board and remove the checkers from the Minicomputer.

Exercise 3: Subtraction Problems

Write this subtraction problem on the board.

$$535 - 284 =$$

T: The problem is to find what number 535 - 284 is.
Is it more than 100? How do you know?
Is it more than 200? How do you know?
Is it more than 300? How do you know?

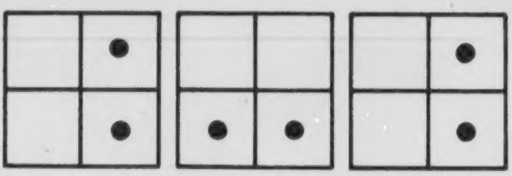
Let the students react freely to these questions. Accept their explanations, even if they are not well-formed. Allow students to guess what the number is and record their guesses on the board.

T: How can we find out what number 535 - 284 is?

S: Put 535 on the Minicomputer and then take off 284.

NOTE: One of your students might suggest calculating "535 + $\widehat{284}$". In this case, let the class decide which calculation they prefer to make. The lesson description does not include a discussion of the calculation "535 + $\widehat{284}$", but only minor changes need be made.

Invite someone to put 535 on the Minicomputer.



[*]

Ask for a volunteer to point to the squares where checkers are needed for 284. There are many possibilities, but very likely the student will indicate the standard configuration for 284 by pointing to the 200-square, to the 80-square, and to the 4-square.

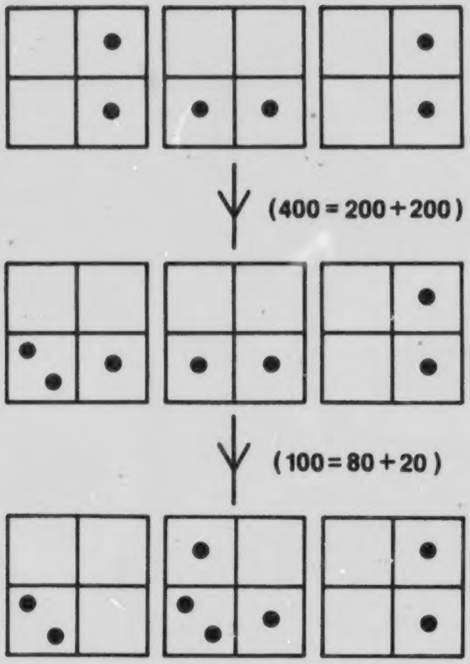
T: Who can make a play which will help us get checkers on these squares?

Only two plays are needed, although students may make other unnecessary plays.

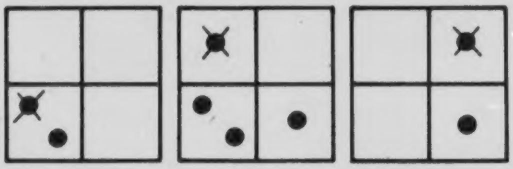
---

[*] When students are asked to put numbers on the Minicomputer, they may or may not present standard configurations for the numbers. The lesson description will assume standard configurations for numbers are put on the Minicomputer. Adjust the lesson for the configurations your students display.

N 1



T: Who can take 284 off the Minicomputer?



*

T: What number is 535 - 284?

S: 251.

Complete the number story on the board.

$$535 - 284 = 251$$

---

[*] An "X" through a checker indicates removing the checker from the Minicomputer.

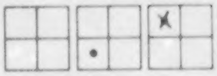
Let the students determine which guess is the closest.

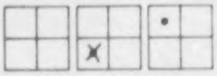
Worksheets N1*, **, ***, and **** are available for individual work. Some students may wish to use individual Minicomputers.

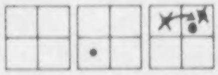
Name

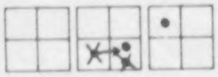
N 1 *

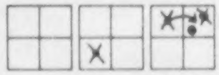
Complete


28 - 8 = 20


28 - 20 = 8


28 - 4 = 24

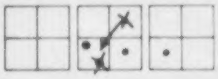

28 - 10 = 18

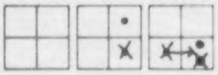

28 - 24 = 4

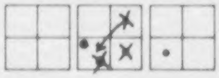
Name

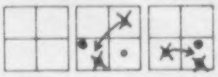
N 1 **

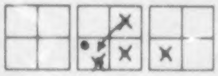
Complete


52 - 20 = 32


52 - 11 = 41


52 - 30 = 22

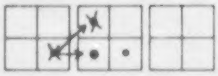

52 - 21 = 31

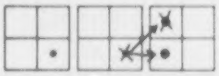

52 - 32 = 20

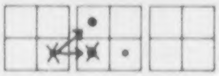
Name

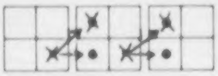
N 1 ***

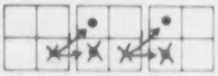
Complete


110 - 80 = 30


110 - 8 = 102


110 - 20 = 90

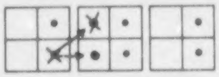

110 - 88 = 22

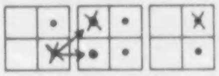

110 - 22 = 88

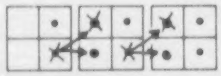
Name

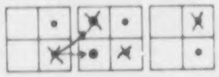
N 1 ****

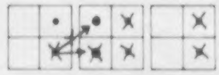
Complete


555 - 80 = 475


555 - 84 = 471


555 - 88 = 467


555 - 94 = 461


555 - 75 = 480

N2 The Functions 2x and 1/2x

CAPSULE LESSON SUMMARY

Ask: How many ears are present today in the classroom? Relate the functions $2x$ and $\frac{1}{2}x$ through the use of a simple arrow diagram; the arrow for one is the return arrow for the other. Draw $2x$ and $\frac{1}{2}x$ arrows whenever possible between the dots for three numbers. In the same picture, ask if any $4x$ arrows or $\frac{1}{4}x$ arrows can be drawn.

MATERIALS

Teacher: Colored chalk; Minicomputer set
Student: Worksheets N2* and **; pencil

DESCRIPTION OF LESSON

Begin this lesson with a few minutes of mental arithmetic emphasizing doubling and halving.

Exercise 1

T: How many students are here today?

The lesson description will assume there are 26 students present. Adjust the lesson to the number of students present in your class.

T: How many ears do 26 (use the number of students in your class) students have? (52) How do you know?

Commend any clear explanation and repeat it for the benefit of the class. If no one answers 52, suggest using the Minicomputer to calculate "26 + 26". When your class is aware that 26 students have 52 ears, draw the arrow picture in the next illustration on the board.

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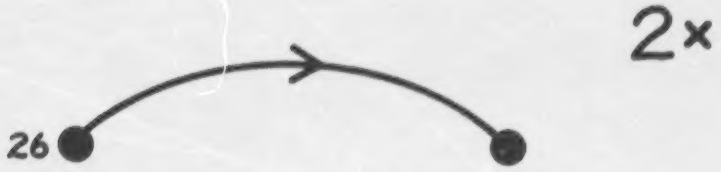
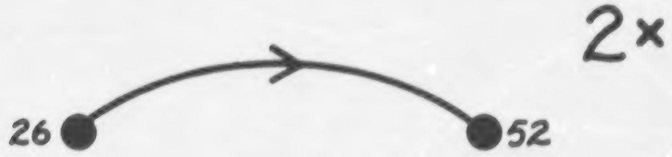
N 2



T: What number is here (point to the dot on the right)?

S: 52.

Label the dot on the right "52".



T: What number story is told by this arrow picture?

S: "2 x 26 = 52".

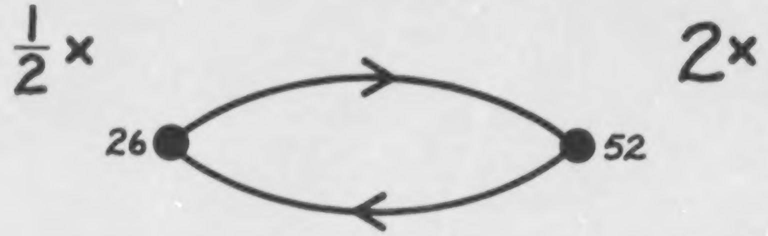
Trace, but do not draw, an arrow from 52 to 26.

T: If we draw an arrow from 52 to 26, what could it be for?

S: $\frac{1}{2}$x (read "one-half of").

NOTE: The arrow also could be for - 26 or "I am more than you" or other relations, but in this case we are only interested in $\frac{1}{2}$x .

Draw this arrow in blue and write "$\frac{1}{2}\times$" near the arrow picture.



T: What number story is told by this blue arrow?

S: "$\frac{1}{2} \times 52 = 26$" (read: one-half of fifty-two equals twenty-six).

Erase "26" and "52" and point to the dot on the left. Do not label the dots during this next activity. Use these or similar problems more appropriate for your students. Occasionally ask students to explain how they found the answer.

T: If this number were 3, what number would the other dot be for? (6)

Continue with several other starting numbers at the left dot such as the following. Corresponding numbers at the right are given in parentheses.

5 (10)	20 (40)	100 (200)
6 (12)	25 (50)	200 (400)
9 (18)	30 (60)	500 (1,000)

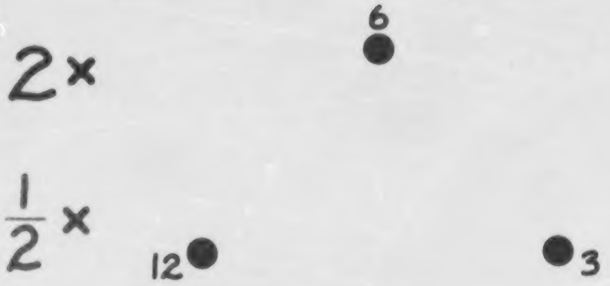
When a student correctly doubles 500, ask someone to write "1,000" on the board away from the arrow picture.

T (pointing to the dot on the right): If this number were 1,000, what number
would the other dot be for? (500)
If it were 100 . . . ? (50)
If it were 60 . . . ? (30)
If it were 2,000 . . . ? (1,000)

N 2

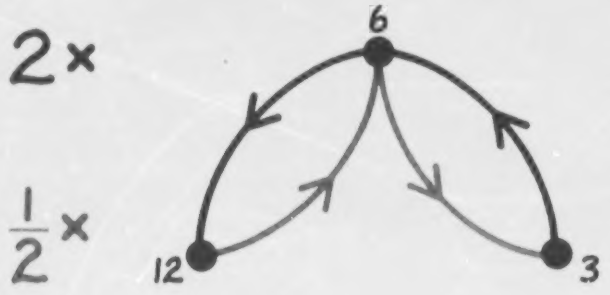
Exercise 2

Erase the board and then draw this picture.



T: How many 2x arrows can we draw in this arrow picture? (Two)
How many $\frac{1}{2}$x arrows can we draw in this arrow picture? (Two)

Ask for volunteers to draw the missing arrows and to tell the appropriate number stories for the arrows. Continue until the arrow picture is complete.



T (tracing an arrow from 3 to 12): What could this arrow be for?

S: 4x.

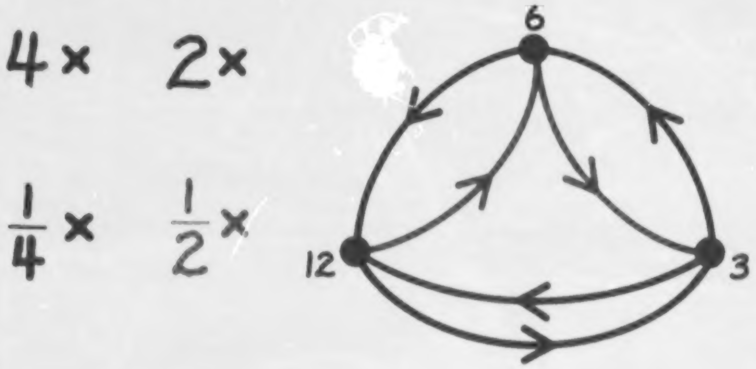
Write "4x" in green near the arrow picture and let a student draw an arrow from 3 to 12 in green. If no one suggests this, proceed to Exercise 3.

T (tracing an arrow from 12 to 3): What could this arrow be for?

S: $\frac{1}{4}x$.

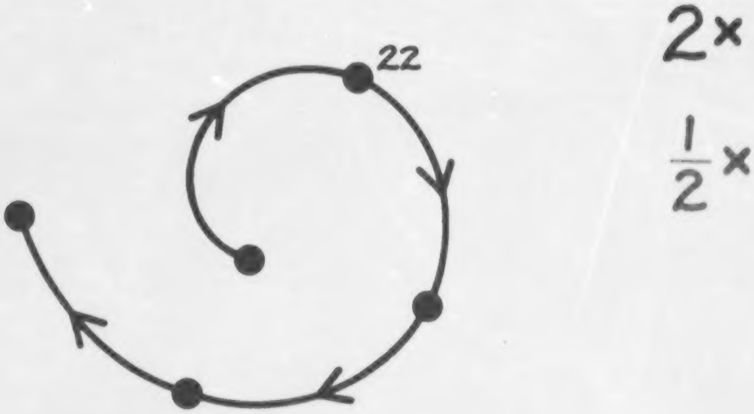
Write "$\frac{1}{4}x$" in orange near the arrow picture and let a student draw an arrow from 12 to 3 in orange. If no one suggests this, do not include the arrow in your picture.

If all four types of arrows are drawn, this is the completed arrow picture.



Exercise 3

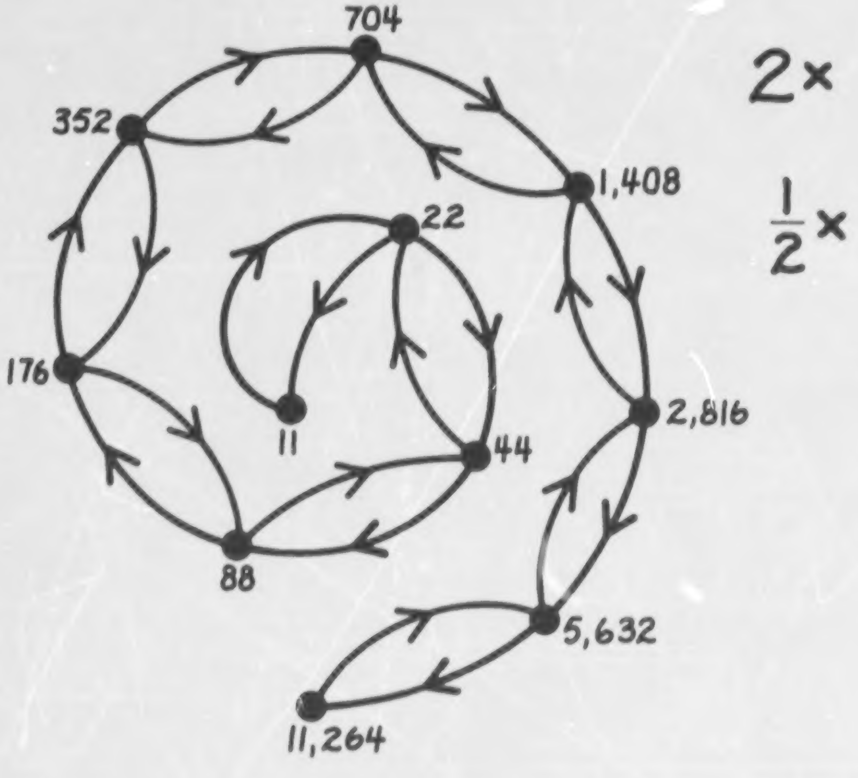
Erase the board and then draw this arrow picture.



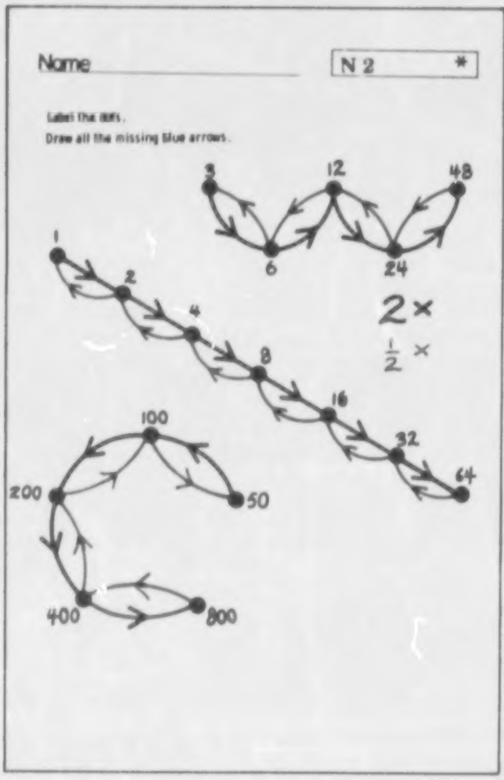
N 2

Ask volunteers to either label a dot or draw a missing $\frac{1}{2}$x arrow. Use the Minicomputer to calculate "2 × 88" or to check a student's mental calculation of "2 × 88".

Continue the spiral using the Minicomputer for the calculations as long as there is student interest. If you have several students who are good calculators, ask the class to predict the next number before it is calculated on the Minicomputer. Save approximately ten minutes for individual work on Worksheets N2* and **.



76



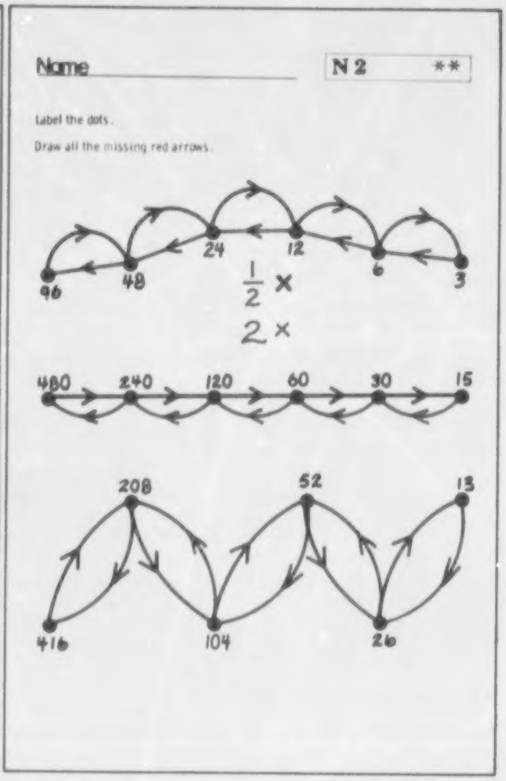
Name ______________________

N 2 *

Label the dots.

Draw all the missing blue arrows.

3
12
48
6
24
1
2
4
8
16
32
64
2×
$\frac{1}{2}$×
100
200
50
400
800



Name ______________________

N 2 **

Label the dots.

Draw all the missing red arrows.

96
48
24
12
6
3
$\frac{1}{2}$×
2×
480
240
120
60
30
15
208
52
13
416
104
26

N 3

N3 Doubling and Halving #1

CAPSULE LESSON SUMMARY

Write number stories about an array of dots in which half of the dots are blue and half of the dots are red. Consider a configuration on the Minicomputer for some number in which the checkers are grouped in pairs and in which both checkers in each pair are on the same square. Recognize what one-half of the number is and then make plays to find the number. Record the situation in an arrow picture. Do several 2 × calculations on individual Minicomputers.

MATERIALS

Teacher: Colored chalk; Minicomputer set
Student: Minicomputer set

DESCRIPTION OF LESSON

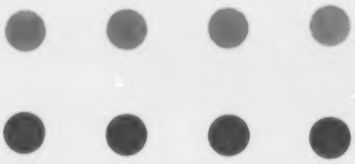
Exercise 1

Begin this lesson with five minutes of mental arithmetic doing calculations involving the functions $2\times$ and $\frac{1}{2}\times$; for example, ask students to calculate:

2×10 (20)	2×7 (14)	2×23 (46)
$\frac{1}{2} \times 20$ (10)	$\frac{1}{2} \times 14$ (7)	$\frac{1}{2} \times 46$ (23)

NOTE: "$\frac{1}{2} \times$" should be read "one-half of"; for example, "$\frac{1}{2} \times 20$" is read "one-half of twenty".

Draw this dot picture on the board.



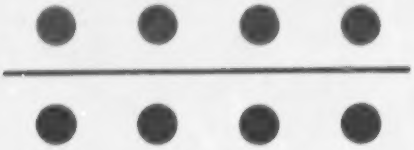
T: Look at this picture of dots. What do you see?

S: I see four blue dots and four red dots.

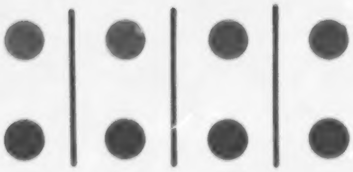
S: I see a blue dot and a red dot; a blue dot and a red dot; a blue dot and a red dot; and a blue dot and a red dot.

S: I see eight dots – half are red and half are blue.

Ask students to express what they see with number stories. You may wish to use a ruler to help illustrate the different number stories; for example,



$$4+4=8$$



$$4\times 2=8$$

Students may suggest some of these number stories. If no one mentions "$\frac{1}{2} \times 8 = 4$", suggest it yourself.

$4 + 4 = 8$

$8 = 2 \times 4$

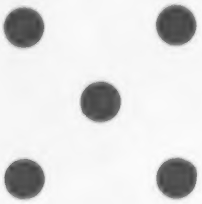
$8 - 4 = 4$

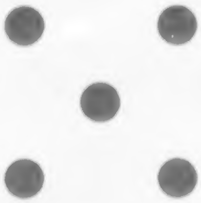
$\frac{1}{2} \times 8 = 4$

$4 \times 2 = 8$

$2 + 2 + 2 + 2 = 8$

Repeat this activity with another dot picture.

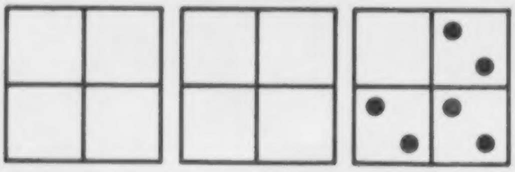




N 3

Exercise 2

Put this configuration on the Minicomputer.



T: What can you tell me about this number?

What number is on the Minicomputer in red checkers? (7)

What number is on the Minicomputer in blue checkers? (7)

What number stories can we write about this number?

Accept and record on the board any appropriate number stories for the configuration. These are some number stories your students might suggest.

$2 \times 7 = 14$

$\frac{1}{2} \times 14 = 7$

$7 + 7 = 14$

$14 - 7 = 7$

$(2 \times 4) + (2 \times 3) = 8 + 6 = 14$

$(2 \times 2) + (2 \times 5) = 4 + 10 = 14$

$(2 \times 6) + (2 \times 1) = 12 + 2 = 14$

$(2 \times 4) + (2 \times 2) + (2 \times 1) = 14$

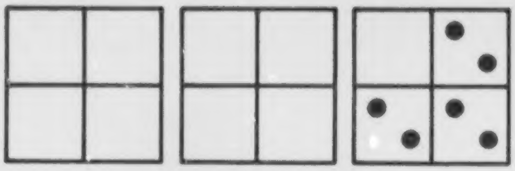
Ask someone to show one-half of this number on the Minicomputer. A student should remove one checker from each pair of checkers on the Minicomputer.

T: What number is $\frac{1}{2} \times 14$? (7)

If the number story "$\frac{1}{2} \times 14 = 7$" has not been suggested previously, record it on the board at this time.

T: If we double 7 (put back on the Minicomputer the three checkers which had been removed), what number do we get?

S: We get 14.



Erase all the number stories except "$\frac{1}{2} \times 14 = 7$" and "$2 \times 7 = 14$". If these number stories are not close to one another, rewrite them so they are.

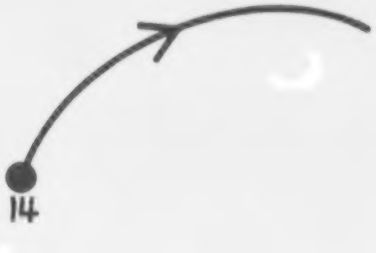
T: Let's show these number stories in an arrow picture. What kind of arrows should we use?

S: $2\times$ and $\frac{1}{2}\times$ arrows.

Draw a large dot on the board as you say:

T: This dot is for the number on the Minicomputer.

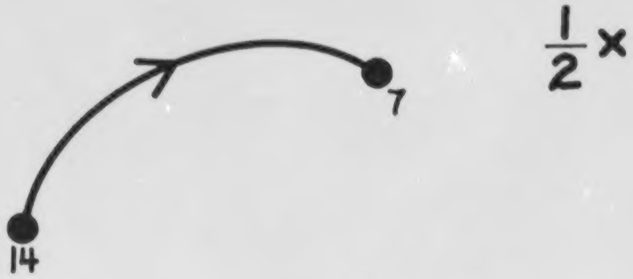
Label the dot "14" and then draw a red arrow starting from that dot.



T (drawing the ending dot of the red arrow): This dot is for one-half of that number.

N 3

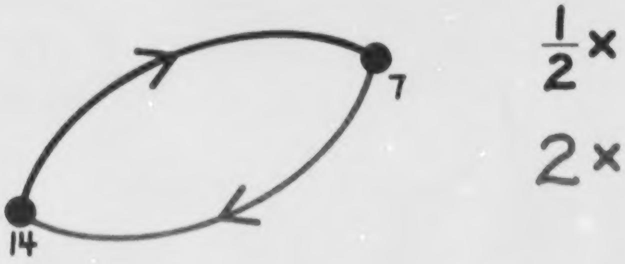
Label the ending dot "7" and then write "$\frac{1}{2}$x" near the arrow picture.



T (tracing an imaginary arrow from 7 to 14): What could this arrow be for?

S: 2x.

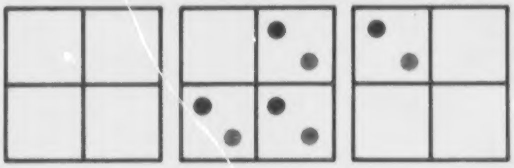
Invite someone to draw a blue arrow for 2x. Write "2x" under "$\frac{1}{2}$x".



T: What number stories are told by this arrow picture?

S: "$\frac{1}{2} \times 14 = 7$" and "$2 \times 7 = 14$".

Erase the dot labels and then put this configuration on the Minicomputer.



T (pointing to the dot on the left in the arrow picture): This dot is for the number on the Minicomputer.

Trace the red arrow.

T (pointing to the dot on the right): What is one-half of this number?

S: 78.

T: How did you calculate one-half of this number so quickly?

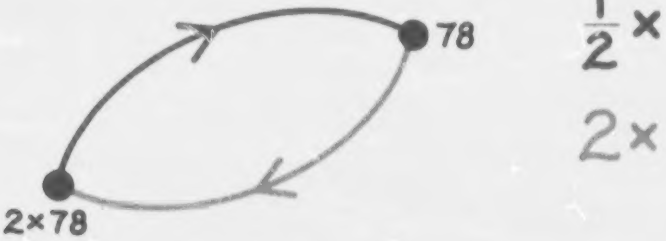
S: All the checkers are in pairs.

S: I only looked at the red (or the blue) checkers.

Trace the blue arrow.

T (pointing to the dot on the left): So the number on the Minicomputer is 2 x 78.

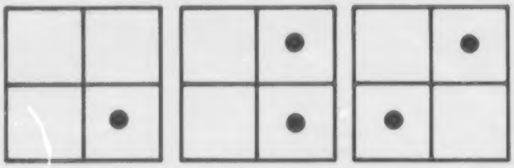
Label the dot on the left "2 x 78".



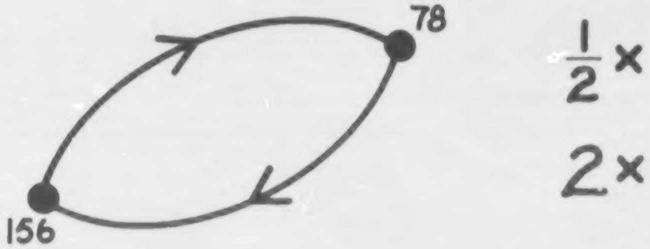
T: Tell me with whispers what number is 2 x 78.

N 3

After many students have whispered their answers to you, invite students to make plays until this configuration is obtained.



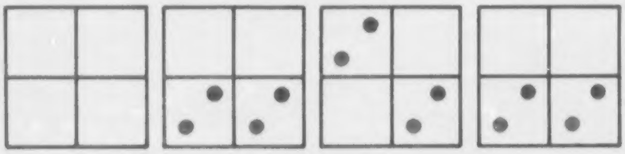
Conclude that 2 x 78 = 156 and change the label of the left dot from "2 x 78" to "156".



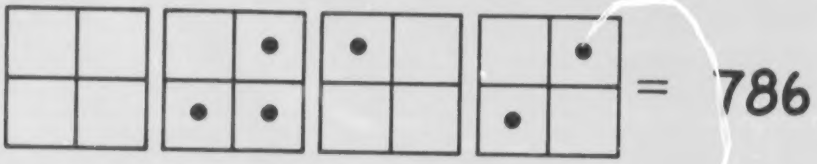
T: What number stories are told by this arrow picture?

S: "$\frac{1}{2}$ x 156 = 78" and "2 x 78 = 156".

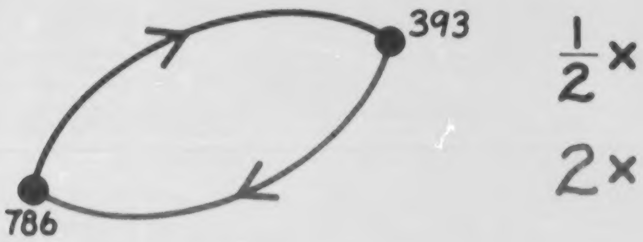
Erase the dot labels and repeat the activity with this configuration. You will need to display a fourth Minicomputer board.



After plays are made, this configuration will be on the Minicomputer . . .



and the arrow picture will have these labels.



Exercise 3: Individual Minicomputers

Each student should have two Minicomputer sheets and a set of checkers. Write "2 × 19 = ?" on the board.

T: Put 2 × 19 on your Minicomputer.

Invite a student who completes this task quickly to put 2 × 19 on the demonstration Minicomputer.

T: What number is 2 × 19?

Allow several students to predict the answer, and then encourage the students to make plays on their individual Minicomputers until they obtain standard configuration for 38.

N 3

Let someone make plays on the demonstration Minicomputer for the benefit of students who are having difficulty. Those students should be encouraged to watch closely and then to make the same plays on their individual Minicomputers. Complete the number story on the board. Continue this activity with calculations which are appropriate for your students. Calculations of varying difficulty are given here.

*

Two to four forward plays are needed.

2 x 14
2 x 23
2 x 45
2 x 140
2 x 409
2 x 510
2 x 1,244
2 x 2,045

**

More than four forward plays are needed.

2 x 47
2 x 35
2 x 59
2 x 374
2 x 199
2 x 570
2 x 2,096
2 x 4,350

***

Backward plays may be necessary.

2 x 16
2 x 36
2 x 261
2 x 656
2 x 18
2 x 78
2 x 856
2 x 2,807

N4 Consecutive Jumps on the Number Line

CAPSULE LESSON SUMMARY

Locate numbers on the number line. Make some consecutive + 2 and - 3 jumps on the number line and label the ending points of the corresponding arrows. Play the Number Line Game.

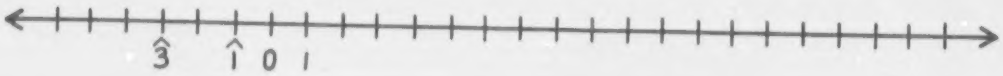
MATERIALS

Teacher: Colored chalk
Student: None

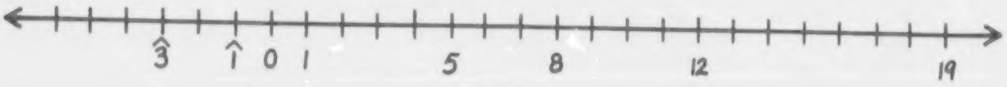
DESCRIPTION OF LESSON

Exercise 1

Draw a line on the board with equally spaced marks at least six centimeters apart. Label some of the marks as in the next illustration.



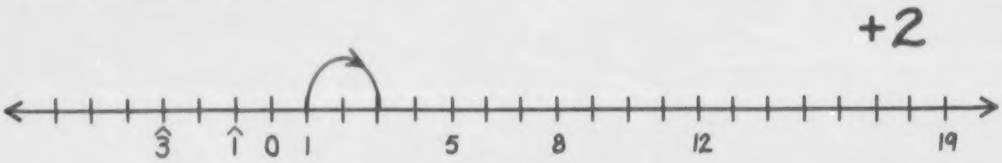
Point to some of the marks to the right of "1", each time moving further to the right. When you point to a mark, ask the students, "What number is here? ", and then label the mark when the correct number is given. Continue until students have helped label four or five marks.



T: Let's start at 1 on the number line and make jumps to the right moving two spaces each time. (Write "**+ 2**" near the number line.) I'll make the first + 2 jump.

N 4

Draw a + 2 arrow starting at 1.

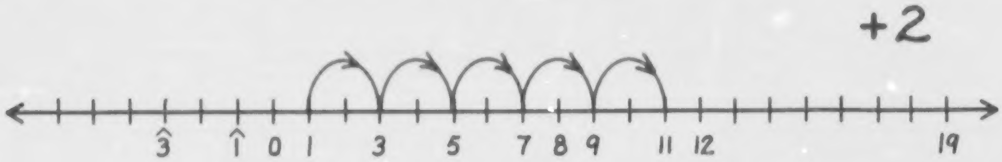


T (pointing to the ending point of the red arrow): 1 + 2 = . . . ?

S: 3.

Ask a student to label the appropriate mark "3".

Invite students to make some consecutive + 2 jumps, to draw the appropriate red arrows, and to label the ending points of the red arrows.



When you have about five + 2 jumps, you should ask some of the following questions or others similar to them. The questions and answers given below are based on the preceding illustration.

- How many jumps have we made so far? (Five jumps)
- How many spaces to the right of "1" have we moved altogether? (Ten spaces)
- Which numbers have we landed on? (1, 3, 5, 7, 9, 11)
- Which numbers have we jumped over? (2, 4, 6, 8, 10)
- Do you see a pattern? (We land on odd numbers and jump over even numbers.)

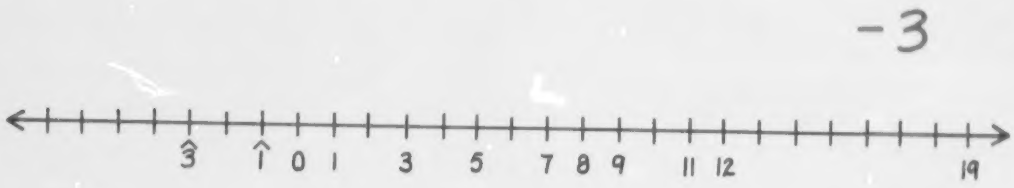
- If we were to continue making jumps from 11, we would land on many more numbers. Name some of them. (Any odd integer which is more than 11 is correct.)
- Would we ever land on "100"? (No, 100 is an even number.)
  What is a number larger than 100 that we would land on?
  What is a number larger than 100 that we would jump over?
  What is the smallest number larger than 100 that we would land on? (101)

NOTE: Numbers such as 102.5 and 117 $\frac{1}{4}$ are jumped over as well as all the even numbers. Also 0 and any negative number will not be landed on because the + 2 jumps started at 1.

Erase only the arrows and the key, leaving the labeled number line.

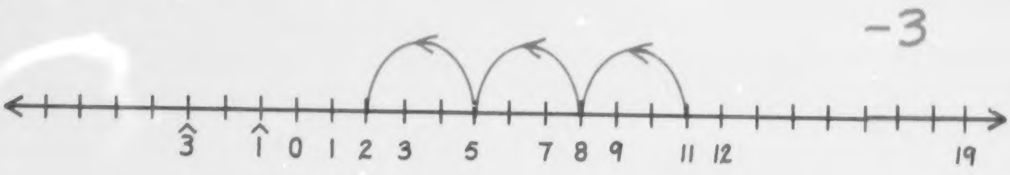
Exercise 2

Write " – 3 " near the number line.



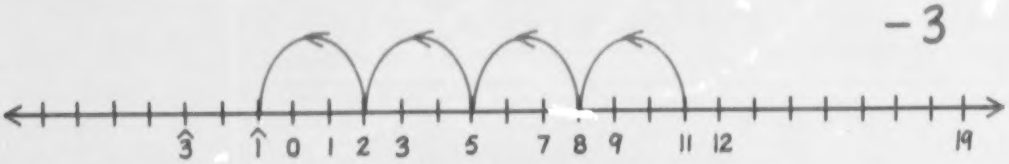
T: This time, we will make – 3 jumps (read: minus three jumps) starting at 11.

Ask for volunteers to make some consecutive – 3 jumps and to draw the appropriate arrows. Your picture should look similar to this one.

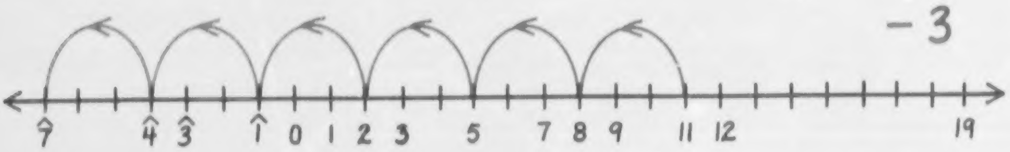


N 4

Students should have little trouble making the - 3 jump from 2 to $\hat{1}$. Those who seem puzzled should be reminded that the - 3 jump from 2 goes three spaces to the left, just as the other - 3 jumps.



Continue to ask students to draw - 3 jumps from $\hat{1}$ until you have six or seven jumps altogether.



You may want to ask questions similar to those for the + 2 jumps. Other suggestions follow.

- How is this picture different from the one showing + 2 jumps? (Possible answers: The jumps move to the left instead of to the right. These jumps are longer. Each of these jumps took us to a smaller number; each + 2 jump took us to a larger number.)
- The last - 3 jump drawn on the board ends at $\hat{7}$. If we made another - 3 jump, where would we land? ($\hat{10}$)

If your class is interested and enjoying the lesson, vary the activity by making + 5 (or - 4, + 10, - 1, . . .) jumps or by changing the starting points so that you begin sometimes at a relatively large number, such as 59, and sometimes at a relatively small positive number, such as 18. You may even wish to start with a negative number. Otherwise, proceed to Exercise 3.

NOTE: The problems in these exercises are directly related, of course, to simple addition and subtraction calculations. For example,

labeling the landing point of a − 3 jump beginning at 2	corresponds to	doing the calculation "$2 - 3 = \hat{1}$"
labeling the landing point of a − 3 jump beginning at $\hat{5}$	corresponds to	doing the calculation "$\hat{5} - 3 = \hat{8}$"
labeling the landing point of a + 2 jump beginning at 11	corresponds to	doing the calculation "$11 + 2 = 13$"
labeling the landing point of a + 2 jump beginning at $\hat{9}$	corresponds to	doing the calculation "$\hat{9} + 2 = \hat{7}$"

But this information is just for you. Do not make any formal statement to your students about rules for addition or subtraction of positive and negative integers.

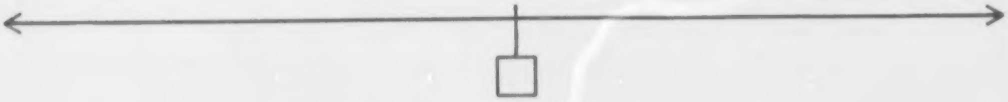
Exercise 3: The Number Line Game

The Number Line Game is one which may be played frequently throughout the year whenever you finish a lesson early or have some extra time. Each game takes approximately five to ten minutes. Several variations of the game are given at the end of this lesson. Use these variations when your class is ready for them.

T: Today we are going to play the Number Line Game. I am thinking of a number between 0 and 200. I'll show you where it is on the number line; my number is hidden in this box.

N 4

Draw a line on the board and position a box for the secret number near the center of your picture.

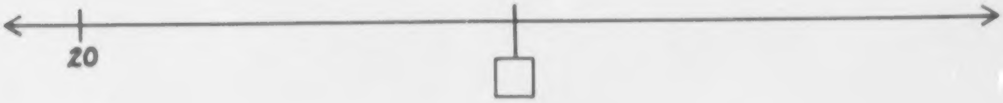


T: Guess what my secret number is.

This is a description of a possible game. Suppose the secret number is 114.

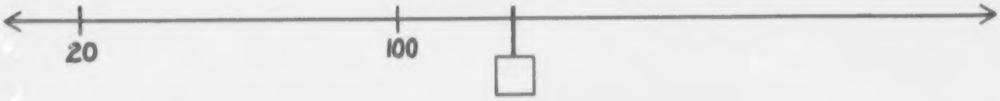
S: 20.

T: I'll show you where 20 is on this number line. It might not be in exactly the right place, but I'll try to make it as close as possible.



T: 20 is less than my secret number. Who has another guess?

S: 100.



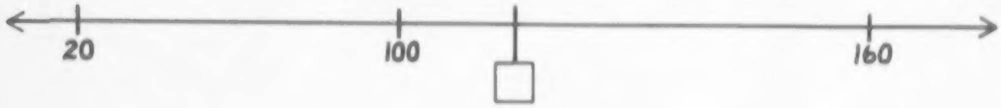
T: 100 is less than my number. Who has another guess?

S: 200.

T: My number is between 0 and 200. 200 cannot be my number.

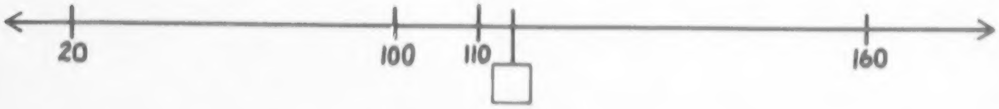
Anytime the guess given is a number which is not between 0 and 200, do not write anything. Just remind the students that your number is between 0 and 200, and ask someone else to guess.

S: 160.



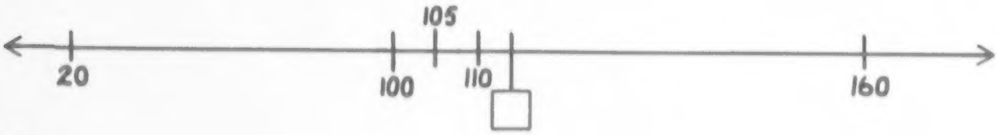
T: 160 is more than my secret number.

S: 110.



T: 110 is less than my secret number.

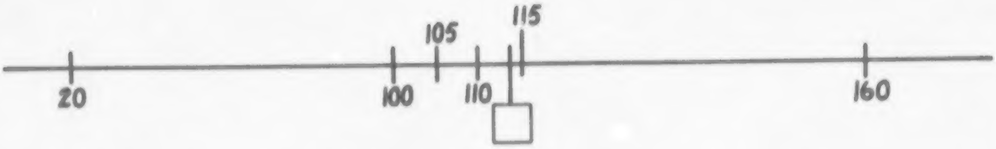
S: 105.



T: 105 is less than my secret number.

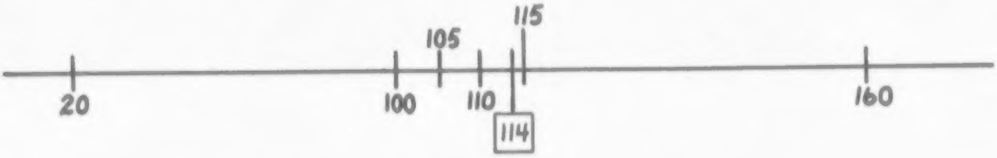
NOTE: Although a guess may give no new information, such as 105 in this sample game, do not criticize or comment except to say that it is more or less than your number.

S: 115.


105
115
20
100
110
160

T: 115 is more than my number.

S: 114.


105
115
20
100
110
114
160

T: You have discovered my secret number.

Play the game a few more times. You may wish to let the student who guesses your number in a game choose the secret number for the next game and whisper it to you. Then ask that student to keep the number a secret and not play in that game.

VARIATIONS OF THE NUMBER LINE GAME

Variation 1

When your class is comfortable with the game using numbers between 0 and 200, you can expand the range of possible numbers. Some possibilities are:

- numbers between 0 and 500;
- numbers between 500 and 1,000;
- numbers between $\widehat{50}$ and 50;
- numbers between $\widehat{100}$ and 100.

Variation 2

As the students become familiar with numbers such as $2\frac{1}{2}$, occasionally you can choose a non-integer as your secret number. It is best to limit the range of possible numbers to those between 0 and 10 when you use this variation.

Variation 3

When you are using only whole numbers (0, 1, 2, 3, . . .) for your range of possible numbers, ask a student to choose a secret number and to judge the guesses of the other students. The student who chooses the secret number tells the class whether a guess is more or less than the secret number. You draw and label a mark on the number line for each guess and, if necessary, help the student to judge. Remind the student who is judging not to give additional clues.

Variation 4

Divide the class into two teams. The teams alternate turns and the members of each team take turns guessing the secret number.

N 5

N5 Doubling and Halving #2

CAPSULE LESSON SUMMARY

Consider a configuration on the Minicomputer in which the checkers are grouped in pairs and in which both checkers in each pair are on the same square. Recognize what one-half of the number is and then make plays to find the number. Record the situation in an arrow picture. Repeat this activity with variations. In one variation, the number on the Minicomputer can be recognized easily, but one-half of it cannot because the checkers are not in pairs. In another variation, students use individual Minicomputers.

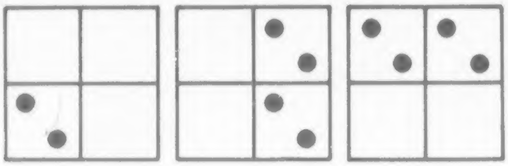
MATERIALS

Teacher: Minicomputer set; colored chalk
Student: Minicomputer set

DESCRIPTION OF LESSON

Exercise 1

Put this configuration on the Minicomputer.



T: What can you tell me about this number?

Is it more than 100? How do you know?

Is it more than 400? How do you know?

Is it more than 500? How do you know?

Is it more than 1,000? How do you know?

Let the students react freely to these questions which should help them to arrive at a good estimate of the number on the Minicomputer. Do not ask the students to tell you what this number is yet.

T: What is one-half of this number? How can you tell?

Commend any student who suggests that one-half of the number on the hundreds' board is 200, one-half of the number on the tens' board is 50, and one-half of the number on the ones' board is 12. Otherwise, guide your class to these conclusions. Write this number story on the board.

$$200 + 50 + 12 = ?$$

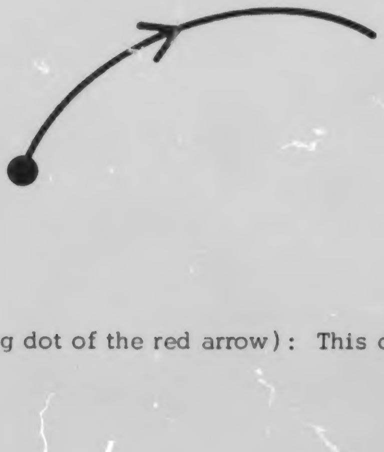
T: What is one-half of the number on the Minicomputer?

S: 262.

Draw a large dot on the board as you say:

T: This dot is for the number on the Minicomputer.

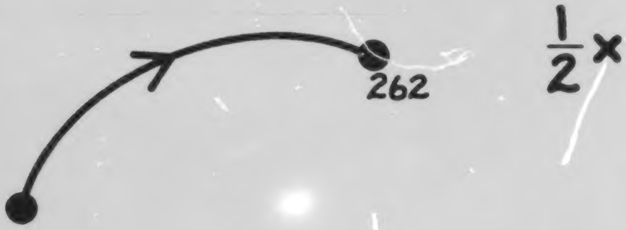
Draw a red arrow starting at that dot.



T (drawing the ending dot of the red arrow): This dot is for one-half of that number.

N 5

Write "$\frac{1}{2}$x" near the arrow picture and then label the ending dot "262".

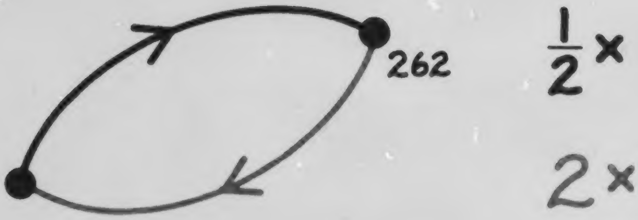


Draw the return arrow in blue.

T: What could the blue arrow be for?

S: 2x.

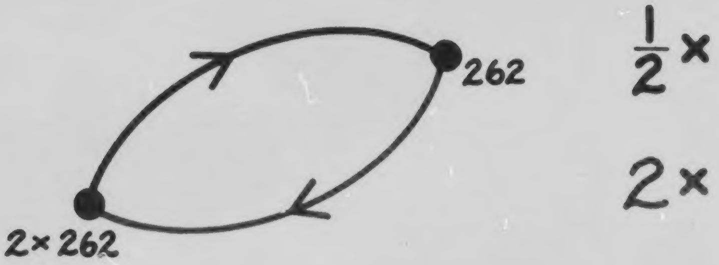
Write "2x" under "$\frac{1}{2}$x".



Trace the blue arrow.

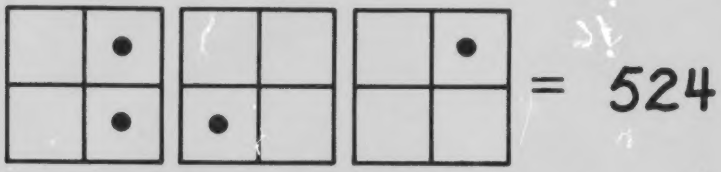
T (pointing to the dot on the left): So the number on the Minicomputer is 2 x 262.

Label the dot on the left "2 × 262".

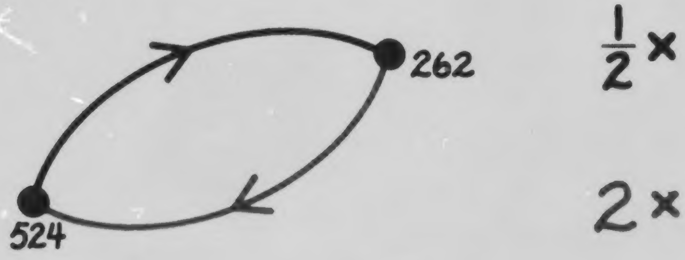

262
2×262
$\frac{1}{2}$×
2×

T: Tell me with whispers what number is 2 × 262.

After many students have whispered their answers to you, invite students to make plays to obtain standard configuration.



Conclude that 2 × 262 = 524 and label the dot on the left "524" (erase "2 × 262").

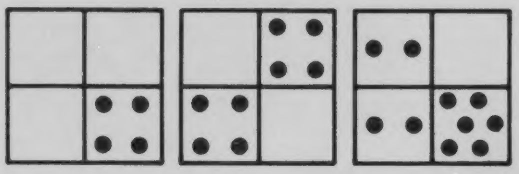

262
524
$\frac{1}{2}$×
2×

T: What number stories are told by this arrow picture?

N5

S: "$\frac{1}{2}$ × 524 = 262" and "2 × 262 = 524".

Erase "524" and "262". Put this configuration on the Minicomputer.



T: What can you tell me about this number?
Is it more than 100? How do you know?
Is it more than 400? How do you know?
Is it more than 500? How do you know?
Is it more than 600? How do you know?

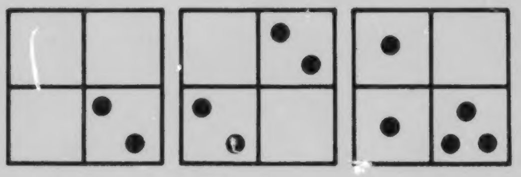
Let the students react freely to these questions. Accept their explanations, even if they are not well-formed. Do not ask the students to tell you what the number is at this time in the lesson.

T: We aren't sure what number this is, but can you tell me immediately what one-half of this number is?

Students may be confused by there being many checkers.

T: Who can remove some checkers so that one-half of this number will be on the Minicomputer?

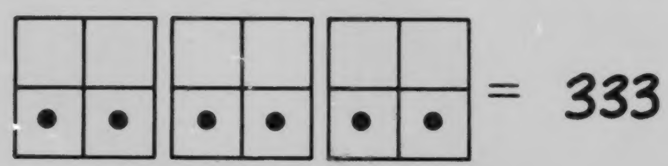
A student should remove one checker from each pair of checkers.



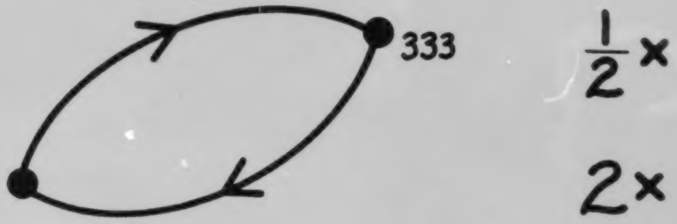
T: This is one-half of the number that was on the Minicomputer. How can we make the number easier to read?

S: Make some plays.

Invite students to make plays until the number is in standard configuration.



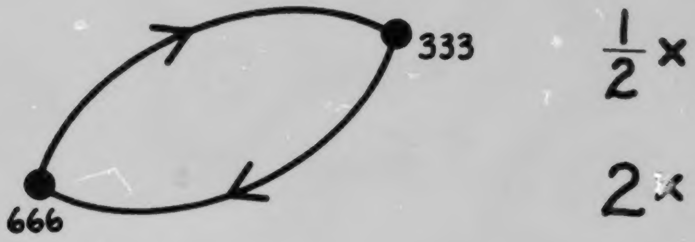
Label the dot on the right in the arrow picture "333".



T (tracing the blue arrow): So the number that was on the Minicomputer is 2 x 333. 2 x 333 = . . . ?

S: 666.

Label the dot on the left "666".

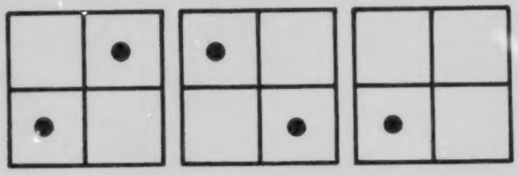


N 5

T: What number stories are told by this arrow picture?

S: "$\frac{1}{2} \times 666 = 333$" and "$2 \times 333 = 666$".

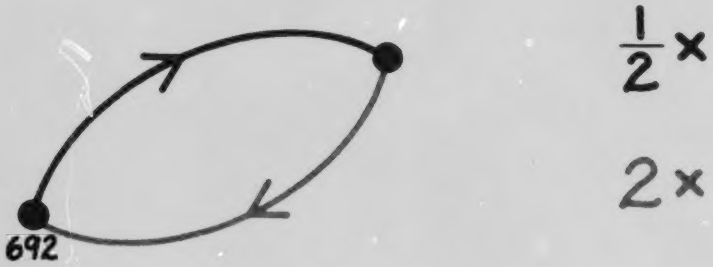
Erase "666" and "333" from the arrow picture. Put this configuration on the Minicomputer.



T: What number is this?

S: 692.

Label the dot on the left in the arrow picture "692".



T (tracing the red arrow): What is one-half of this number?

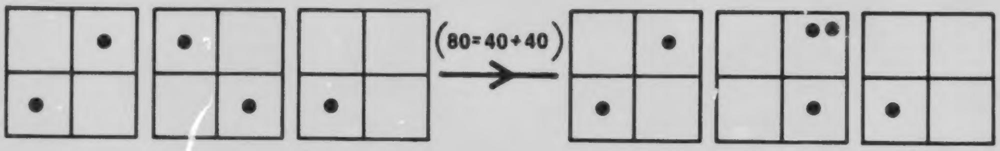
Some students might guess the answer, but others very likely will say they do not know because the checkers are not in pairs.

T: How can we put these checkers in pairs without changing the number?

S: Make some backward plays.

Invite someone to make a play. Any backward play which could be made except "10 = 8 + 2" will result in a pair of checkers on one of the squares. You may need to guide the discussion by pointing to the checker on the 80-square and asking what play you could make with that checker which would result in a pair of checkers.

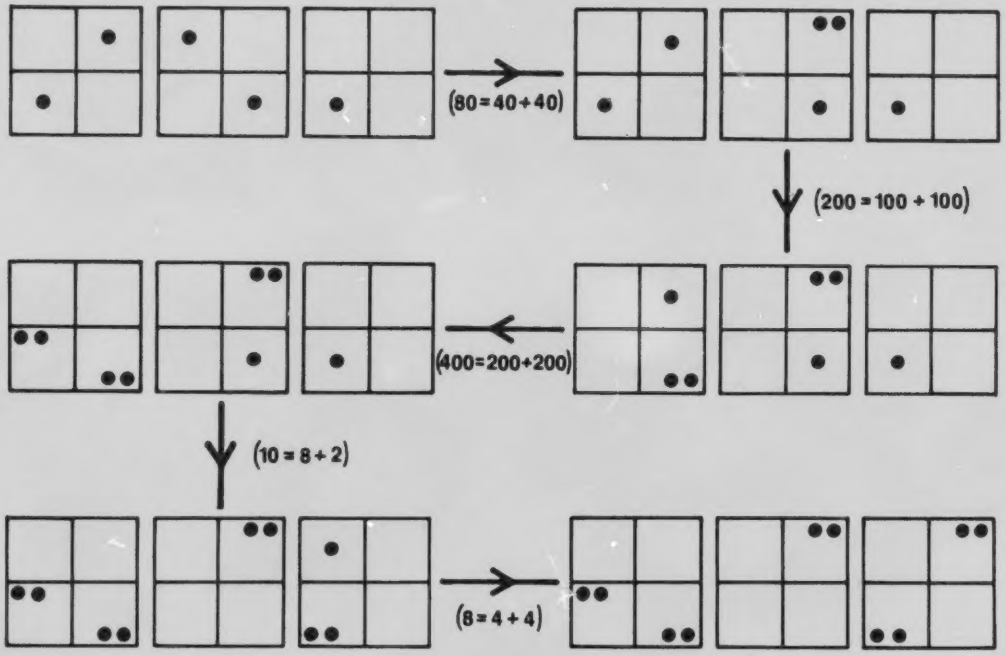
Whenever you get a pair of checkers on the same square, push the two checkers into a corner of that square. For example,



T: We now have a pair of checkers. Can someone make a play which will give us another pair of checkers?

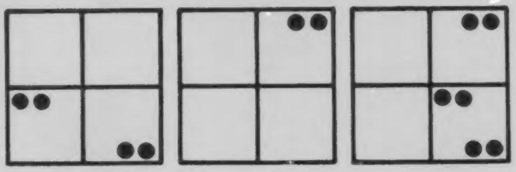
Continue this activity until all the checkers are in pairs. Discourage any student who wants to make a play with checkers which are already paired. Tell the student it is a correct play but not very useful for the goal of getting pairs.

One possible sequence of plays is given below.



N 5

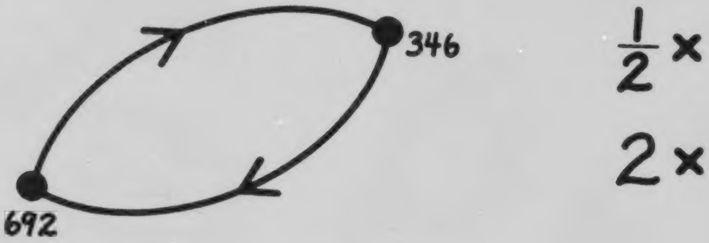
NOTE: With another sequence of plays, your students could arrive at this configuration.



T: All the checkers are in pairs. What is one-half of this number?

S: 346.

Label the dot on the right in the arrow picture "346".

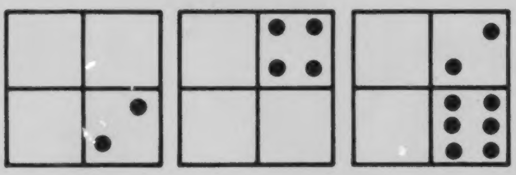


T: What number stories are told by this arrow picture?

S: "2 x 346 = 692" and "$\frac{1}{2}$ x 692 = 346".

Exercise 2: Individual Minicomputers

Erase the board and put this configuration on the Minicomputer.



T: What can you tell me about this number?
Is it more than 100? How do you know?
Is it more than 200? How do you know?
Is it more than 300? How do you know?
Is it more than 400? How do you know?

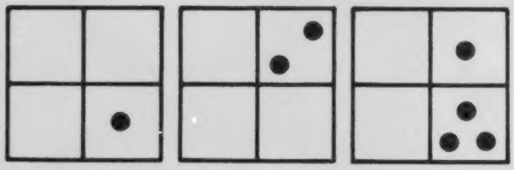
These questions should lead the students to make a good estimate of the number on the Minicomputer. Do not ask the students to tell you what the number is at this time in the lesson.

Each student should have two Minicomputer sheets and a set of checkers.

T: Put one-half of this number on your Minicomputer.

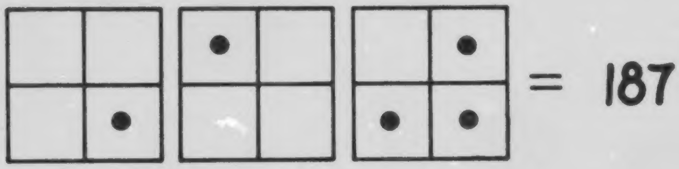
Do not change the number on the demonstration Minicomputer. Some students may not know what to do if there are more than two checkers on a square. If this is a common difficulty, ask students to explain their methods to the class. One student might pair all the checkers and take off one out of every two, another might count the checkers on a square and take off half that many. Either approach is correct.

Each student should have this configuration displayed.

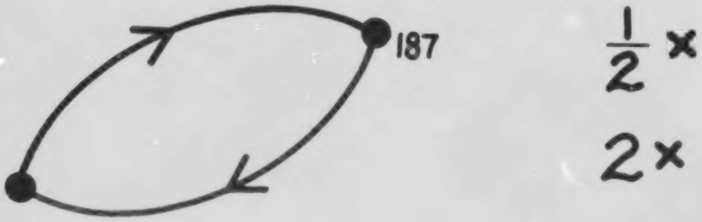


T: Is the number on your Minicomputer more than 100? How do you know? Is this number more than 200? How do you know? What is this number?

Allow several students to answer; then let the students make plays on their individual Minicomputers to determine what the number is. You may need to help a few students make the plays. Conclude that 187 is one-half of the number on the (demonstration) Minicomputer.



Label the dot on the right in the arrow picture "187".



T (tracing the blue arrow): 2 x 187. How can we calculate this number without using the Minicomputer?

Let them suggest their own methods of calculation. Perhaps many students will do it this way.

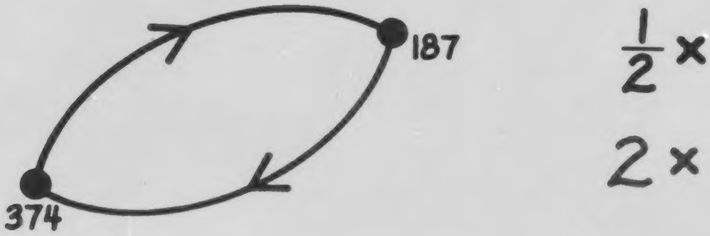
$$100 + 100 = 200 \text{ and } 80 + 80 = 160 \text{ and } 7 + 7 = 14$$

so

$$2 \times 187 = 200 + 160 + 14 = 374$$

Afterward let them check this answer by calculating "2 x 187" on their individual Minicomputers.

Label the dot on the left "374".



T: What number stories are told by this arrow picture?

S: "$\frac{1}{2}$ x 374 = 187" and "2 x 187 = 374".

If there is still time remaining for the lesson, continue with other similar problems.

N 6

N6 Roads #1

CAPSULE LESSON SUMMARY

Consider four arrow roads which start at the same number and which have only +10 and +1 arrows. Determine which road has the largest ending number and which has the smallest. Label the dots in all the roads. Draw roads which have three +10 and two +1 arrows and discover that the order in which the arrows are drawn does not change the ending number, 32. Build a road from 0 to 24 using +10 and +1 arrows and decide which is the shortest possible road and which is the longest. Do some mental arithmetic asking for the ending number of a road described in terms of the number of +10 and +1 arrows.

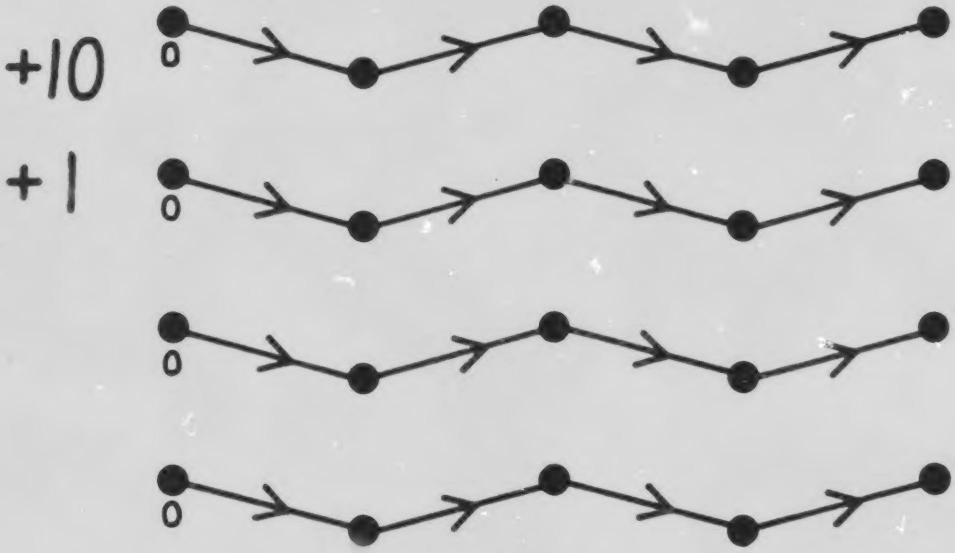
MATERIALS

Teacher: Colored chalk
Student: Colored pencils*; paper; Worksheets N6* and **

DESCRIPTION OF LESSON

Exercise 1

Draw this arrow picture on the board.



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* Crayons can be substituted for colored pencils in any of the lessons.

T: Look at the four arrow roads on the board. How are these roads alike?

S: They all have four arrows.

S: They all start at 0.

T: What is different about these four roads?

S: Three of the roads start with a blue arrow and one road starts with a red arrow.

S: There is a different number of red (or blue) arrows in each road.

T: Which of these roads has the largest ending number?

S: The second one.

T: How do you know?

S: Because it has the most +10 arrows.

T: What is the ending number of this road? How do you know?

S: 40. I counted by tens.

S: 40, because $4 \times 10 = 40$.

Ask for a volunteer to label the dots in the second road.

T: Which of these roads has the smallest ending number?

S: The last one.

T: How do you know?

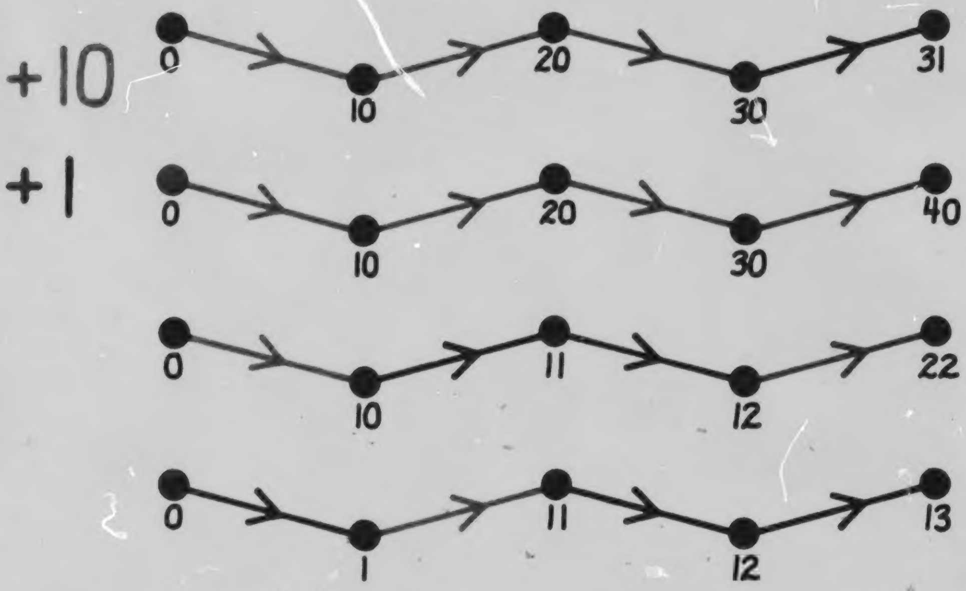
S: It has the fewest +10 arrows.

T: What is the ending number of this road?

N 6

S: 13.

Let students label the dots in this road and in the other roads.



T: Can you think of a way to get a number smaller than 13 by starting at 0 and using only four arrows?

S: Use four +1 arrows and the ending number will be 4.

Distribute colored pencils and unlined paper. Ask the students to build roads which have exactly three +10 arrows and two +1 arrows in any order they wish, starting at 0. Summarize this information on the board.

Start at 0.
three +10 arrows
two +1 arrows

As students are working, invite several of them to each draw a road on the board. Continue until there are several different solutions on the board. The students very likely will notice that all of the roads end at 32; the order in which you draw + 10 and + 1 arrows does not matter.

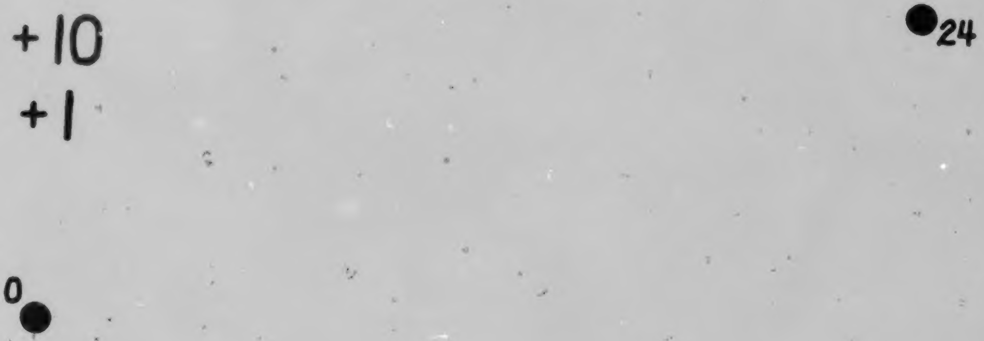
T: These roads always end in 32. What does this tell us?

S: Three 10's and two 1's is 32.

Erase the board.

Exercise 2

Put this information on the board.



Students should have paper and colored pencils from the first exercise.

T: On your paper build a road from 0 to 24 using blue arrows and red arrows.

Let the students work individually for a few minutes. Invite students who finish quickly to draw their roads on the board.

T: Which of the roads on the board has the most arrows?
Did any of you build a longer road?
What is the longest road we can build?

S: The longest road is the one which has twenty-four +1 arrows.

T: Which of the roads on the board has the fewest arrows?
Did any of you build a shorter road?
What is the shortest road we can build?

S: The shortest road is the one which has two +10 arrows and four +1 arrows.

Exercise 3: Mental Arithmetic

Ask the students for the ending number of each of the roads described below.

- Start at 0. Use five +10 arrows and three +1 arrows.
  (Ending number is 53.)
- Start at 0. Use seven +10 arrows and nine +1 arrows.
  (Ending number is 79.)
- Start at 0. Use nine +10 arrows.
  (Ending number is 90.)
- Start at 0. Use nine +10 arrows and ten +1 arrows.
  (Ending number is 100.)
- Start at 0. Use one hundred thirty-three +1 arrows.
  (Ending number is 133.)
- Start at 0. Use one hundred thirty-three +1 arrows and one +10 arrow.
  (Ending number is 143.)

T: Imagine a road which starts at 0 and which has exactly seven arrows. Again we are only using +10 and +1 arrows. Which road are you thinking of? What is its ending number?

Let several students describe possible roads. Record their ending numbers on the board. If your students find all such roads, your list of ending numbers will look like (except for order) the one on the right.

70
61
52
43
34
25
16
7

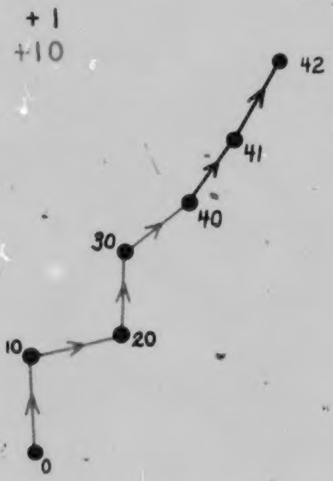
T (pointing to the list on the board): Does anyone see a pattern to these numbers?

S: 7 + 0 = 7; 6 + 1 = 7; 5 + 2 = 7; 4 + 3 = 7; and so on.

Distribute Worksheets N6* and ** and let the students work independently. Ask students who finish quickly to build a road from 0 to 452 using only +1, +10, and +100 arrows.

Name ______________________ **N 6** *

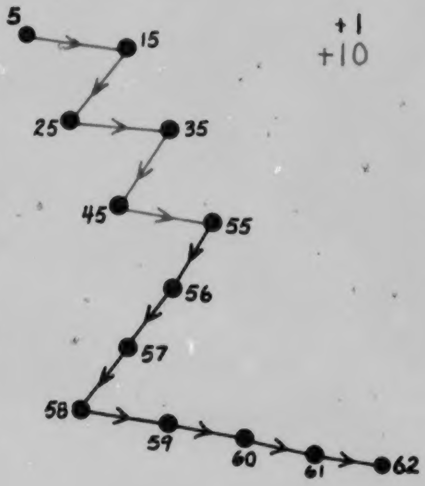
Build a red-blue road from 0 to 42.


+1
+10
42
41
40
30
20
10
0

Other solutions are possible, including some with many more arrows.

Name ______________________ **N 6** **

Build a red-blue road from 5 to 62.


5
15
+1
+10
25
35
45
55
56
57
58
59
60
61
62

Other solutions are possible, including some with many more arrows.

N7 Addition Algorithm #1

CAPSULE LESSON SUMMARY

Put a number on the Minicomputer with many checkers. Do mental calculations to find out which number it is and discuss several approaches. Make plays to obtain the standard configuration for the number. Given a configuration consisting of ten checkers on the same square, make plays until the standard configuration is obtained. Suggest the idea of replacing ten checkers on the same square with one checker on the square of the same color on the next board to the left. Practice the addition algorithm previously introduced in Lesson N31 of UPG, Part I to do addition calculations such as "453 + 276".

MATERIALS

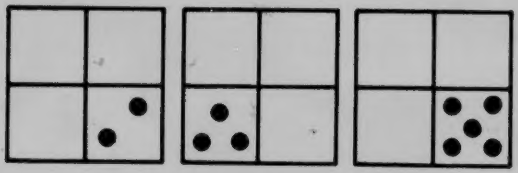
Teacher: Minicomputer set

Student: Paper; pencil; Addition Problems booklet (optional)

DESCRIPTION OF LESSON

Exercise 1

Put this configuration on the Minicomputer.



T: How many checkers are on the Minicomputer? (Ten)

Without making any plays can you figure out which number this is? Write it on a piece of paper.

Some students will calculate mentally what the number is; others will need to do some addition calculations with paper and pencil. Look at many of the students' answers and then invite some students to explain how they calculated the number. Record some of the calculations on the board.

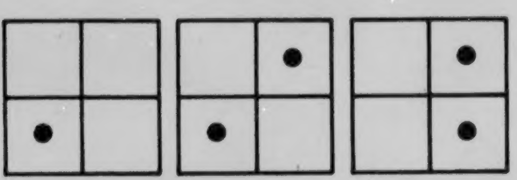
N 7

These are some calculations your students might suggest.

$$(2\times100)+(3\times20)+(5\times1)=200+60+5=265$$

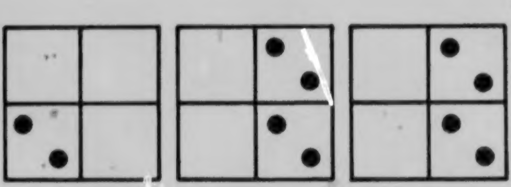
$$(2\times100)+20+20+20+5=265$$

Afterward invite students to make plays until the number is in standard configuration.

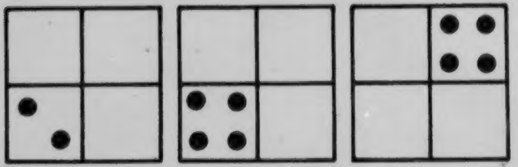


= 265

Repeat this activity with these configurations.



= 510



= 496

T: Is 496 more or less than 500? (Less)
How much less?

Write this number story on the board.

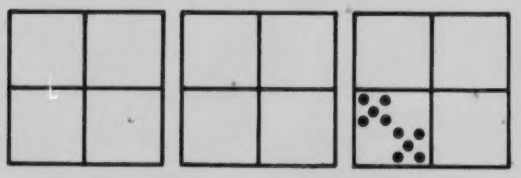
$$496 + \boxed{} = 500$$

With the students, count by ones from 496 to 500 and conclude that 496 is 4 less than 500. Complete the number story.

$$496 + \boxed{4} = 500$$

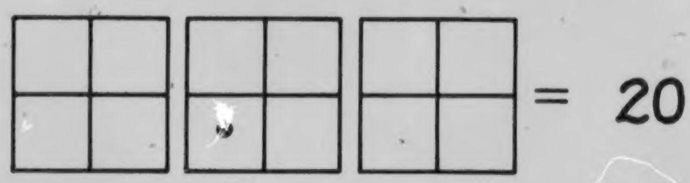
Exercise 2: Multiplying by 10

Put this configuration on the Minicomputer.



T: Ten checkers on the same square! What is this number?

Many students will recognize immediately that this number is 20 because 10 x 2 = 20. Ask for a volunteer to make plays until the standard configuration for 20 is obtained.

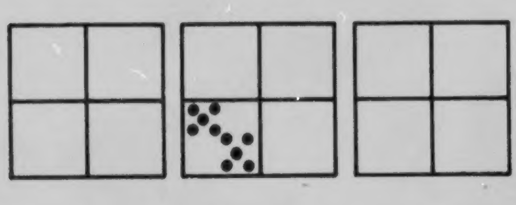


N 7

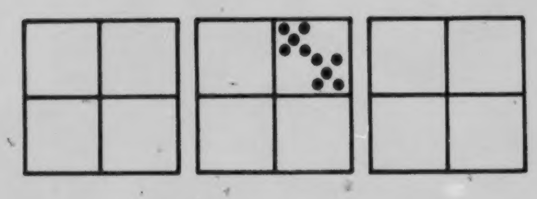
Emphasize that there were ten checkers on the 2-square and now there is one checker on the 20-square. Record this number story on the board.

$$10 \times 2 = 20$$

Repeat this activity with these configurations.



$= \quad 10 \times 20 = 200$



$= \quad 10 \times 40 = 400$

Exercise 3: Addition Algorithm

Write this problem on the board.

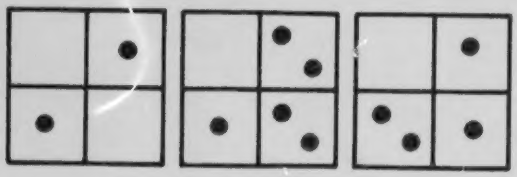
$$\begin{array}{r} 453 \\ +\,276 \\ \hline \end{array}$$

T: What number is 453 + 276?

Write some of the students' guesses on the board.

T: Let's do the calculation on the Minicomputer.

Invite someone to put 453 on the Minicomputer using red checkers and someone else to put 276 using blue checkers.

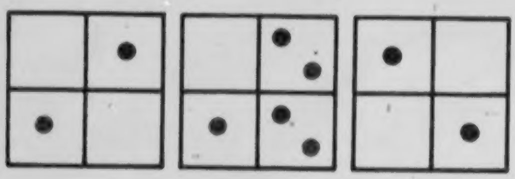


Considering only the checkers on the ones' board, point to the red checkers and then to the blue checkers as you say:

T: 3 (red) + 6 (blue) = . . . ?

S: 9.

Ask a student to make plays on the ones' board until this configuration of checkers is on the Minicomputer.



*

T: What numbers are in the ones' column of this addition problem?

S: 3 and 6.

T: 3 + 6 = . . . ?

---

* In this exercise, the checkers shown in black are those which are on the Minicomputer after certain plays are made; in actuality each of these checkers will be red or blue unless a checker of another color was used in a backward play.

119

N 7

S: 9.

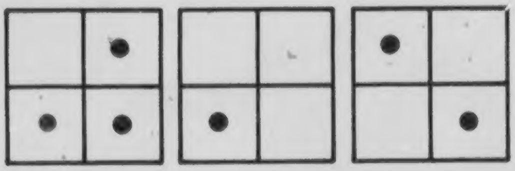
$$\begin{array}{r} 453 \\ +276 \\ \hline 9 \end{array}$$

Considering only the checkers on the tens' board, point to the red checkers and then to the blue checkers as you say:

T: 50 (red) + 70 (blue) = . . . ?

S: 120.

Ask a student to make the plays involving the checkers on the tens' board until this configuration of checkers is on the Minicomputer.



Emphasize that the "80 + 20 = 100" play results in another checker on the hundreds' board.

T: What numbers are in the tens' column of this addition problem?

S: 5 and 7.

T: Five tens plus seven tens is . . . ?

S: Twelve tens.

S: 120.

T: 120; one hundred and two tens. We record the one hundred in the hundreds' column and record the two tens in the tens' column.

$$\begin{array}{r} \overset{1}{4}53 \\ +276 \\ \hline 29 \end{array}$$

Point to the appropriate checkers on the hundreds' board as you say:

T: One hundred + four hundreds + two hundreds = . . . ?

S: Seven hundreds.

T: Do we need to make any plays?

S: No.

T: What numbers are in the hundreds' column of this addition problem?

S: 1, 4, and 2.

T: One hundred + four hundreds + two hundreds = . . . ?

S: Seven hundreds.

$$\begin{array}{r} \overset{1}{4}53 \\ +276 \\ \hline 729 \end{array}$$

Let the students determine which guess was closest to the answer.

N 7

Write the following problems on the board and ask the students to copy them on a piece of paper and to solve them. Allow the students a few minutes to work individually, then ask volunteers to solve the problems at the board and explain what they are doing.

$$\begin{array}{r} 745 \\ +193 \\ \hline \end{array} \qquad \begin{array}{r} 288 \\ +371 \\ \hline \end{array} \qquad \begin{array}{r} 4,376 \\ +2,492 \\ \hline \end{array}$$

You may wish to let students who finish quickly work in the ADDITION PROBLEMS booklet for five to ten minutes[5]. (The students began this booklet in UPG, Part I.)

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[5] The answer key for the ADDITION PROBLEMS booklet is included at the end of the description of Lesson N9, pages 73-76.

N8 The Functions +10 and -1

CAPSULE LESSON SUMMARY

Consider three arrow roads which start at the same number and which each have +10 and -1 arrows. Determine which road has the largest ending number and which has the smallest. Label the dots in all the roads. Do a similar activity with another three roads, but discover that the roads all have the same ending numbers. Introduce other arrows into the picture.

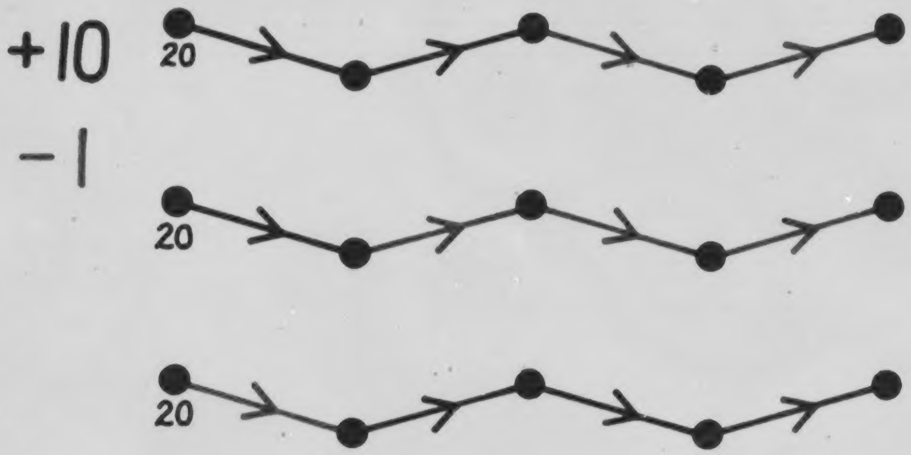
MATERIALS

Teacher: Colored chalk
Student: Colored pencils; paper; Worksheets N8* and **

DESCRIPTION OF LESSON

Exercise 1

Draw this arrow picture on the board.



T: The red arrows are for +10. The blue arrows are for -1. What do you notice about these arrow roads?

S: All the roads start at 20.

N8

S: Each road has four arrows.

T: Which road has the largest ending number?

S: The last road.

T: How do you know?

S: Because that road has the most +10 arrows.

Ask for a volunteer to label the dots in the last arrow road.

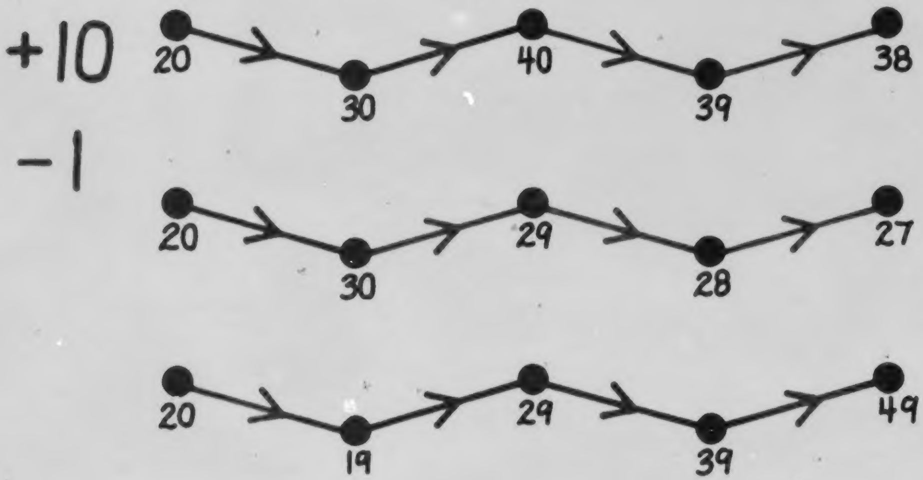
T: Which road has the smallest ending number?

S: The one in the middle.

T: Why?

S: Because it has the fewest +10 arrows.

Invite students to label the dots in the middle arrow road and then in the first one.



T: If we draw another arrow road which starts at 20 and has exactly four of these arrows, could we reach a number larger than 49? How?

S: Yes; use only +10 arrows.

T: What would the ending number be if we used four +10 arrows and started at 20?

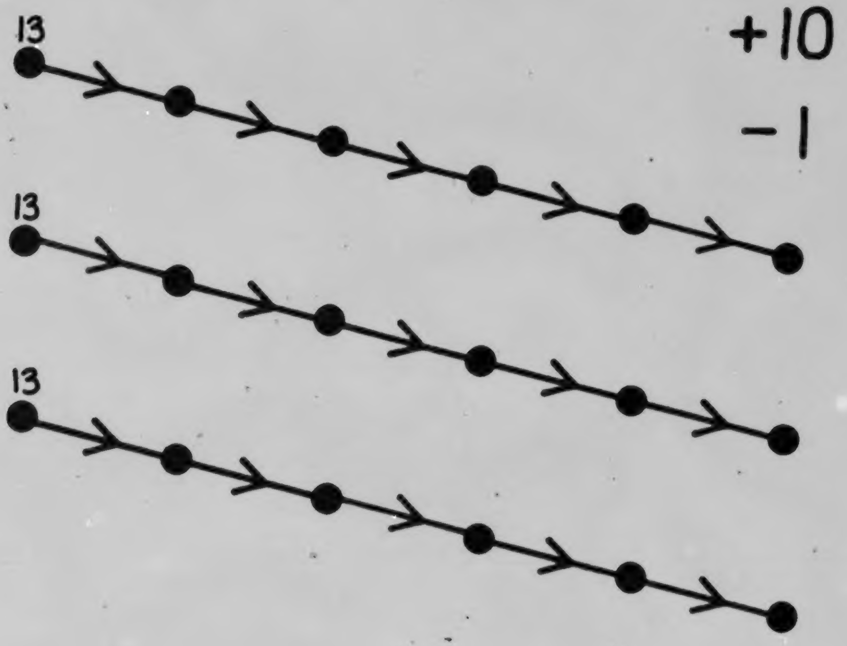
S: 60.

T: What is the smallest number we could reach with exactly four of these arrows? (16)
How could you reach 16?

S: Draw four -1 arrows. 20 - 4 = 16.

Exercise 2

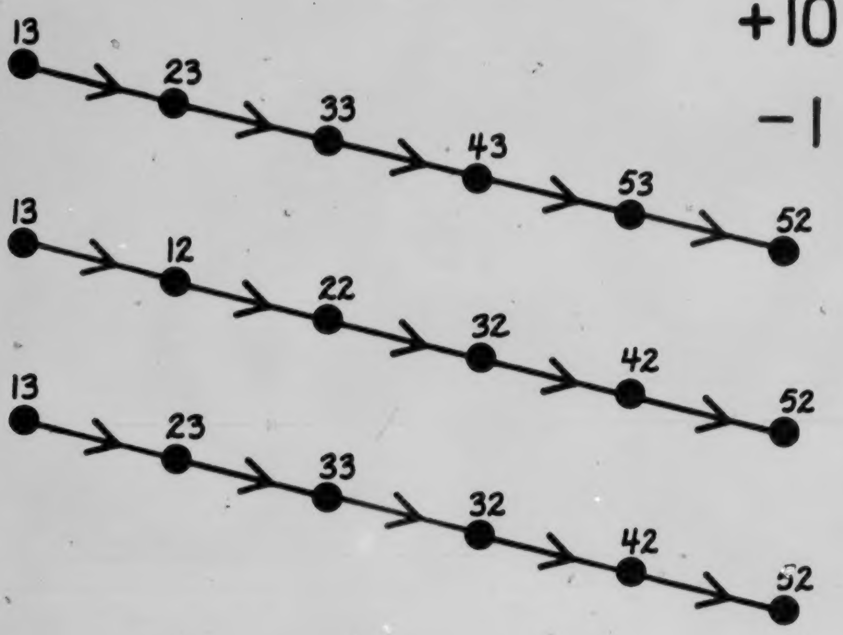
Erase the board and draw these arrow roads.



N 8

T: Which of these roads has the largest ending number?

Allow students to predict which road has the largest ending number; then ask them to copy the roads and to label all the dots. When many students have finished, ask three volunteers to label the dots on the roads at the board.

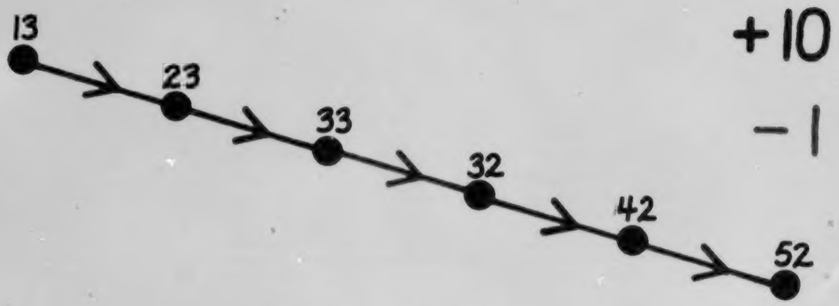

+10
−1
13
23
33
43
53
52
13
12
22
32
42
52
13
23
33
32
42
52

T: What do you notice about these three roads?

S: The ending numbers are the same.

S: They each have four red arrows and one blue arrow.

Erase the top two roads.


+10
−1
13
23
33
32
42
52

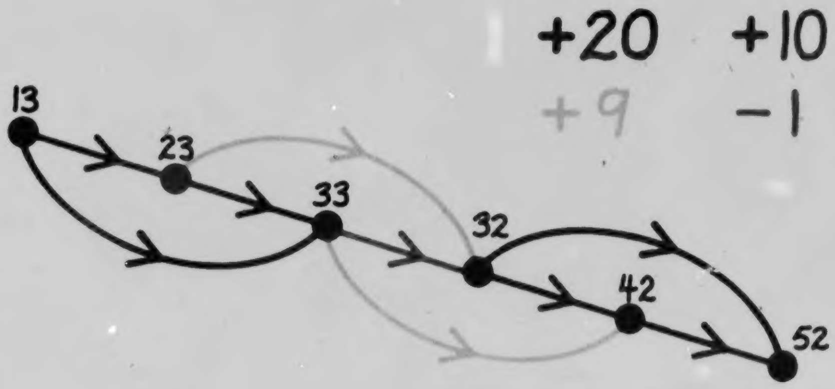
T: Who would like to add a different color arrow to our picture without drawing any more dots? Tell us what the arrow could be for.

Ask each student who volunteers to trace a possible arrow and to tell the class what the arrow could be for. There are many possibilities; some are listed below. Do not expect your class to find more than a few of them.

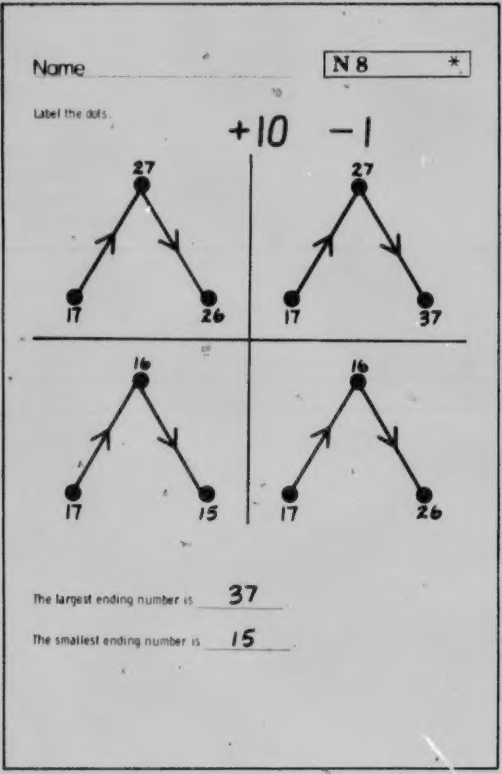
from 13 to 33	+ 20	from 33 to 13	- 20
from 13 to 32	+ 19	from 32 to 13	- 19
from 13 to 42	+ 29	from 42 to 13	- 29
from 13 to 52	+ 39	from 52 to 13	- 39
from 23 to 32	+ 9	from 32 to 23	- 9
from 23 to 42	+ 19	from 42 to 23	- 19
from 23 to 52	+ 29	from 52 to 23	- 29
from 33 to 42	+ 9	from 42 to 33	- 9
from 33 to 52	+ 19	from 52 to 33	- 19
from 32 to 52	+ 20	from 52 to 32	- 20

Afterwards write "+ 20" in green and "+ 9" in yellow on the board. Ask the students to draw all the green and yellow arrows in the picture.

Your picture should look like this one.



Worksheets N8* and ** are available for individual work.



Name ______________ **N 8** *

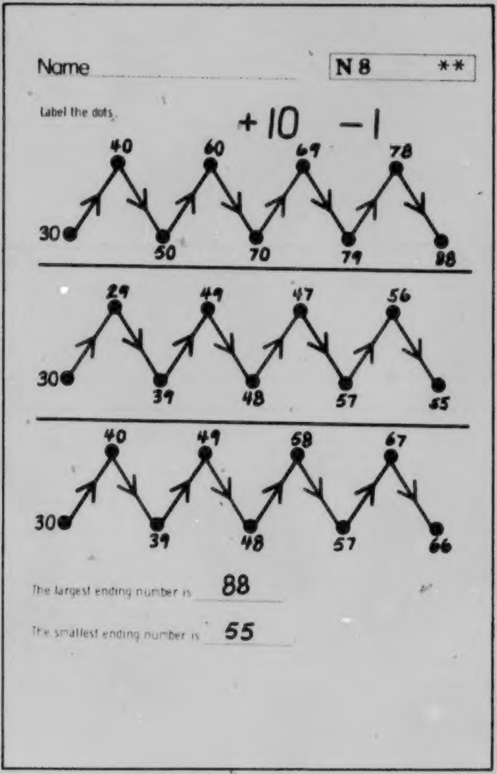
Label the dots.

+10 −1

27
17
26
27
17
37
16
17
15
16
17
26

The largest ending number is 37

The smallest ending number is 15



Name ______________ **N 8** **

Label the dots.

+10 −1

30
40
50
60
70
69
79
78
88
30
29
39
49
48
47
57
56
55
30
40
39
49
48
58
57
67
66

The largest ending number is 88

The smallest ending number is 55

N9 Addition Algorithm #2

CAPSULE LESSON SUMMARY

Given a configuration of ten checkers on the same square of the Minicomputer, make plays to support replacing them with one checker on the square of the same color on the next board to the left. Practice using the addition algorithm previously introduced to do addition calculations such as "576 + 287".

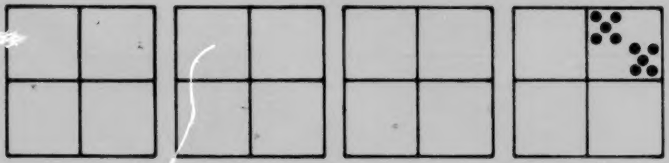
MATERIALS

Teacher: Minicomputer set
Student: ADDITION PROBLEMS booklet; paper; pencil

DESCRIPTION OF LESSON

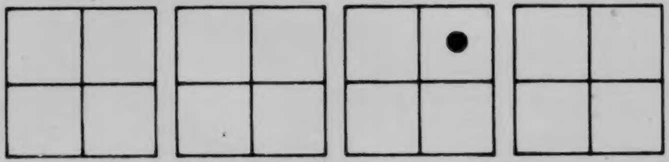
Exercise 1: Multiplying by 10

Put this configuration on the Minicomputer.



T: Ten checkers on the same square! What number is this?

Many students may recognize immediately that this number is 40 because 10 × 4 = 40. Ask for a volunteer to make plays until the standard configuration for 40 is obtained.

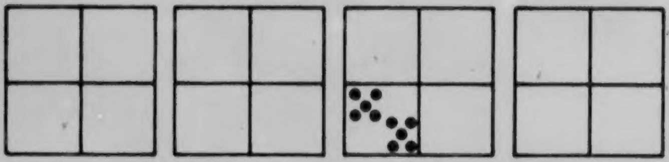


N 9

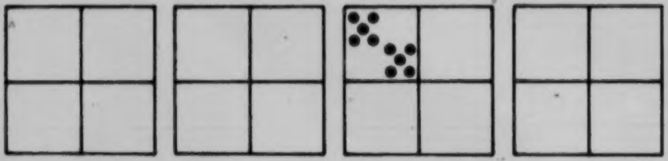
Emphasize that there were ten checkers on the 4-square and now there is one checker on the 40-square. Record this number story on the board.

$$10 \times 4 = 40$$

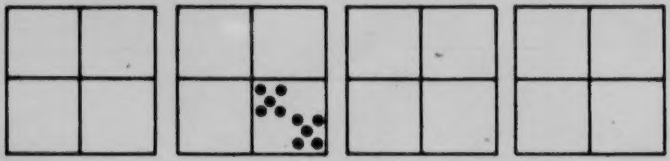
Repeat this activity with these configurations.



$= 10 \times 20 = 200$



$= 10 \times 80 = 800$



$= 10 \times 100 = 1{,}000$

When considering the third of the above configurations, emphasize that "ten hundred" is another name for 1,000. Ask a student to write "1,000" below the Minicomputer after the standard configuration for 1,000 is obtained.

Exercise 2: Addition Algorithm

Write this problem on the board.

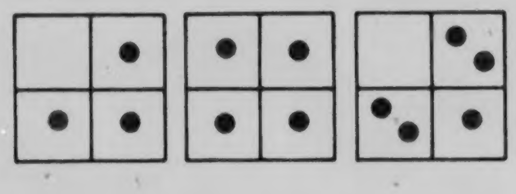
$$\begin{array}{r} 576 \\ +287 \\ \hline \end{array}$$

T: What number is 576 + 287?

Write some of the students' guesses on the board.

T: Let's do the calculation on the Minicomputer.

Invite someone to put 576 on the Minicomputer using red checkers and someone else to put 287 on the Minicomputer using blue checkers.



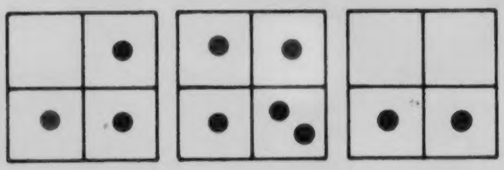
Considering only the checkers on the ones' board, point to the red checkers and then to the blue checkers as you say:

T: 6 (red) + 7 (blue) = . . . ?

S: 13.

S: We need to make some plays.

Ask a student to make the plays involving the checkers on the ones' board until this configuration of checkers is on the Minicomputer.



[*]

S: Now we have another checker on the tens' board.

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[*] In this exercise, the checkers shown in black are those which are on the Minicomputer after certain plays are made; in actuality each of these checkers will be red or blue or another color which might have been used in a backward play.

N9

T: What numbers are in the ones' column of this addition problem?

S: 6 and 7.

T: 6 + 7 = . . . ?

S: 13.

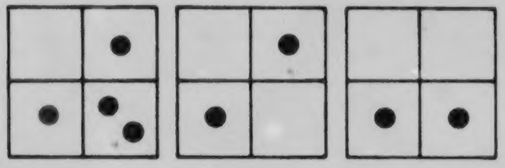
T: 13; one ten and three ones. We record the one ten in the tens' column and record the three ones in the ones' column.

$$\begin{array}{r} 1 \\ 576 \\ +287 \\ \hline 3 \end{array}$$

Point to the appropriate checkers on the tens' board as you ask:

T: What number is 80 + 70? (150) . . . and 10 more is? (160) So 80 + 70 + 10 is 160.

Invite a student to make the plays involving the checkers on the tens' board until this configuration of checkers is on the Minicomputer.



S: Now we have another checker on the hundreds' board.

T: What numbers are in the tens' column of this addition problem?

S: 1 and 7 and 8.

T: One ten + seven tens + eight tens is . . . ?

S: Sixteen tens.

S: 160.

T: 160; one hundred and six tens. We record the one hundred in the hundreds' column and record the six tens in the tens' column.

$$\begin{array}{r} \overset{1}{5}\overset{1}{7}6 \\ +287 \\ \hline 63 \end{array}$$

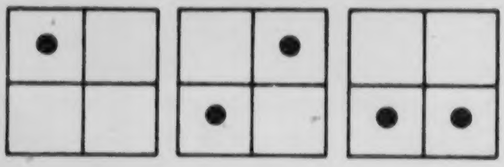
Point to the appropriate checkers on the hundreds' board as you ask:

T: 100 + 500 + 200 = . . . ?

S: 800.

S: We need to make some plays.

Ask a student to make the plays involving the checkers on the hundreds' board until this configuration of checkers is on the Minicomputer.



T: What numbers are in the hundreds' column of this addition problem?

S: 1 and 5 and 2.

T: One hundred + five hundreds + two hundreds = . . . ?

S: Eight hundreds.

$$\begin{array}{cccc} & 1 & 1 & \\ & 5 & 7 & 6 \\ + & 2 & 8 & 7 \\ \hline & 8 & 6 & 3 \end{array}$$

T: What number is on the Minicomputer? (863)

What number is 576 + 287? (863)

Let the students determine who had the best guess.

Write these problems on the board and ask the students to copy them on a piece of paper and to solve them. Allow several minutes for individual work, and then ask for volunteers to do the addition problems at the board and to explain what they are doing.

$$\begin{array}{r} 296 \\ +\,357 \\ \hline \end{array} \qquad \begin{array}{r} 488 \\ +\,376 \\ \hline \end{array}$$

Allow about 10 minutes for independent work in the ADDITION PROBLEMS booklet After the lesson, collect the booklets, check them, and store them for use in Lesson N25.

Addition Problems

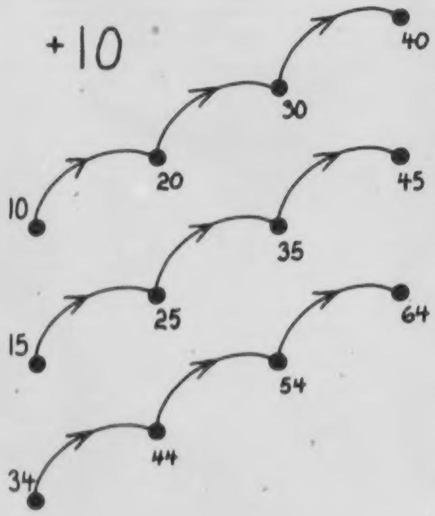
Calculate

22 +43 65	53 +21 74	62 +15 77
47 +12 59	36 +52 88	30 +38 68
43 +46 89	42 +26 68	53 +44 97

Calculate:

42 +31 73	47 +51 98	231 +423 654
51 +16 67	68 +21 89	143 +222 365
59 +30 89	56 +23 79	528 + 21 549

Label the dots.


+10
10
20
30
40
15
25
35
45
34
44
54
64

Calculate

50 + 14 = 64	25 + 24 = 49
65 + 13 = 78	17 + 50 = 67
12 + 72 = 84	27 + 52 = 79
42 + 36 = 78	35 + 24 = 59

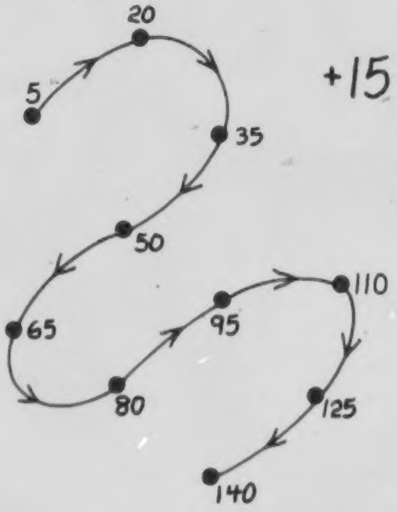
Calculate

63 +17 80	42 +20 62	350 +240 590
68 +24 92	65 +31 96	132 + 67 199
35 +45 80	48 +43 91	207 +177 384

Calculate

20 +36 56	42 +29 71	416 +322 738
19 +57 76	52 +27 79	321 +272 593
48 +32 80	66 +15 81	23 +356 379

Label the dots



Calculate

25 +49 74	42 +25 67	437 + 28 465
63 +12 75	27 +44 71	145 +333 478
24 +52 76	65 +19 84	364 +121 485

Calculate:

35 + 45 = 80

263 + 121 = 384

74 + 26 = 100

343 + 434 = 777

26 + 46 = 72

34 + 27 = 61

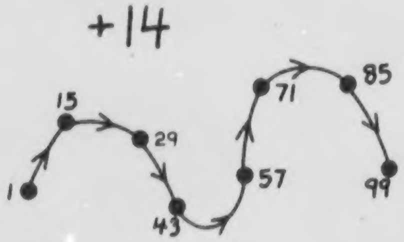
Calculate:

25 +46 71	62 +25 87	37 +28 65
63 +43 106	45 +53 98	47 +44 91
32 +56 88	55 +17 72	20 +76 96
48 +12 60	84 +50 134	59 +23 82

Calculate:

37 +17 54	58 +35 93	424 +355 779	746 + 45 791
37 +36 73	58 +60 118	123 +717 840	328 +533 861

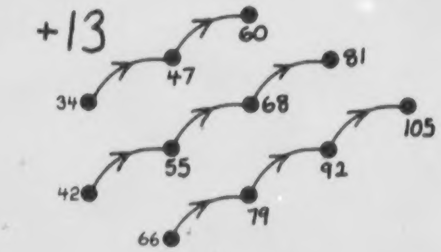
Label the dots.


+14
1
15
29
43
57
71
85
99

Calculate:

450 +139 589	263 +165 428	88 +25 113
356 +244 600	751 + 59 810	89 +88 177

Label the dots.


+13
34
47
60
68
81
42
55
92
105
66
79

Calculate:

271 + 68 339	532 + 516 1,048	258 + 742 1,000
777 + 213 990	74 + 643 717	199 + 144 343
520 + 299 819	426 + 574 1,000	355 + 97 452

Calculate:

234 345 + 213 792	422 255 + 125 802	372 338 + 355 1,065
2,323 56 + 341 2,720	17 455 + 1,255 1,727	8 218 + 88 314

125 + 355 + 465 = 945

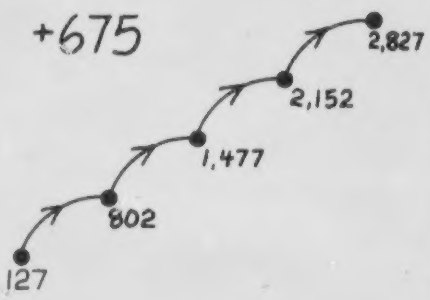
45 + 125 + 75 = 245

7 + 77 + 223 = 307

Calculate:

5,432 + 3,076 8,508	4,327 + 2,498 6,825	6,566 + 5,354 11,920

Label the dots.



N10 Midpoints on the Number Line

CAPSULE LESSON SUMMARY

Find the number halfway between two numbers on the number line. Make some consecutive +2 and +3 jumps on the number line and label the ending points of the corresponding arrows. Decide which numbers are ending points of both the +2 and +3 arrows. Play the Number Line Game.

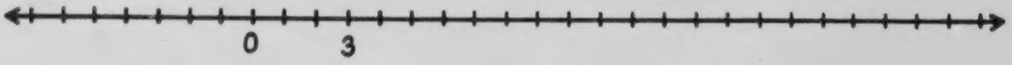
MATERIALS

Teacher : Colored chalk
Student : Worksheet N10

DESCRIPTION OF LESSON

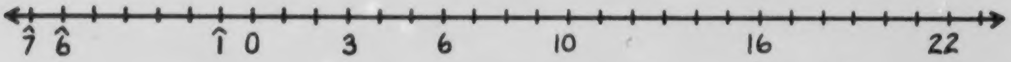
Exercise 1

Draw a line on the board with marks equally spaced at least six centimeters apart. Label one of the marks "0" and label the mark three spaces to the right of it "3".



Point to some marks on the number line in random order; include some marks to the left of 0. When you point to a mark, ask the students, "What number is here?" and then label the mark when the correct number is given. Continue until approximately ten marks are labeled.

Point to the middle of a space between two consecutive marks on the number line and ask what number is there. Repeat this activity if appropriate for your students. Do not draw or label any marks for these non-integer numbers.

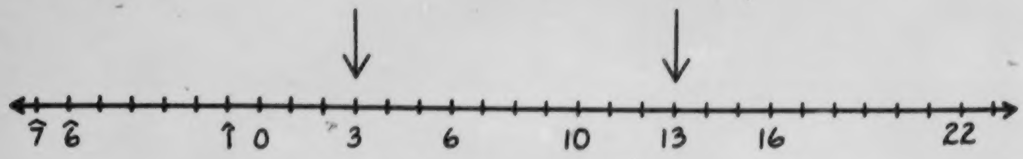


N 10

Distribute copies of Worksheet N10 for the students' reference.

T: Where is 3 on this number line?
Where is 13 on this number line?

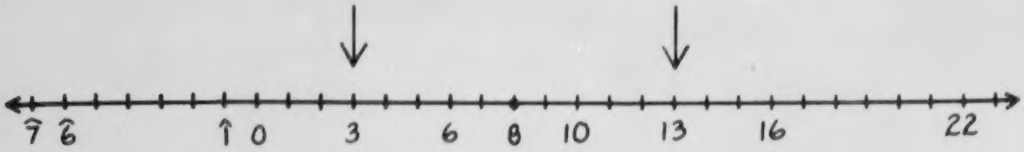
Let a student point to the marks for 3 and 13 on the number line on the board. Draw red arrows which point to the marks for 3 and 13. You may need to label the dot for 13.



Ask the students to find 3 and 13 on their number lines.

T: What are some numbers which are between 3 and 13 on the number line? (4, 5, 6, . . . 12)*
Which number is exactly halfway between 3 and 13 on the number line? (8)

Draw a large dot for 8 on the number line. If necessary, label the dot for 8.

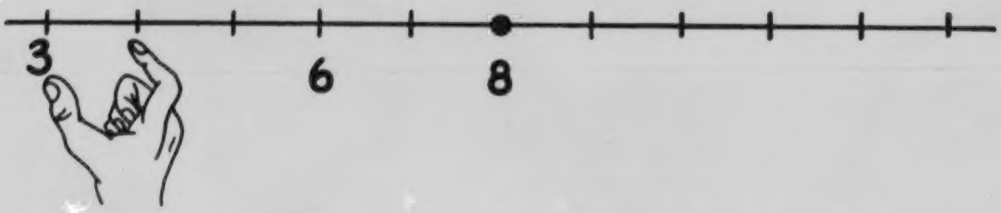


T: How can we be sure that 8 is exactly halfway between 3 and 13 on the number line?

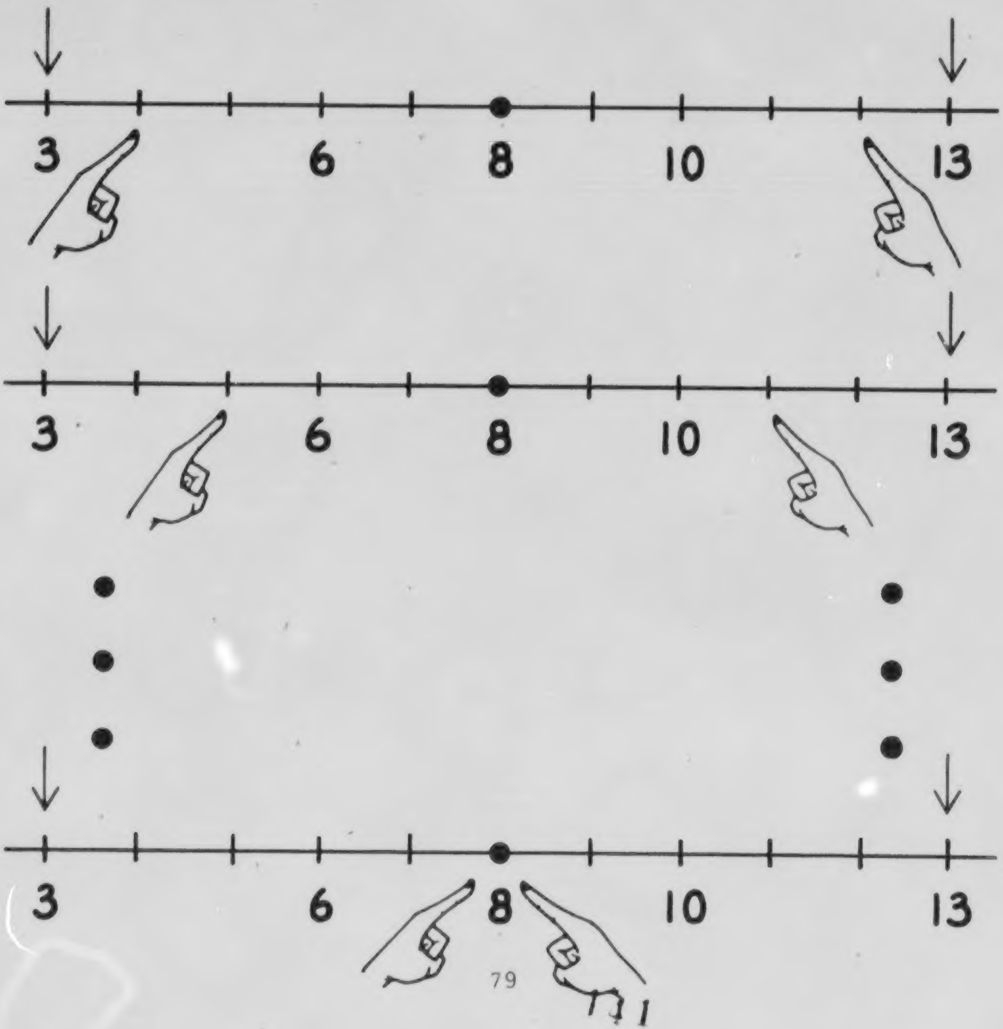
Encourage several students to make suggestions. If no one proposes counting the spaces from 3 to 8 and from 13 to 8, suggest this yourself. Ask the students to count the spaces using their thumbs and index fingers to show each space as they count. Conclude that 8 is the same distance from 3 and from 13 (five spaces).

---

* Of course there are also non-integer numbers between 3 and 13, but do not expect the students to mention these numbers.



Another way to show that 8 is exactly halfway between 3 and 13 on the number line is to point to "3" with your left index finger and to "13" with your right index finger. Then simultaneously move your index fingers toward each other in one unit steps until both fingers point to the same mark. This method is shown in the following illustration.



N 10

When the class is convinced that 8 is halfway between 3 and 13, erase the large dot and the colored arrows; then redraw the mark for 8.

Repeat this activity for other pairs of numbers where both numbers are even or where both numbers are odd.[*] You can use negative as well as positive numbers; you can choose numbers far apart on your number line or very close. These are some suggestions.

Find the number halfway between:

10 and 14 (12)
5 and 15 (10)
$\widehat{1}$ and 5 (2)
20 and 26 (23)
$\widehat{20}$ and $\widehat{10}$ ($\widehat{15}$)

NOTE: You may need to extend the number line on the board, but remember that the number line on the worksheet only extends from $\widehat{2}$ to 15.

Ask the students to find (using mental calculations) the number halfway between:

0 and 10 (5)
0 and 50 (25)
0 and 100 (50)
0 and 200 (100)
0 and 2 million (1 million)

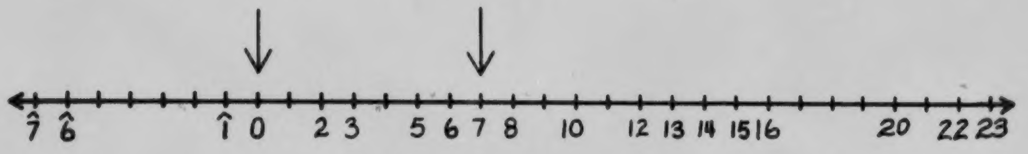
. . . or choose similar problems appropriate for your students' abilities and interests. Occasionally ask students to explain how they found a particular answer. Encourage the idea that the number halfway between a number and 0 is one-half of that number.

---

[*] The number halfway between an even number and an odd number is a non-integer number. A problem where one number is odd and one number is even is presented later in this lesson.

If your students are doing well with this exercise, continue by asking which number is halfway between 0 and 7. (A description follows.) You may choose to go on to Exercise 2 at this point.

Draw red arrows pointing to the marks for 0 and 7. You may need to label the dot for 7.



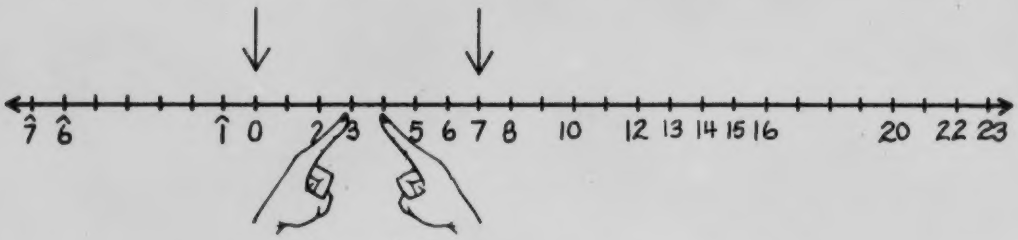
Ask the students to find 0 and 7 on their number lines.

T: Which number is halfway between 0 and 7?

Students might suggest incorrectly that 3 (or 4) is halfway between 0 and 7.

T: How could we check to see which number is exactly halfway between 0 and 7?

Perhaps your students will suggest pointing to "0" and "7" and moving your fingers toward one another simultaneously. In this case, stop when you are pointing to the marks for 3 and 4.



T: What should I do now?

S: Move each of your fingers in just a little bit more.

T: What number is halfway between 0 and 7?

S: It is the number halfway between 3 and 4.

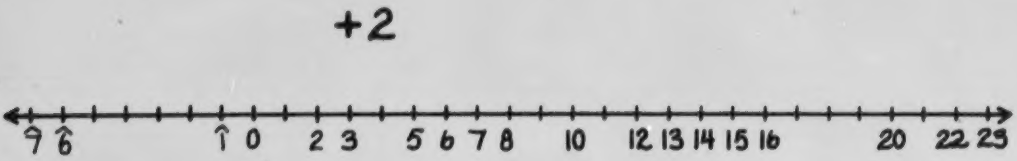
S: $3\frac{1}{2}$ (read: three and one-half).

You may need to suggest $3\frac{1}{2}$ yourself. Draw and label a mark for $3\frac{1}{2}$ on the number line. If no one suggests counting the spaces from 0 to $3\frac{1}{2}$ and from 7 to $3\frac{1}{2}$ suggest this yourself. Conclude that $3\frac{1}{2}$ is $3\frac{1}{2}$ spaces from 0 and from 7.

Erase the mark for $3\frac{1}{2}$ before continuing with Exercise 2.

Exercise 2

Erase everything on the board except the number line. On the board write "+2" above the number line.



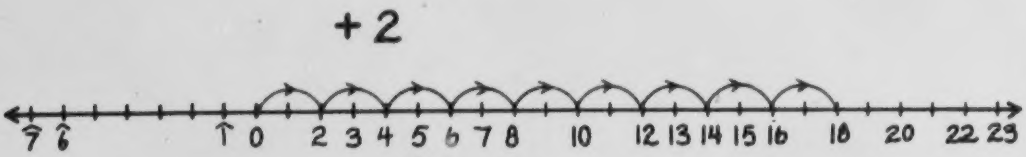
T: If we start at 0 and draw a +2 arrow, what number will we land on? (2)

If we draw two more +2 arrows, what is the last number we will land on? (6)

If we draw three more arrows . . . ? (12)

And three more arrows . . . ? (18)

As the students answer the above questions, draw the appropriate arrows above the number line and label any ending points not already labeled.



T: Do you see a pattern?

S: We skip every other number.

S: We only land on even numbers.

T: What are some numbers we would land on if we drew more red arrows?

Let students make suggestions.

T: Would we land on 100? (Yes)
. . . 101? (No)
. . . 528? (Yes)
. . . 825? (No)
. . . 1,000,000 (Yes)

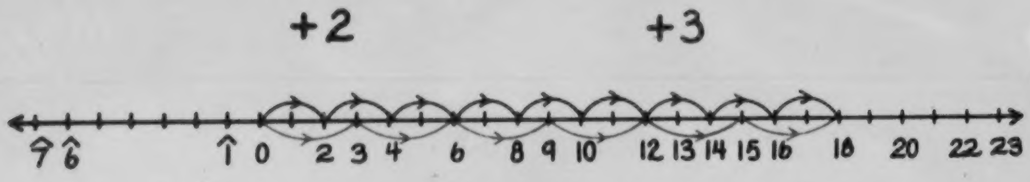
Vary these questions according to your students' abilities and interests.

T: How many spaces did we move in two +2 jumps? (Four spaces)
How many spaces did we move in five +2 jumps? (Ten spaces)
How many spaces did we move in nine +2 jumps? (Eighteen spaces)

Write "+3" above the number line.

T: If we start at 0 and draw a +3 arrow, what number do we land on? (3)
If we draw another +3 arrow, what number do we land on? (6)
If we draw two more +3 arrows, what is the last number we land on? (12)
And two more +3 arrows . . . ? (18)

As the students answer the above questions, draw the appropriate arrows below the number line.



T: When we made +3 jumps, what numbers did we land on?
(0, 3, 6, 9, 12, 15, 18)
What are some numbers we would land on if we drew some more blue arrows? (21, 24, 27, 30, . . .)
Which numbers did we land on with both the +2 arrows and the +3 arrows? (0, 6, 12, 18)

Exercise 3: The Number Line Game

Play the Number Line Game with a secret number between 0 and 200. Repeat this activity two or more times. Then play the Number Line Game with a secret number between $\widehat{50}$ and 50 one or more times. A description of the Number Line is given in Lesson N4 Consecutive Jumps on the Number Line, Exercise 3.

N11 Doubling and Halving #3

CAPSULE LESSON SUMMARY

Consider a configuration on the Minicomputer for some number in which the checkers are grouped in pairs and in which both checkers in each pair are on the same square. Find one-half of the number and then the number. Record the situation in an arrow picture. Put a number on the Minicomputer, calculate one-half of it, and discover the answer is the present year.

MATERIALS

Teacher: Minicomputer set
Student : None

DESCRIPTION OF LESSON

Exercise 1

Begin this lesson with five minutes of mental arithmetic doing calculations involving the functions 2 × and $\frac{1}{2}$ ×; for example, ask students to calculate:

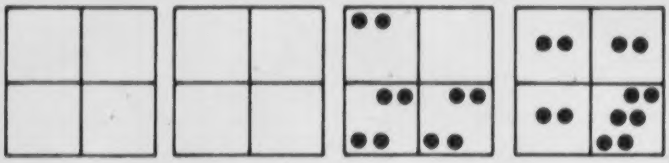
2×10 (20) 2×12 (24) 2×15 (30)

$\frac{1}{2} \times 20$ (10) $\frac{1}{2} \times 24$ (12) $\frac{1}{2} \times 30$ (15)

NOTE: "$\frac{1}{2}$ ×" should be read "one-half of"; for example, "$\frac{1}{2} \times 20$" is read "one-half of twenty."

Exercise 2

Put this configuration on the Minicomputer.



N 11

T: What can you tell me about this number?
Is it more than 50? How do you know?
Is it more than 100? How do you know?
Is it more than 200? How do you know?
Is it more than 300? How do you know?
Is it more than 1,000? How do you know?

Let the students react freely to these questions which should help them progressively arrive at a good estimate of the number on the Minicomputer. Do not ask the students to tell you exactly what this number is yet.

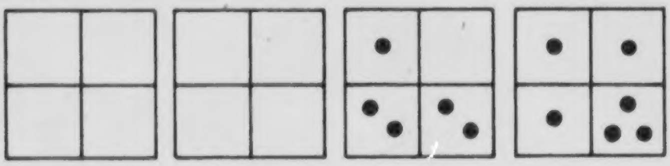
T: Can you tell me what one-half of this number is?

S: There are too many checkers.

T: How can we find one-half of this number?

S: Take off some checkers.

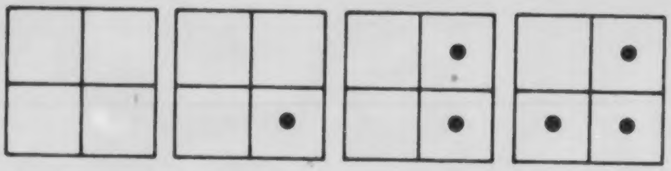
Invite someone to show one-half of this number by taking some checkers off the Minicomputer. Emphasize that the checkers are in pairs and that the student is removing one checker from each pair of checkers on a square.



T: This is one-half of the number that was on the Minicomputer.
What would make it easier to read?

S: Make some plays.

Invite students to make plays until the number is in standard configuration.

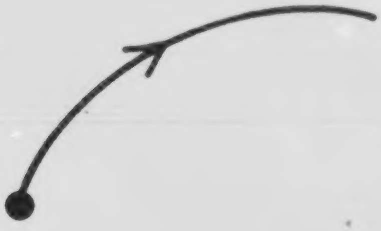


= 157

Draw a large dot on the board near the Minicomputer as you say:

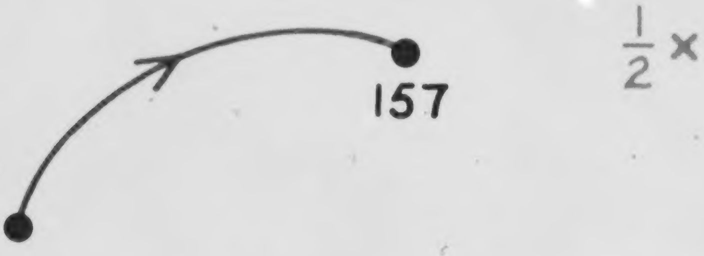
T: This dot is for the number we had on the Minicomputer before we took off one-half of it.

Draw a red arrow starting from that dot.



T (drawing the ending dot of the red arrow): This dot is for one-half of that number, which we found to be 157.

Write "$\frac{1}{2}\times$" near the arrow picture and then label the ending dot "157".



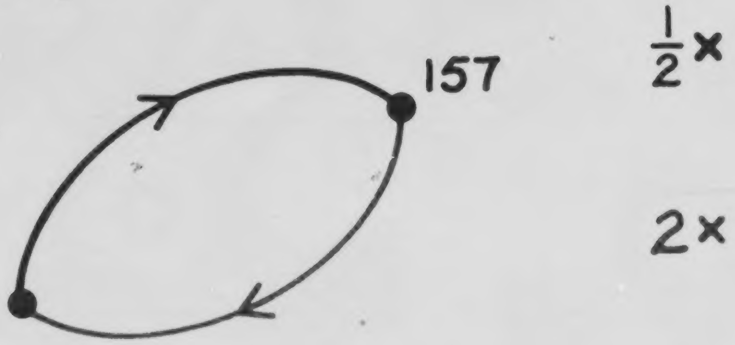
N 11

Draw the return arrow in blue.

T: What could the blue arrow be for?

S: 2×.

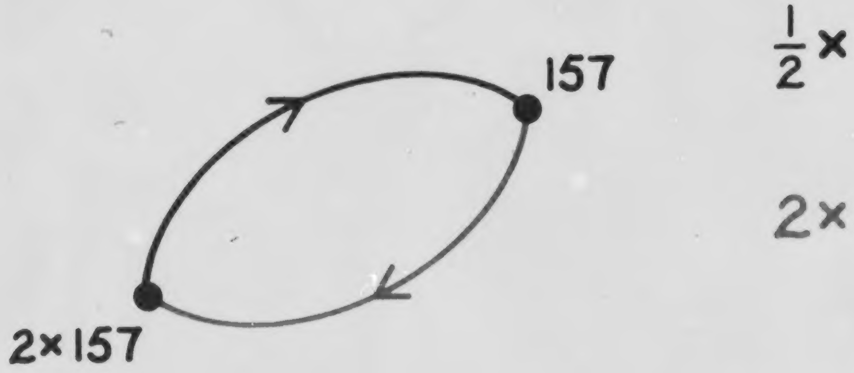
Write " 2× " under " $\frac{1}{2}$× ".


157
$\frac{1}{2}$×
2×

Trace the blue arrow.

T (pointing to the dot on the left): So the number on the Minicomputer is 2 × 157.

Label the dot on the left " 2 × 157".


157
$\frac{1}{2}$×
2×
2×157

T: How can we calculate "2 × 157"?

Accept and follow any correct method your class agrees upon using. Perhaps they will suggest one of these methods.

1) Do mental calculations; for example,

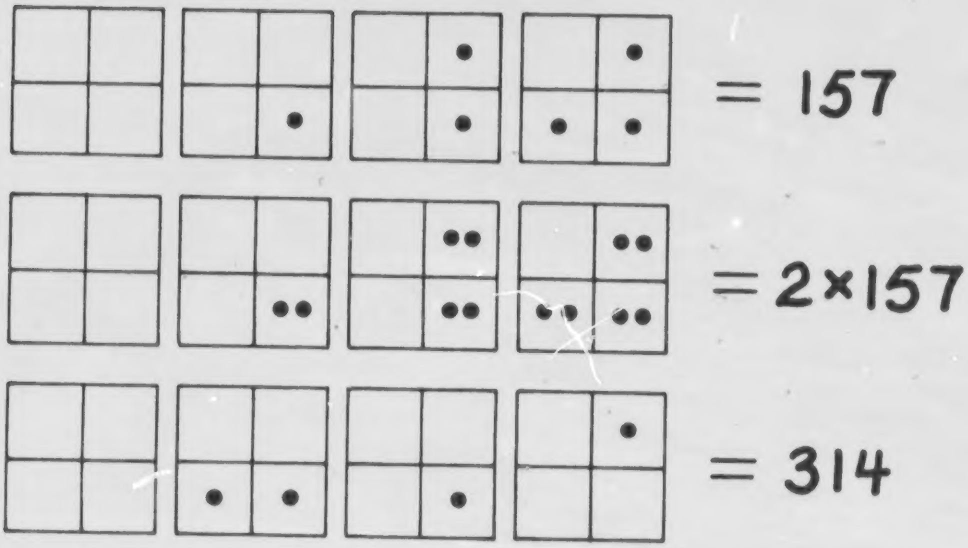
$$\begin{aligned} 2 \times 100 &= 200 \\ 2 \times 50 &= 100 \\ 2 \times 7 &= 14 \end{aligned}$$

$$200 + 100 + 14 = 314$$

2) Use the addition algorithm:

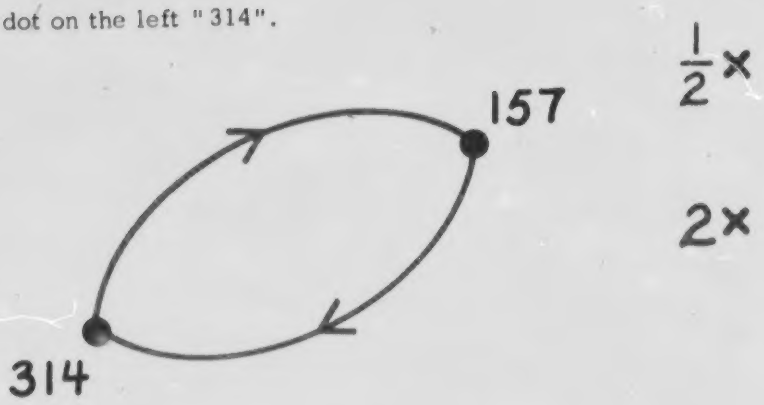
$$\begin{array}{r} {\scriptstyle 1\,1} \\ 157 \\ +\,157 \\ \hline 314 \end{array}$$

3) Use the Minicomputer: put another 157 on the Minicomputer and then make plays until the answer is in standard configuration.



N 11

Label the dot on the left "314".



T: What number stories are told by this arrow picture?

S: "2 x 157 = 314".

S: "$\frac{1}{2}$ x 314 = 157".

Exercise 3

NOTE: Calculate "2 x (the present year)" yourself and write only the resulting number on the board. In the lesson description, we will assume the present year is 1979 and will use "2 x 1979 = 3,958".

3,958

T: Who can read this number?

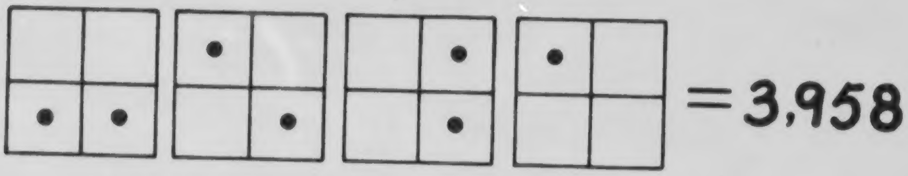
Allow several students to read this number aloud.

Indicate the calculation "$\frac{1}{2}$ x 3,958".

$$\frac{1}{2} \times 3{,}958 =$$

T: Do you think that $\frac{1}{2} \times 3{,}958$ is more than 1,000? (Yes)
Is it more than 2,000? (No)
What number do you think $\frac{1}{2} \times 3{,}958$ is?

Record some of the guesses on the board and then suggest using the Minicomputer to find the answer. Invite someone to put 3,958 on the Minicomputer.



T: Is it easy to decide what one-half of this number is by looking at the checkers?

S: No; we need to put the checkers in pairs.

T: How can we put these checkers in pairs without changing the number?

S: Make backward plays.

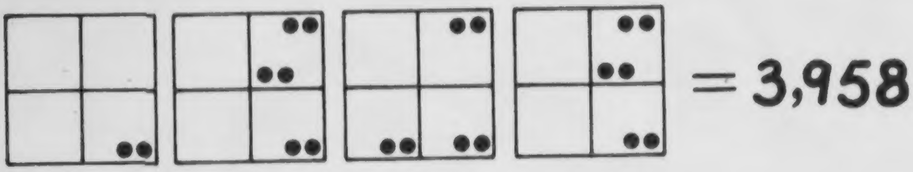
Invite someone to make a backward play. Any backward play which could be made except "100 = 80 + 20" results in a pair of checkers on one of the squares.

T: We now have a pair of checkers so we won't make any more plays with these checkers. Can someone make a play which will give us another pair of checkers?

Continue this activity until all the checkers are in pairs and the checkers in each pair are on the same square. Whenever you get a pair of checkers on one of the squares, push them into a corner of that square. Discourage any student who wants to make a play with checkers which are already paired. Tell the student it is a correct play but not very useful for creating pairs of checkers.

N 11

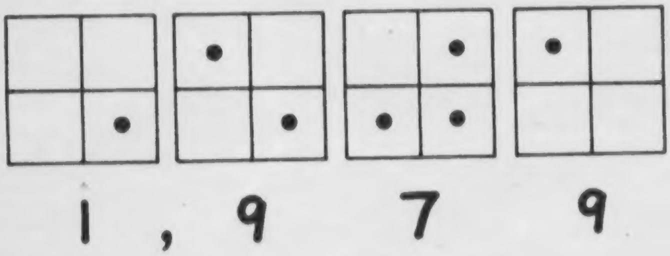
It is possible that your students will have this configuration when all the checkers are paired.



T: All the checkers are in pairs. What should we do if we want to calculate "$\frac{1}{2}$ × 3,958"?

Students very likely will suggest taking off half of the checkers. If so, emphasis that they should take off half of the checkers on each square. After the checkers are removed, erase "=3958" and let the students make any forward plays needed to obtain the standard configuration for 1,979.

Ask for volunteer to write the number below the Minicomputer.



T: Who can read this number?

S: One thousand, nine hundred, seventy-nine.

If no one recognizes that this number is the present year, write it again to one side of the Minicomputer without the comma.

Ask a student to complete the number story on the board.

$$\frac{1}{2} \times 3{,}958 = 1{,}979$$

Allow the students to determine which guess was closest to 1,979.

N12 Roads #2

CAPSULE LESSON SUMMARY

Consider four arrow roads which start at the same number and which each have three +1 arrows and one 2× arrow. Observe that the order in which the arrows occur affects the ending number of the road. Build a road from 0 to 21 using 2× and +1 arrows.

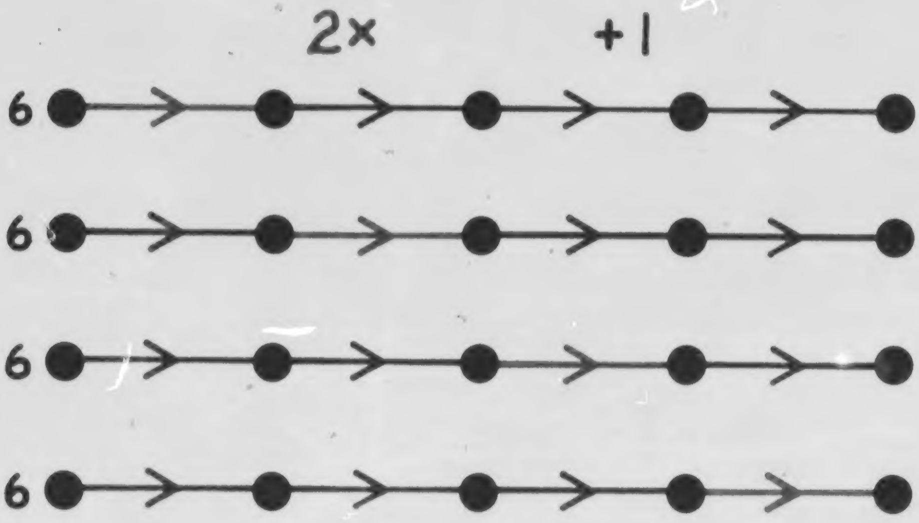
MATERIALS

Teacher: Colored chalk
Student: Colored pencils; Worksheets N12* and **

DESCRIPTION OF LESSON

Exercise 1

Before the lesson begins, draw this arrow picture on the board.



T: Look at the four arrow roads on the board. How are these roads alike?

S: They all have four arrows.

S: They all start at 6.

S: They all have three red arrows and one blue arrow.

T: What is different about these roads?

S: The blue arrow is in a different place each time.

T: Which of these roads has the largest ending number?

Some students may think that all the roads end at the same number. Ask these students to look again at the arrow picture.

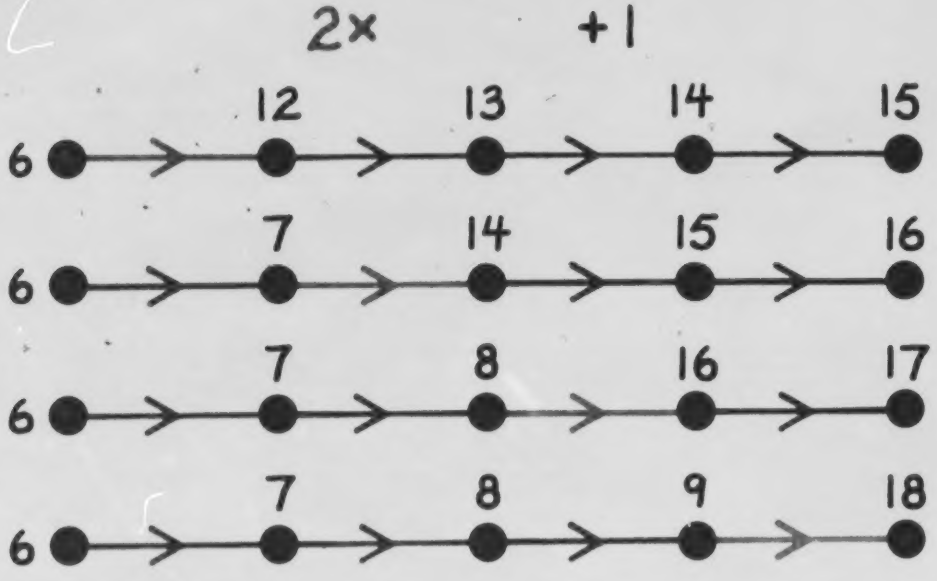
S: The last one — it ends in 18.

Ask a student to label the dots in the last arrow road.

T: Which of these roads has the smallest ending number?

S: The first one — it ends in 15.

Ask for volunteers to label the dots in the first three roads.



T: Each of these roads has three +1 arrows and one 2x arrow and they all start at 6. Why don't they all end at the same number?

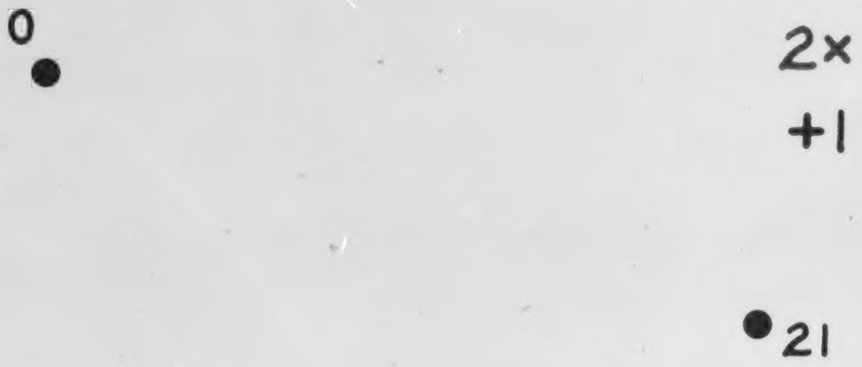
Do not expect the students' explanations to be well-formed; accept their attempts without comment. Perhaps one of your students will give this or a similar explanation.

S: In each arrow road we double a different number and that changes the ending number.

Erase the board before going on to Exercise 2.

Exercise 2

Write this information on the board.



T: We are going to build a road from 0 to 21 with 2x and +1 arrows. Which kind of arrow do you want to use first?

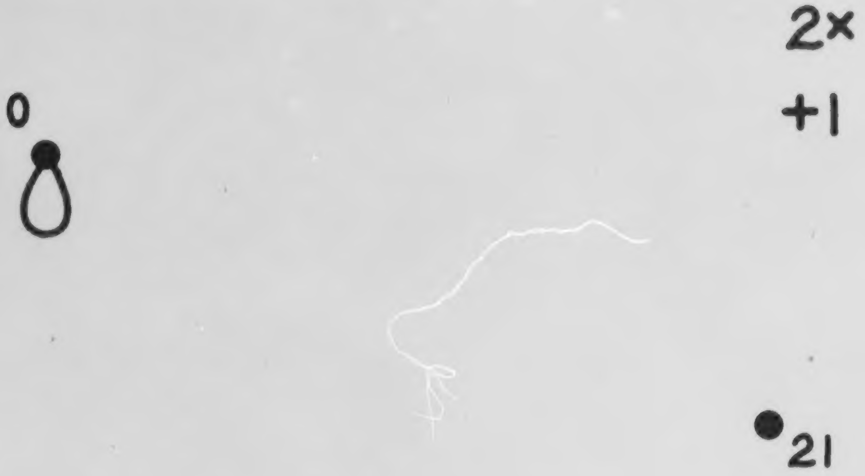
Accept the suggestion of one of your students. A possible dialogue is given here.

S: Use a blue arrow first.

T: The blue arrows are for 2x. What number is 2 x 0? How should I draw the arrow?

N 12

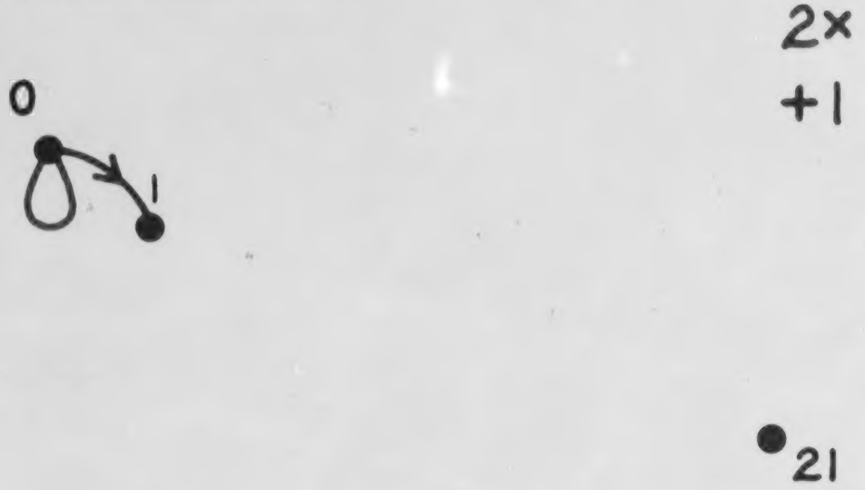
S: You need to draw a loop, because 2 × 0 = 0.



T: Which kind of arrow do you want to use now?

S: A red arrow. 0 + 1 = 1.

Draw a red arrow which starts at 0.



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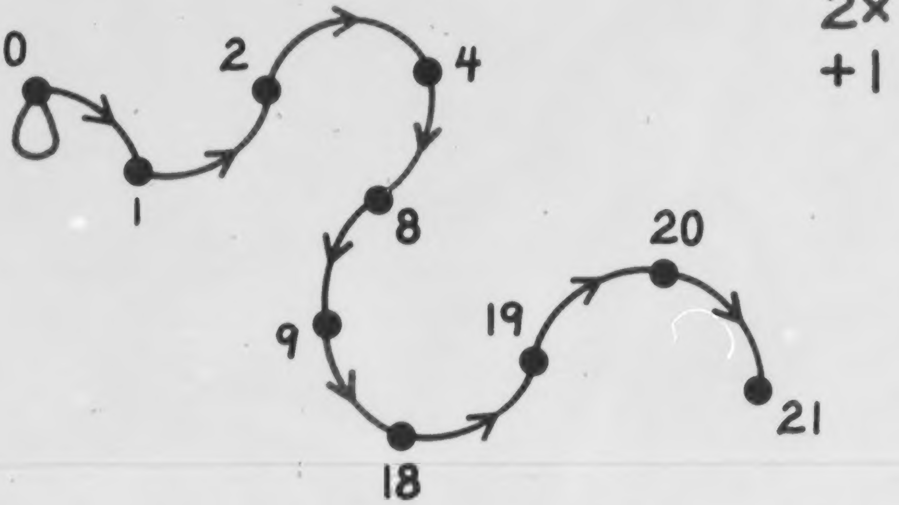
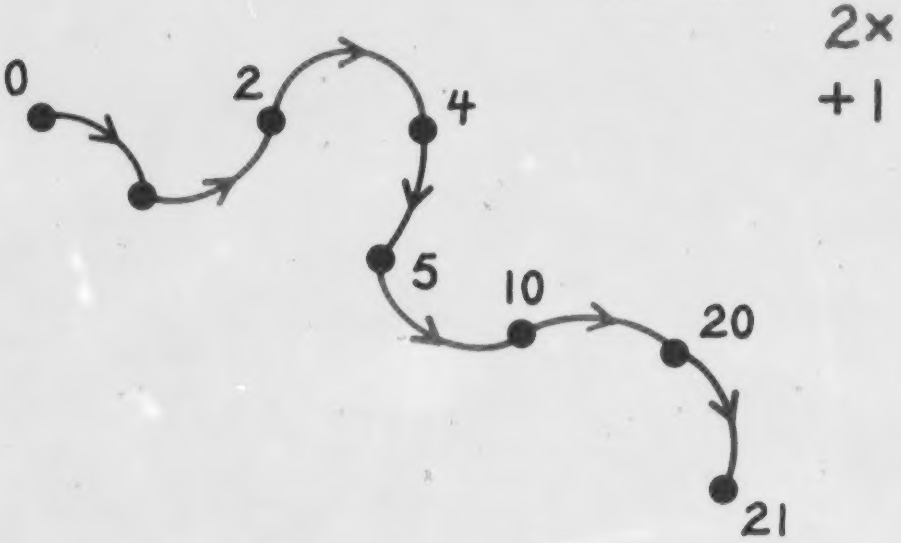
Continue building the red-blue road from 0 to 21 until it is complete. There are many possible solutions. One solution is given here.



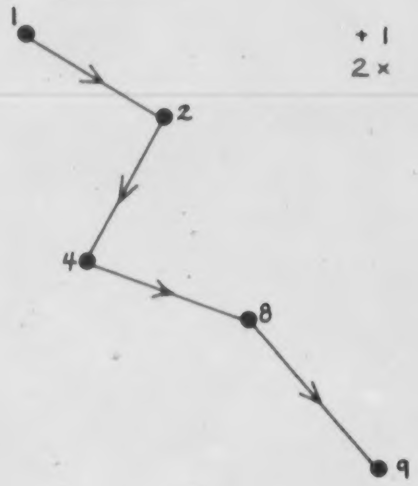
If this activity was easy for your class, you may wish to ask if they could build a shorter road. The shortest possible road is shown below.



Worksheets N12* and ** are available for individual work. Students who finish quickly should be encouraged to build a road with 2× and +1 arrows from 0 to 100.

Name ______________ **N 12 ***

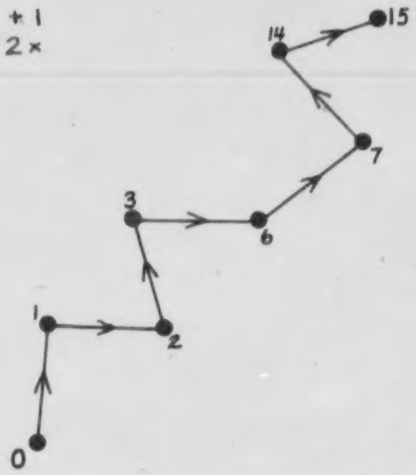
Build a red-blue road from 1 to 9.



This road has the fewest possible number of arrows. Other solutions are possible.

Name ______________ **N 12 ****

Build a red-blue road from 0 to 15.



This road has the fewest possible number of arrows. Other solutions are possible.

N13 Doubling and Halving #4

CAPSULE LESSON SUMMARY

Do some mental arithmetic involving the functions $2\times$ and $\frac{1}{2}\times$. Calculate one-half of some numbers with the help of the Minicomputer.

MATERIALS

Teacher: Minicomputer set
Student: Worksheets N13 *, **, ***, and ****; pencil; Minicomputer set (optional)

DESCRIPTION OF LESSON

Exercise 1

Begin the lesson with some mental arithmetic.

T: What number is 2 × 2? (4)
What number is 2 × 20? (40)
What number is 2 × 200? (400)
What number is 2 × 2,000? (4,000)
What number is 2 × 2 million? (4 million)

What number is 2 × 10? (20)
What number is 2 × 3? (6)
2 × 10 = 20 and 2 × 3 = 6, so what number is 2 × 13? (26)
What number is 2 × 100? (200)
What number is 2 × 103? (206)

Continue this activity with calculations which are appropriate for the abilities of your students. After a few minutes change to mental arithmetic problems involving the function $\frac{1}{2}\times$ (read: one-half of).

N 13

T: What number is $\frac{1}{2} \times 10$? (5)

What number is $\frac{1}{2} \times 100$? (50)

What number is $\frac{1}{2} \times 1000$? (500)

What number is $\frac{1}{2} \times 10{,}000$? (5,000)

What number is $\frac{1}{2} \times 40$? (20)

What number is $\frac{1}{2} \times 6$? (3)

$\frac{1}{2} \times 40 = 20$ and $\frac{1}{2} \times 6 = 3$, so what number is $\frac{1}{2} \times 46$? (23)

What number is $\frac{1}{2} \times 400$? (200)

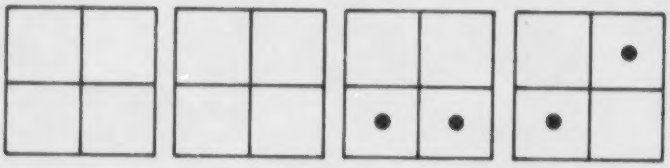
What number is $\frac{1}{2} \times 406$? (203)

Continue with other "$\frac{1}{2}\times$" calculations.

$\frac{1}{2} \times 20$ (10)
$\frac{1}{2} \times 30$ (15)
$\frac{1}{2} \times 40$ (20)
$\frac{1}{2} \times 50$ (25)
$\frac{1}{2} \times 60$ (30)

$\frac{1}{2} \times 6$ (3)
$\frac{1}{2} \times 16$ (8)
$\frac{1}{2} \times 26$ (13)
$\frac{1}{2} \times 36$ (18)

Extend the last sequence until your class is uncertain of the correct answer to one of your questions. Perhaps a student will suggest using the Minicomputer to calculate the answer to that question; otherwise, suggest using it yourself. For the purpose of this lesson description, we will assume the class had difficulty calculating "$\frac{1}{2} \times 36$" mentally. Ask a student to put 36 on the Minicomputer.



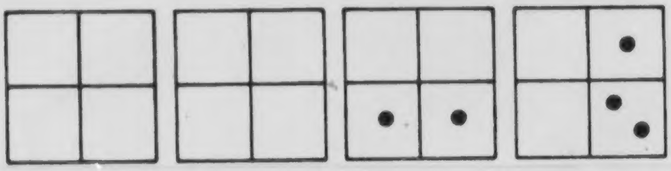
T: How can we calculate "$\frac{1}{2} \times 36$" on the Minicomputer?

S: Put the checkers in pairs and then on each square take off one out of each pair.

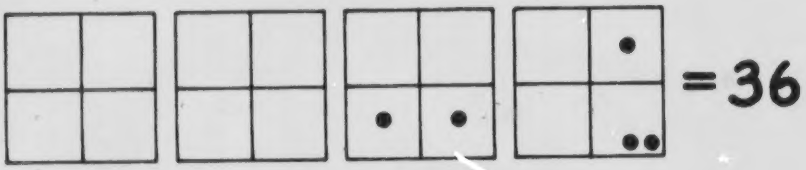
S: We need to make some backward plays.

T: Who can make a backward play so we will have two checkers on the same square?

One possibility is that a student will make the play "2 = 1 + 1".

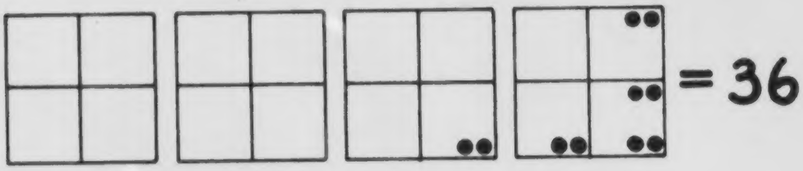


T: Now we have two checkers on the 1-square. I'm going to put these two checkers into the corner of the 1-square because they make a pair.



T: Who can make a play which will help us get another pair of checkers?

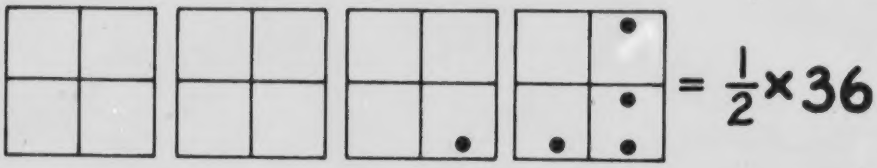
Continue this activity until all the checkers are paired.



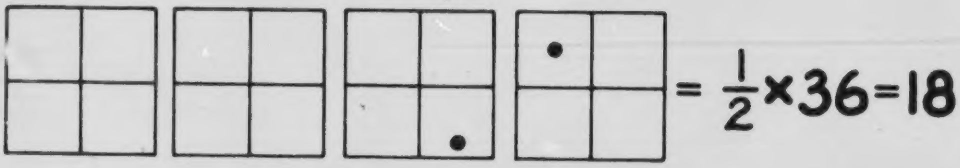
N 13

Erase "36" and write "$= \frac{1}{2} \times 36$".

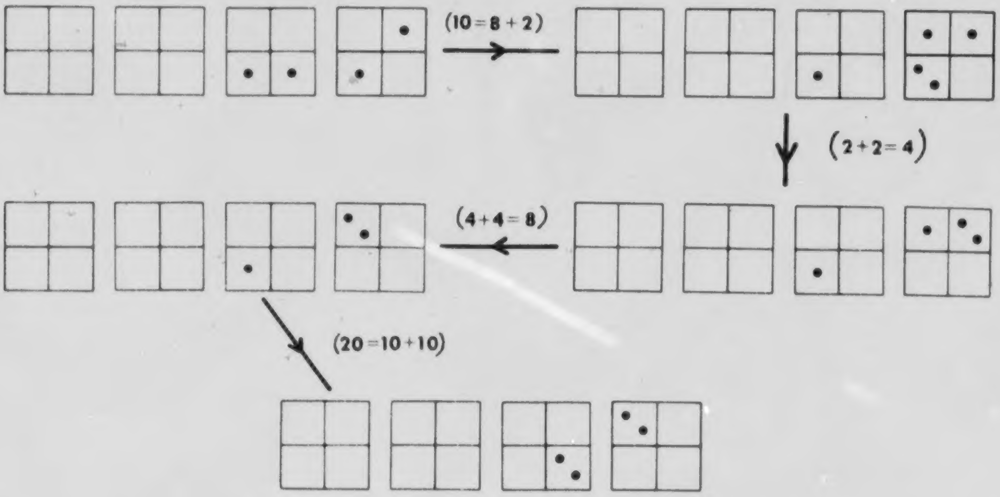
T: All the checkers are in pairs. Who can remove some checkers so that one-half of this number will be on the Minicomputer?


$= \frac{1}{2} \times 36$

If necessary, ask some students to make plays until the standard configuration for 18 is obtained. Write "= 18" next to "$\frac{1}{2} \times 36$".


$= \frac{1}{2} \times 36 = 18$

NOTE: If is also possible to do some backward and some forward plays to get pairs of checkers. For example,


(10 = 8 + 2)
(2 + 2 = 4)
(4 + 4 = 8)
(20 = 10 + 10)

Exercise 2

Write this problem on the board.

$$\frac{1}{2} \times 1{,}310 =$$

T: Is $\frac{1}{2}$ × 1,310 more or less than 100? How do you know?

S: It is more than 100 because 2 × 100 = 200.

T: Is $\frac{1}{2}$ × 1,310 more than 500?

S: Yes, because 2 × 500 = 1,000.

T: Is it more than 1,000?

S: No; 2 × 1,000 = 2,000.

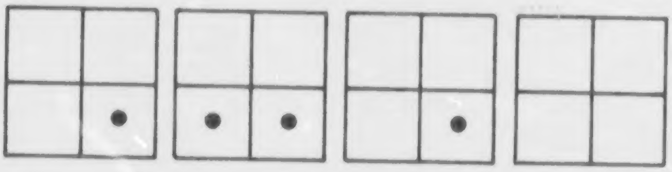
T: So we know that $\frac{1}{2}$ × 1,310 is between 500 and 1,000. What number do you think $\frac{1}{2}$ × 1,310 is?

Record the students' guesses on the board.

T: How can we calculate "$\frac{1}{2}$ × 1,310"?

S: Use the Minicomputer.

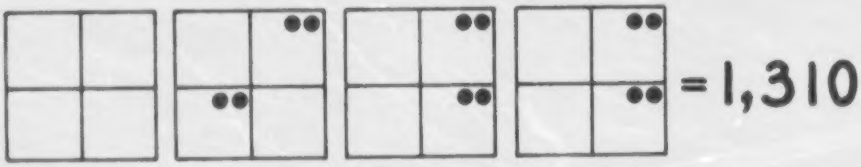
Ask a student to put 1,310 on the Minicomputer and record "=1,310" to the side of the Minicomputer.



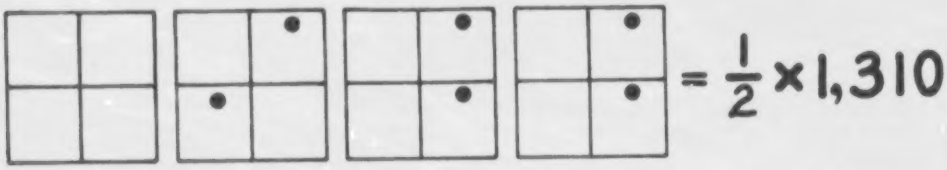
= 1,310

N 13

Ask students to make backward plays to get pairs of checkers as in the next illustration. Whenever a pair of checkers is obtained on a square, push them to the corner of that square to discourage students from making another play with those checkers.



Invite someone to take off some checkers so that one-half of the number is on the Minicomputer.



T: What number is $\frac{1}{2} \times 1{,}310$?

S: 655.

Complete the number story and let the students determine which guess was the closest.

Worksheets N13*, **, ***, and **** are available for individual work. Allow students to use individual Minicomputers if they wish.

Name ____________ **N 13** *

$\frac{1}{2} \times 4 =$ 2

$\frac{1}{2} \times 6 =$ 3

$\frac{1}{2} \times 8 =$ 4

$\frac{1}{2} \times 2 =$ 1

$\frac{1}{2} \times 20 =$ 10

$\frac{1}{2} \times 200 =$ 100

$\frac{1}{2} \times 12 =$ 6

$\frac{1}{2} \times 14 =$ 7

$\frac{1}{2} \times 16 =$ 8

$\frac{1}{2} \times 10 =$ 5

$\frac{1}{2} \times 40 =$ 20

$\frac{1}{2} \times 100 =$ 50

Name ____________ **N 13** **

$\frac{1}{2} \times 24 =$ 12

$\frac{1}{2} \times 28 =$ 14

$\frac{1}{2} \times 26 =$ 13

$\frac{1}{2} \times 42 =$ 21

$\frac{1}{2} \times 48 =$ 24

$\frac{1}{2} \times 82 =$ 41

$\frac{1}{2} \times 64 =$ 32

$\frac{1}{2} \times 46 =$ 23

$\frac{1}{2} \times 60 =$ 30

$\frac{1}{2} \times 66 =$ 33

$\frac{1}{2} \times 68 =$ 34

$\frac{1}{2} \times 86 =$ 43

$\frac{1}{2} \times 240 =$ 120

$\frac{1}{2} \times 608 =$ 304

$\frac{1}{2} \times 206 =$ 103

$\frac{1}{2} \times 480 =$ 240

Name ____________ **N 13** ***

$\frac{1}{2} \times 286 =$ 143

$\frac{1}{2} \times 828 =$ 414

$\frac{1}{2} \times 262 =$ 131

$\frac{1}{2} \times 684 =$ 342

$\frac{1}{2} \times 200 =$ 100

$\frac{1}{2} \times 208 =$ 104

$\frac{1}{2} \times 210 =$ 105

$\frac{1}{2} \times 212 =$ 106

$\frac{1}{2} \times 108 =$ 54

$\frac{1}{2} \times 110 =$ 55

$\frac{1}{2} \times 112 =$ 56

$\frac{1}{2} \times 118 =$ 59

$\frac{1}{2} \times 30 =$ 15

$\frac{1}{2} \times 32 =$ 16

$\frac{1}{2} \times 34 =$ 17

$\frac{1}{2} \times 38 =$ 19

Name ____________ **N 13** ****

$\frac{1}{2} \times 50 =$ 25

$\frac{1}{2} \times 52 =$ 26

$\frac{1}{2} \times 56 =$ 28

$\frac{1}{2} \times 58 =$ 29

$\frac{1}{2} \times 70 =$ 35

$\frac{1}{2} \times 74 =$ 37

$\frac{1}{2} \times 90 =$ 45

$\frac{1}{2} \times 96 =$ 48

$\frac{1}{2} \times 120 =$ 60

$\frac{1}{2} \times 140 =$ 70

$\frac{1}{2} \times 160 =$ 80

$\frac{1}{2} \times 180 =$ 90

$\frac{1}{2} \times 142 =$ 71

$\frac{1}{2} \times 186 =$ 93

$\frac{1}{2} \times 320 =$ 160

$\frac{1}{2} \times 512 =$ 256

N 14

N14 The Function 3 ×

CAPSULE LESSON SUMMARY

Calculate how much money Chuck will receive on each day of a one-week-stay at his Uncle Albert's if his Uncle gives him 4¢ on the first day and then each day after triples the amount of the day before.

MATERIALS

Teacher: Colored chalk; Minicomputer set

Student: Paper; colored pencils; Minicomputer set (optional)

DESCRIPTION OF LESSON

Begin this lesson with five minutes of mental arithmetic activities involving 2 × and 3 × calculations.

Tell the following story to your class. Choose one of your students to be the star of this story.

T: Chuck often visits his Uncle Albert whom he likes very much. Uncle Albert knows lots of tricks and sometimes plays games with Chuck. Chuck is to stay with his Uncle Albert for one week. How many days are there in a week?

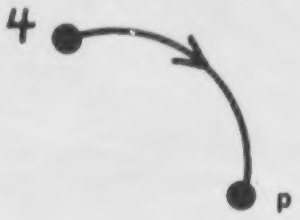
S: Seven.

T: Uncle Albert has a big surprise for Chuck when he arrives. He gives Chuck four pennies and promises: on the second day, to give him three times that much money; on the third day, to give him three times as much as on the second day; and so on for each day of Chuck's visit.

Looking at the four pennies, Chuck isn't sure that this is such a big surprise. Let's figure out how much money Chuck will receive on each day of his visit.

On the first day Chuck receives 4¢.

Draw this arrow picture on the board.

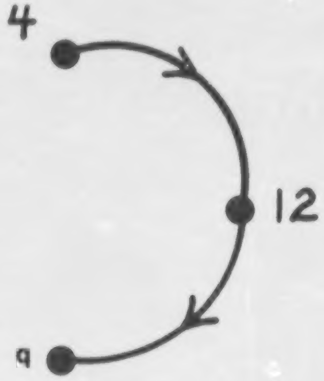


Do not write the letter on the board. It is here just to make the description of the lesson easier to follow.

T: On the second day Uncle Albert will give him three times as much money. How much is 3 × 4? How do you know?

S: 12, because $4 + 4 = 8$ and $8 + 4 = 12$.

Label p "12" and then extend the spiral by adding another arrow.



Do not write the letter on the board. It is here just to make the description of the lesson easier to follow.

T: How much money will Uncle Albert give Chuck on the third day? What number is "3 x 12"? How do you know?

Record several of the students' calculations on the board. Perhaps they will suggest these number stories.

$(10+10+10) + (2+2+2) = 36$

$12+12=24$ $24+12 = 36$

$2\times 12=24$ $24+10 = 34$ $34+2=36$

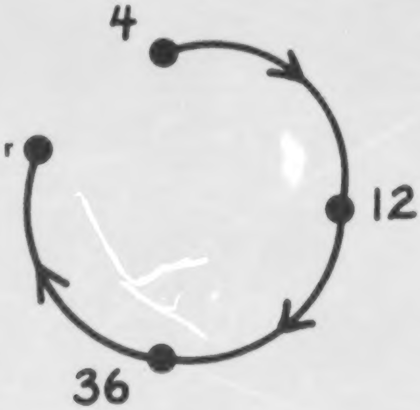
N 14

Label q "36" and extend the spiral by adding another arrow.

T: What could these red arrows be for?

S: 3 ×.

Write "3 ×" near the spiral.



Do not write the letter on the board. It is here just to make the description of the lesson easier to follow.

T: What number is 3 × 36?

Record any reasonable guesses on the board. If some numbers suggested are not good guesses, discuss why not and do not record those numbers on the board; for example,

S: 18.

T: 18 is less than 36. Could 3 × 36 be less than 36?

S: No.

T: 3 × 36 is not 18. Does anyone else have a suggestion?

S: 72, because 36 + 36 = 72.

T: But Uncle Albert will give Chuck 3 × 36¢. You calculated "2 × 36".

T: How can we find out for sure what number 3 × 36 is?

S: Use the Minicomputer.

NOTE: If most of your students are convinced that 3 × 36 = 108, do not insist that the class find (or check) the answer on the Minicomputer.

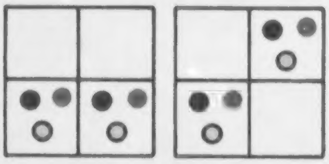
T: How many Minicomputer boards do we need?

S: At least two.

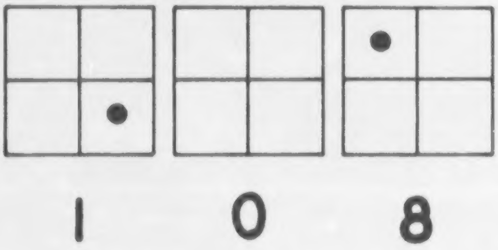
T: What should we do to calculate "3 × 36" on the Minicomputer?

S: Put 36 on the Minicomputer three times.

Invite a student to put 36 on the Minicomputer in red and another student to put 36 on the Minicomputer in blue. Remind the class that the number which is on the Minicomputer at that time is 2 × 36 and that you want to calculate 3 × 36. Invite a third student to put another 36 on the Minicomputer in yellow.



Let students make plays until the standard configuration for 108 is obtained. The class will note that a third Minicomputer board is needed. Ask a student to write the number below the Minicomputer and another to read the number aloud.



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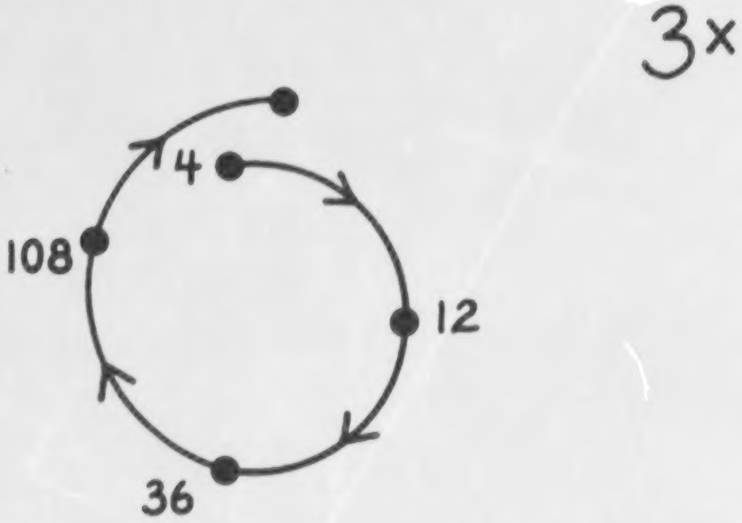
N 14

Label r "108" in the arrow picture.

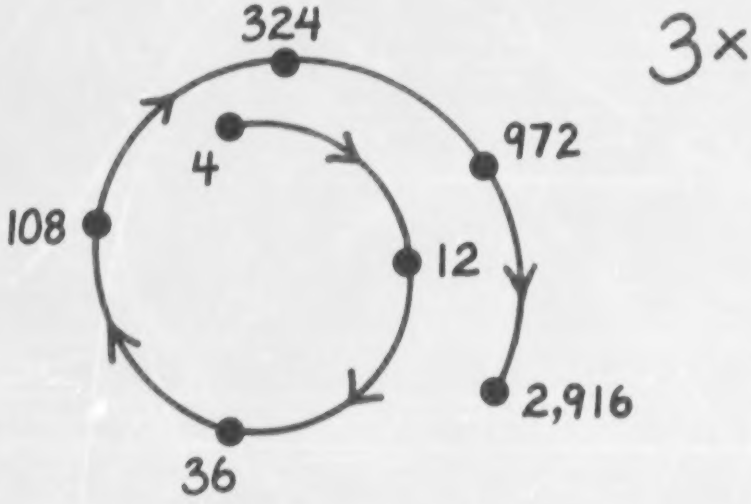
T: On the fourth day Chuck's uncle will give him 108 cents. Do you know another name for 108 cents?

S: One dollar and eight cents.

Extend the spiral.



In the same manner, continue labeling dots and extending the spiral until all seven days are accounted for. You should have this picture on the board.



Ask the students to draw their own 3 × arrow picture starting at 1, 2, or 5. Encourage students to keep extending their pictures until the numbers are too large for them to do the calculations; when this happens they should draw another 3 × arrow picture but starting with another number. Some students may wish to use individual Minicomputers during this activity.

N 15

N15 Subtraction Problems # 2

CAPSULE LESSON SUMMARY

Put on the Minicomputer ten times a number which has two checkers in its standard configuration. Make standard plays to support a play which replaces ten checkers on a square with one checker on the square of the same color on the next board to the left. Draw string pictures for two similar story situations and conclude that $13 - 5 = 13 + \hat{5}$. Find the solution to a subtraction calculation on the Minicomputer by adding the negative number corresponding to the number to be subtracted.

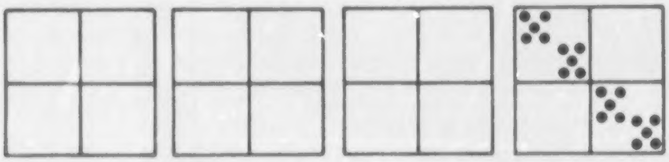
MATERIALS

Teacher: Minicomputer set; colored chalk
Student: None

DESCRIPTION OF LESSON

Exercise 1: Multiplying by 10

Put 10 × 9 on the Minicomputer in the following configuration.



T: What number is this?

What multiplication calculation does the Minicomputer suggest for this number? ("10 × 9")

Do we need to make some plays before we know which number this is?

Let the students react freely to these questions. Many of them already know that 10 × 9 = 90 and no longer need to make plays on the Minicomputer in order to find the number. Nevertheless, ask for volunteers to make plays until you obtain the standard configuration for 90. Let each volunteer make several plays in succession. Record this number story on the board.

$$10 \times 9 = 90$$

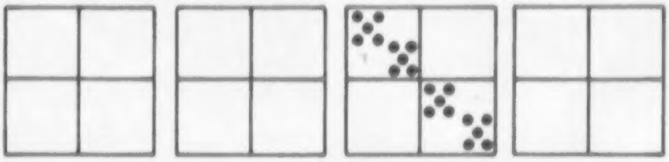
Point to the 8-square and then to the 80-square.

T: Ten checkers on this brown square (8-square) is the same as one checker on this brown square (80-square).

Point to the 1-square and then the 10-square.

T: Ten checkers on this white square (1-square) is the same as one checker on this white square (10-square).

Put 10 × 90 on the Minicomputer in the following configuration.



T: What multiplication calculation does the Minicomputer suggest for this number? ("10 × 90")
Is this calculation very different from the previous multiplication story (point to "10 × 9 = 90")?

The students should notice that this problem is very similar to the first one. Many students may predict that 10 × 90 = 900.

T: Do we need to make plays on the Minicomputer in order to find the number? Could we find an easy way to solve this problem?

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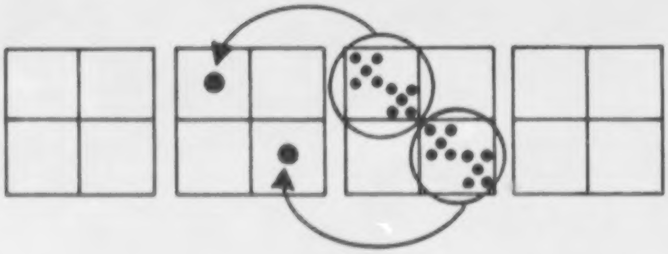
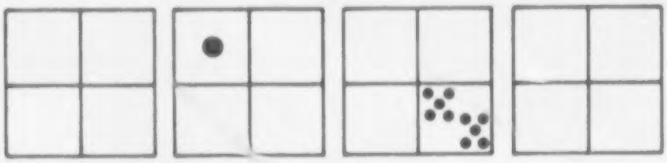
Students may observe that these two plays could be made; otherwise, suggest them yourself.



Allow the students to discuss these plays. If many of your students are not sure that these plays are correct, let the class make standard plays until they obtain the standard configuration for 900. Then put the original configuration for 10 × 90 on the Minicomputer again and continue with this dialogue.

T: Who can make a play with all of the checkers on the 80-square?

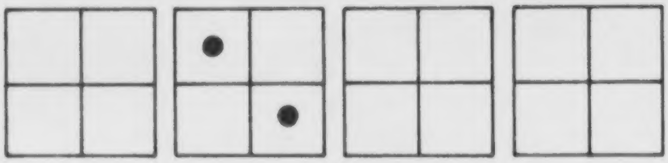
A student should remove all the checkers from the 80-square and put one checker on the 800-square.



Point to the 80-square and then the 800-square.

T: Ten checkers on this brown square (80-square) is the same as one checker on this brown square (800-square). 10 × 80 = 800.
Who can make a play with all the checkers on the 10-square?

A student should remove all the checkers from the 10-square and put one checker on the 100-square.



Point to the 10-square and then the 100-square.

T: Ten checkers on this white square (10-square) is the same as one checker on this white square (100-square). 10 × 10 = 100.

Conclude that 10 × 90 = 900 and record the number story on the board.

10 × 9 = 90
10 × 90 = 900

In the same manner ask the students to calculate "10 × 900". When they conclude that 10 × 900 = 9,000, ask if anyone can see a pattern in the number stories you have written on the board.

10 × 9 = 90
10 × 90 = 900
10 × 90 = 9,000

In the same manner do these calculations.

10 × 12
10 × 120
10 × 1,200

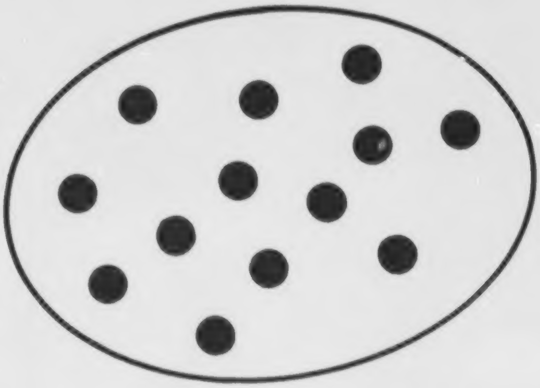
N 15

NOTE: Calculating "10 x 1,200" will require a fifth Minicomputer board which you can draw on the chalkboard to the left of all your Minicomputer boards.

Exercise 2: Subtraction Problem

T: Nick put thirteen red checkers in his pocket.

Draw this string picture on the board.



T: If Nick takes out five of these checkers, how many checkers will be left in his pocket?

S: Eight.

T: How can we show this on our picture?

S: Erase five of the red checkers.

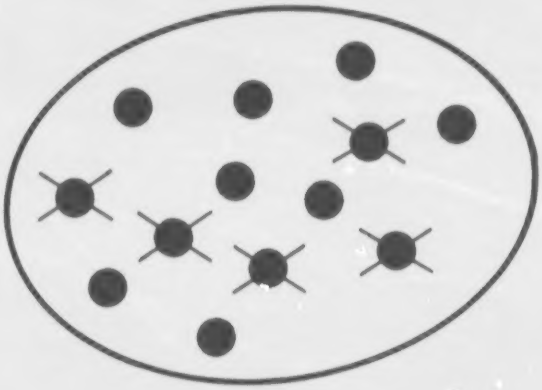
S: Cross out five of the red checkers.

Let a student cross out five of the red checkers.

T: What number story could we write about this picture?

S: "13 - 5 = 8".

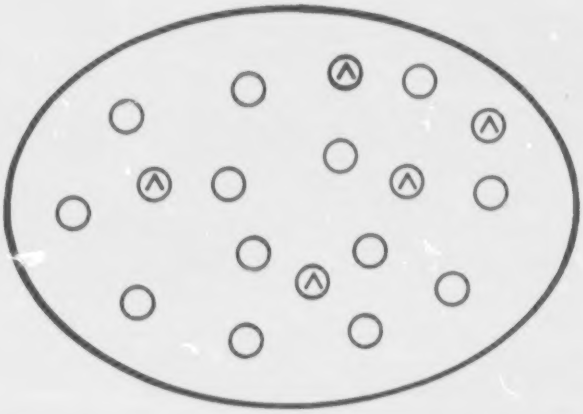
Write "13 - 5 = 8" below the string picture.



13 - 5 = 8

T: Eli the Elephant put 13 regular peanuts and 5 magic peanuts in his bag.

Draw this string picture on the board. Do not erase the first string picture.



N 15

T: How many peanuts will Eli find when he opens his bag?

S: Eight, because the magic peanuts and five of the regular peanuts disappear.

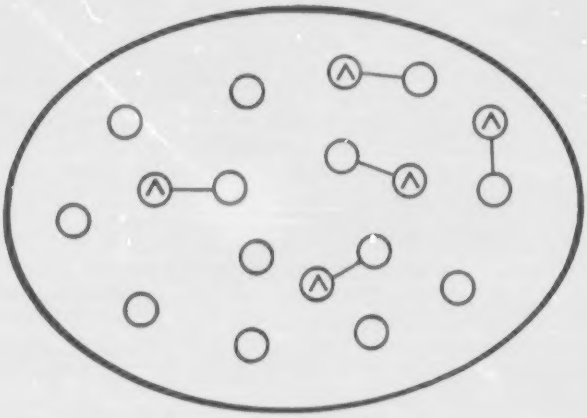
T: Who can show this in our picture?

Let a student pair the regular and magic peanuts in the usual way.

T: What number story can we write about this picture?

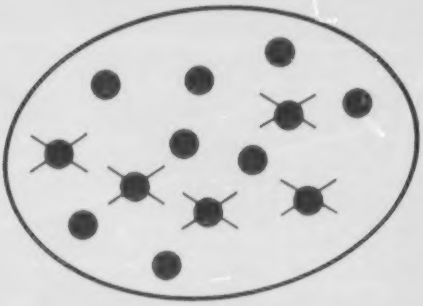
S: "$13 + \hat{5} = 8$".

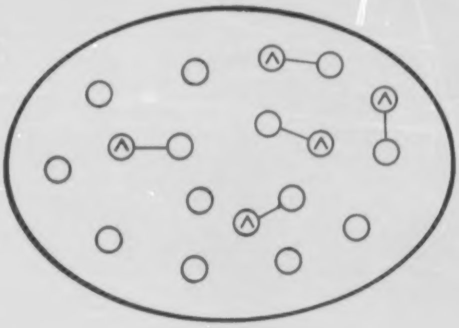
Write "$13 + \hat{5} = 8$" below the second string picture.



$$13 + \hat{5} = 8$$

T: What do you notice about these two pictures?





$$13 - 5 = 8 \qquad 13 + \hat{5} = 8$$

The students should tell you that subtracting 5 is the same as adding $\hat{5}$. Erase the board.

Exercise 3: Subtraction Problem

T: If Nick puts 100 checkers in his pocket and later gives 46 of these checkers to Janet, how many checkers are left?

Some students may be able to calculate the number mentally; others may guess.

T: How can we be sure how many checkers are left?

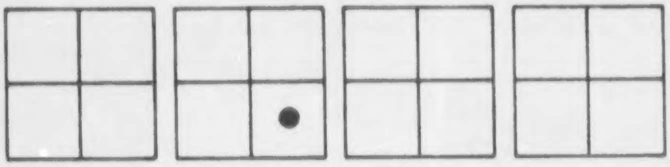
S: Draw one hundred dots and then cross out forty-six of them.

Accept this as a good idea but not a very practical one because it would take a long time.

S: Put 100 on the Minicomputer and then take off 46.

N 15

Invite someone to put 100 on the Minicomputer.



T: Will it be easy to subtract 46?

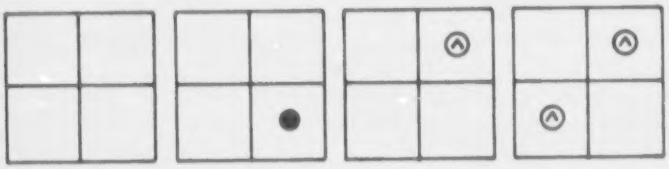
S: We need to make some plays first.

S: We could add $\widehat{46}$.

If no one suggests adding $\widehat{46}$, suggest it yourself and then indicate this equality on the board.

$$100 - 46 = 100 + \widehat{46}$$

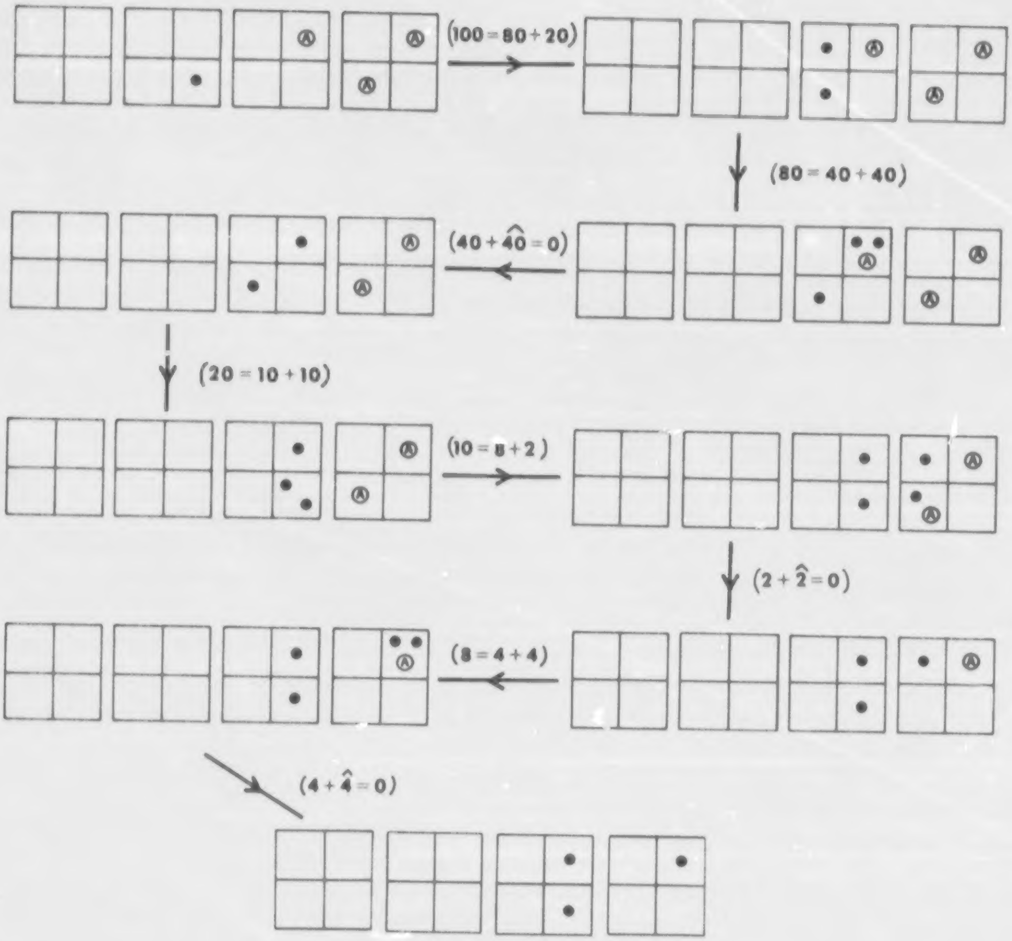
Emphasize that both of the calculations have the same solution. Ask a student to put $\widehat{46}$ on the Minicomputer.



T: What should we do next?

S: We need to make some backward plays so we can get negative and regular checkers on the same squares.

Invite students to make plays on the Minicomputer. Watch closely as plays are made; a student might suggest incorrectly that $\widehat{40} = \widehat{20} + 20$ when the correct play to be made is "$\widehat{40} = \widehat{20} + \widehat{20}$".


(100 = 80 + 20)
(80 = 40 + 40)
(40 + $\widehat{40}$ = 0)
(20 = 10 + 10)
(10 = 8 + 2)
(2 + $\hat{2}$ = 0)
(8 = 4 + 4)
(4 + $\hat{4}$ = 0)

T: What number is 100 - 46?

S: 54.

Erase the board and remove the checkers from the Minicomputer.

Exercise 4: Subtraction Problem

Write this problem on the board.

$$296 - \widehat{165} =$$

T: What number is 296 - 165?

Record some of the students' guesses on the board.

T: How can we calculate "296 - 165"?

Some students may suggest calculating "$296 + \widehat{165}$" and others might suggest calculating "296 - 165". Ask the class which method they prefer and then use that method to solve the problem on the Minicomputer. If there is sufficient time and student interest, repeat this activity to calculate "703 - 458".

N16 Composition of Funtions #1

CAPSULE LESSON SUMMARY

Draw an arrow picture on the board and discover that the effect of four - 10 arrows is the same as the effect of one - 40 arrow. In the same arrow picture, draw - 20 and - 30 arrows. Build many arrow roads using only - 10 arrows. In each road draw an arrow from the first dot to the last dot and discuss what the arrow could be for. Do a similar activity with a road which has three - 10 arrows and one - 3 arrow.

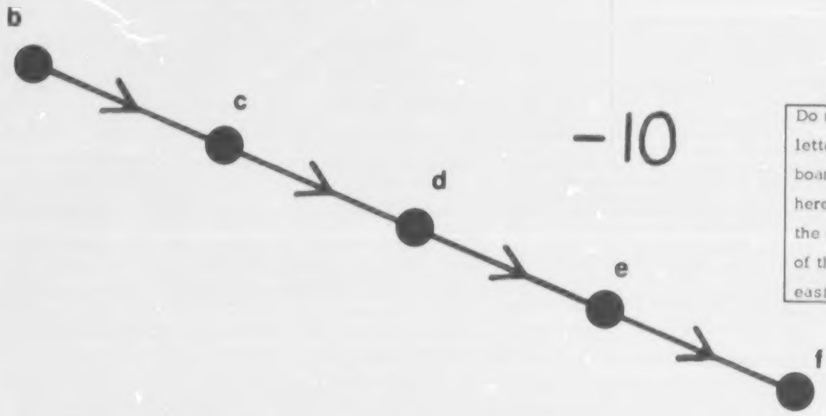
MATERIALS

Teacher: Colored chalk
Student : Paper; colored pencils

DESCRIPTION OF LESSON

Exercise 1

Draw this arrow picture on the board.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: Point to the largest number in this arrow picture.

The students should point to b. Ask a volunteer to explain why this is the largest number.

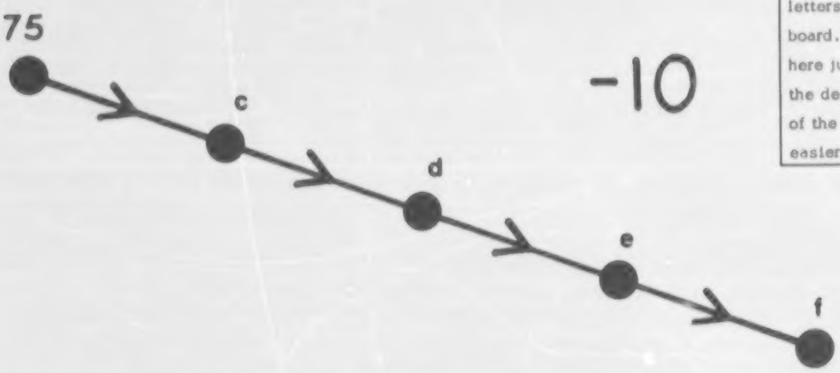
T: Point to the smallest number in this arrow picture.

The students should point to f.

N 16

T: These arrows are for - 10 so when we follow the arrows, the numbers get smaller.

Label b "75".



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

Point to the dot for 75 and then trace the first arrow.

T: If this number is 75, what is this number?

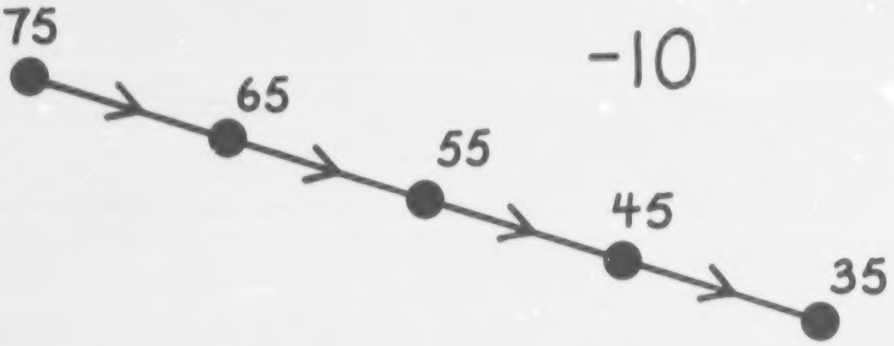
S: 65.

Label c "65". Trace the arrow which starts at 65.

T: If this number is 65, what is this number (d)?

S: 55.

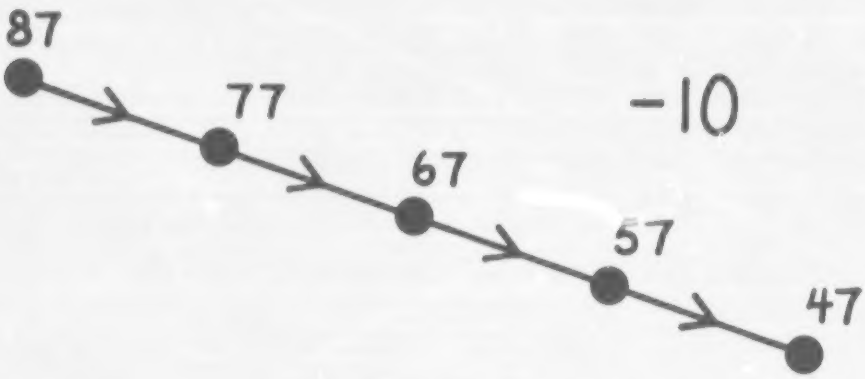
Continue this activity until all the dots are labeled.



Erase all the numbers and then label the dot farthest to the left "87".

T: If this number is 87, which numbers are the other dots for?

Continue this activity until all the dots are labeled.

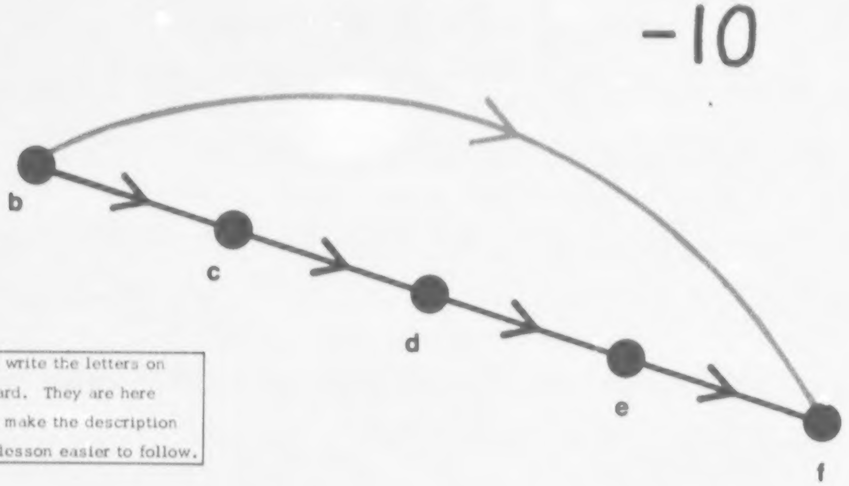


Trace (but do not draw) another arrow starting at 47.

T: If we draw another red arrow, which number would we meet? (37)
And if we continued drawing more red arrows . . . ?

The class should suggest 27, 17, 7 and perhaps $\hat{3}$. If negative numbers are not mentioned, do not suggest them yourself.

Erase only the numbers for the dots. Draw a blue arrow from the dot labeled b in the next illustration to the dot labeled f.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

N 16

T: What could this blue arrow be for? How do you know?

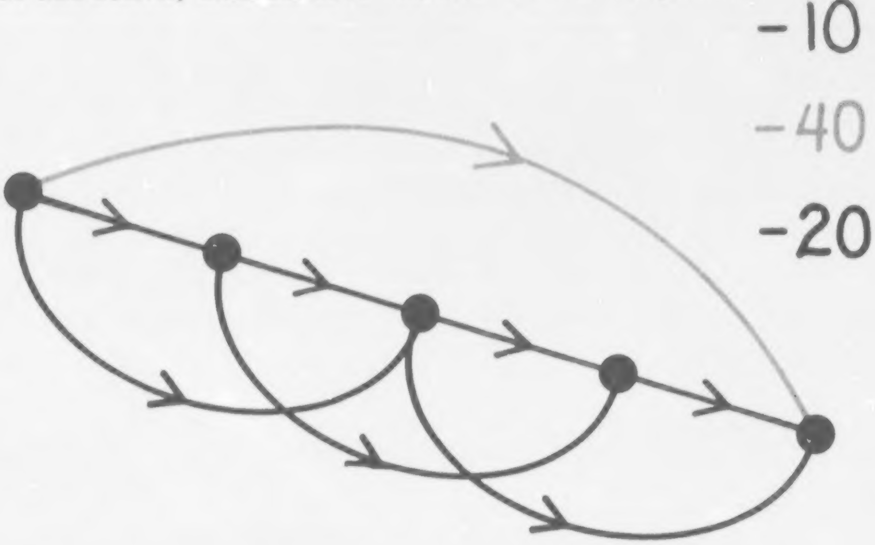
S: It could be for - 40, because 4 × 10 = 40.

NOTE: The blue arrow could be for other relations such as "I am more than you". If a student suggests this, you should reply that the arrow could be for "I am more than you", but this arrow is for something else.

Write "- 40" and "**-20**" near the arrow picture.

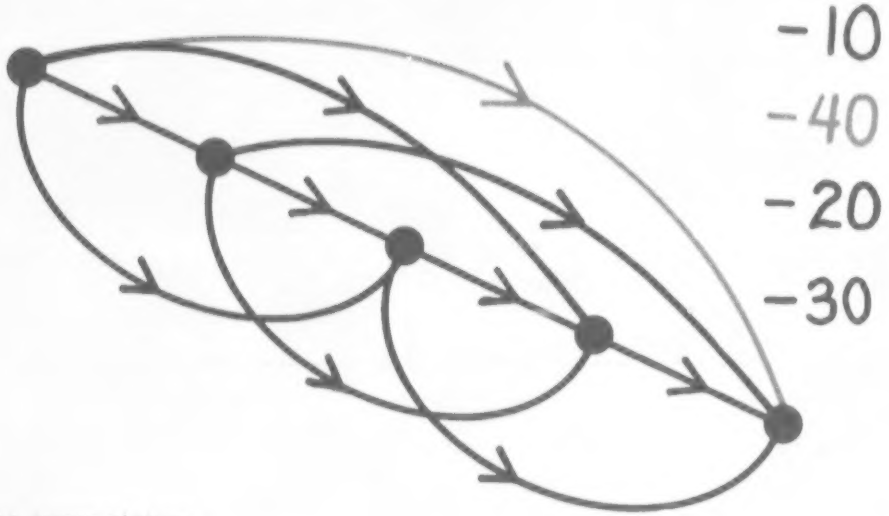
T: Where could we draw a - 20 arrow in this arrow picture without drawing any more dots?

Let the students draw any - 20 arrows which they correctly trace. Continue this activity until all three - 20 arrows are in the picture.



T: Where could we draw a - 30 arrow in this arrow picture without drawing any more dots?

Write "- 30" in orange near the arrow picture and invite students to trace - 30 arrows. You may wish to draw these arrows yourself if your picture is becoming complex. Continue until your class finds both - 30 arrows.



Erase the arrow picture.

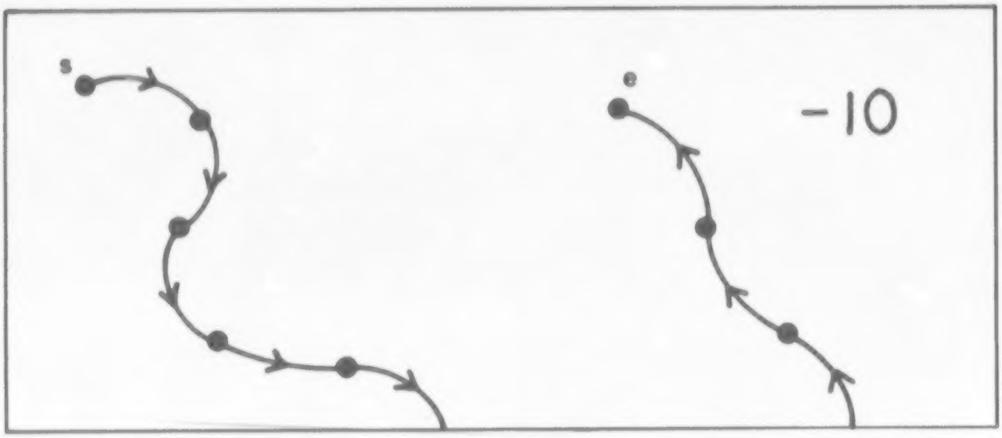
Exercise 2

Distribute unlined paper and colored pencils. Write " -10 " on the board.

T: Draw a red arrow road with as many arrows as you like on your paper. Work quickly. Do not label the dots.

Help some students individually to get started. Then while the students are working, draw a red arrow road which is partially off the board.

N 16



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

Identify a student who has a road with at least eight arrows; we refer to this student as "Ed" in the following dialogue. After all the students have drawn at least three arrows, continue the collective lesson.

T: On the board I have drawn two parts of one red arrow road, but you can't see all of it. How many arrows could be in this red arrow road?

The students should conclude that the arrow road on the board has at least eight arrows.

T: Count the number of red arrows in your arrow road.

Imagine that this is Ed's arrow road. Ed, how many arrows does your road have?

S: Nine.

T: How much smaller is this number (point to e) than this number (point to s)?

Perhaps one of your students will know that the number at e is 90 less than the number at s.

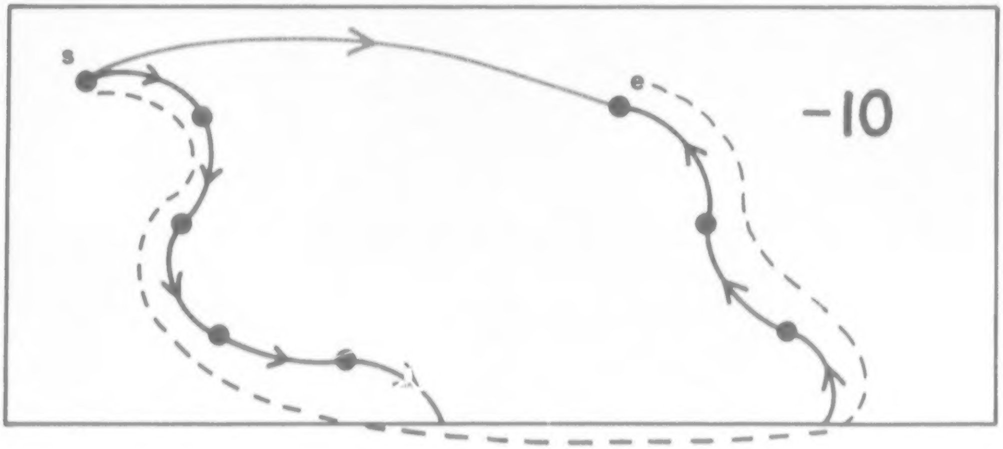
T: Each red arrow is for - 10. How much will we have subtracted from the starting number of this road if it has nine red arrows? Let's count by tens.

Hold up nine fingers and point to them as appropriate.

NOTE: If the road your class is discussing has more than ten arrows you will need the fingers of at least one more person besides yourself. Use students in such cases.

T: Each finger is for a - 10 arrow. How much will we have subtracted with one red arrow? (10) . . . with two red arrows? (20) . . . with three red arrows? (30) . . . with four red arrows? (40) . . . with nine red arrows? (90) So the ending number of this road is 90 less than the starting number since 9 x 10 = 90.

Put your left index finger on s and trace the arrow road with your right index finger until you reach e. The dotted line in the next illustration indicates how to trace the arrow road. Trace and then draw a blue arrow from s to e.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: If this red arrow road has nine arrows, what would this blue arrow be for?

S: -90.

Write "-90" near the arrow picture.

T: Who has an arrow road which has a different number of red arrows than Ed's does?

Choose a student whose road has more than eight arrows. Erase "-90" and imagine that the arrow road on the board is for that student's road. Decide with your students what the blue arrow is for and write the key on the board.

T: On your paper draw a blue arrow from the first dot in your road to the last dot in your road. Decide what your blue arrow is for and write it in blue on your paper.

Help any students who have drawn blue arrows in the wrong direction. Allow the students a few minutes to write their keys on their papers.

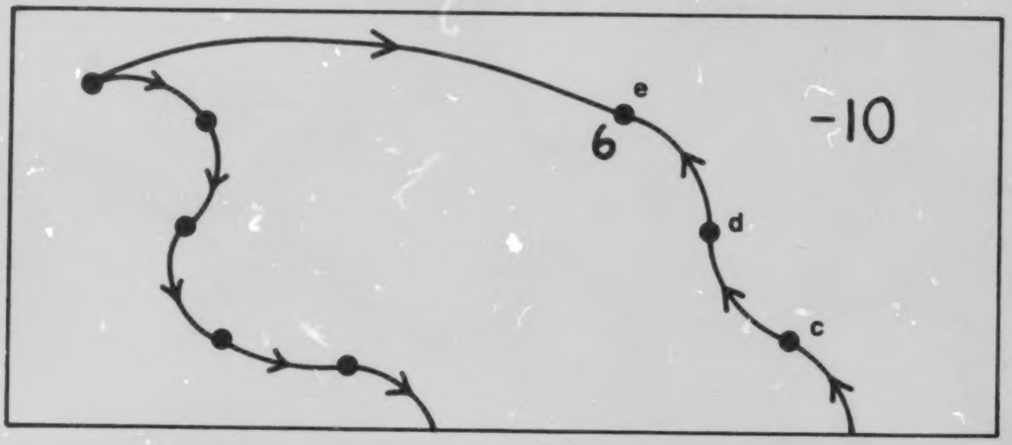
T: Who hasn't labeled their blue arrows yet?

Suppose a student whose road has twenty-six arrows replies. Ask that student:

T: How many red arrows are in your arrow road? (Twenty-six) What number is 26 × 10? (260)
So your blue arrow is for -260. If anyone else has exactly twenty-six arrows, your blue arrow also is for -260.

Continue this activity until most of the students have labeled their blue arrows.

Label e in the arrow road on the board "6". Erase the blue key.



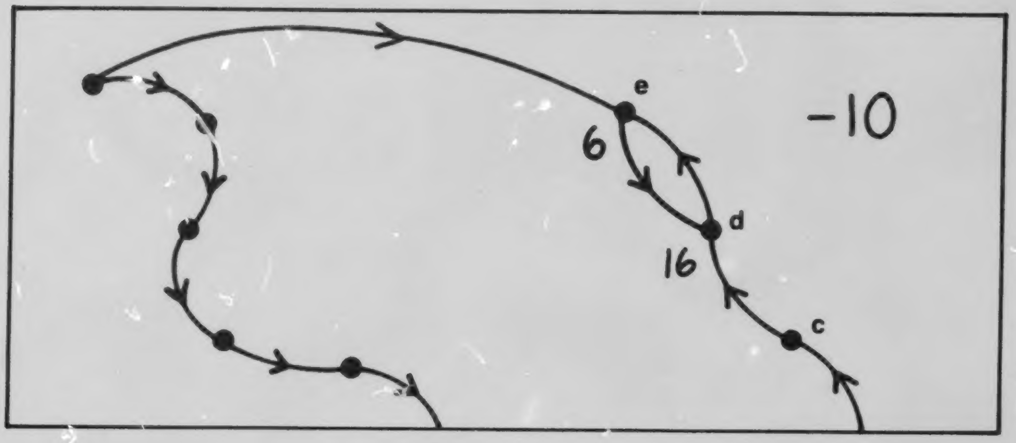
Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

Point to e and then to d as you ask:

T: If this number is 6, what number is this dot (d) for? How do you know?

S: It is for 16, because 6 + 10 = 16 (or because 16 - 10 = 6).

Label d "16" and then draw the return arrow from e to d in green.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

N 16

T: If the red arrows are for - 10, what could this green return arrow be for?

S: +10.

Write "+10" in green near the arrow picture. Decide with the students that c is for the number 26 and then label it.

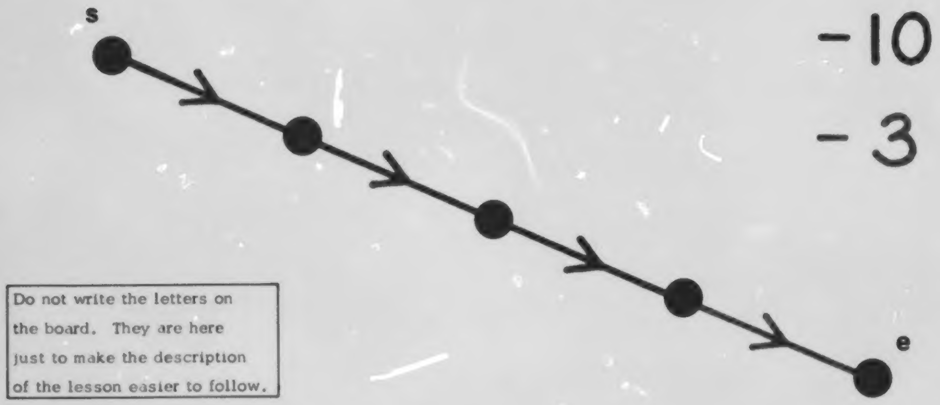
T: Label the ending dot in your arrow road "6" and then label all the dots.

Encourage students who finish quickly to draw all the green arrows in their arrow pictures.

Erase the arrow picture on the board before going on to the next exercise.

Exercise 3

Draw this arrow picture on the board.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: Point to where the largest number is in this arrow picture.

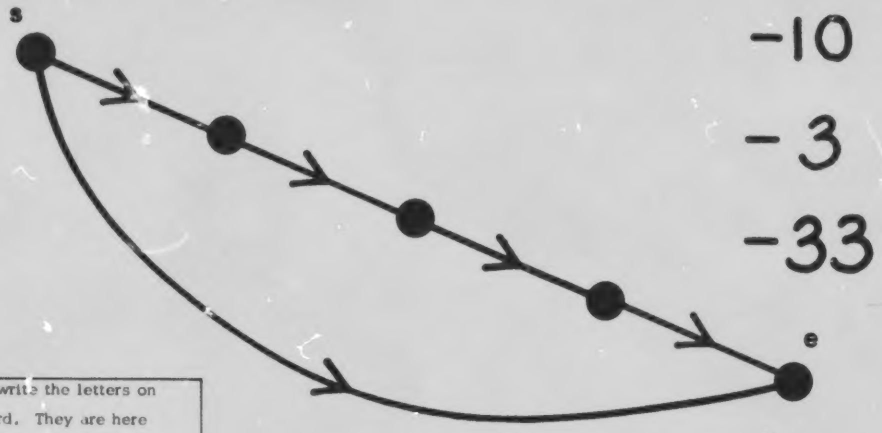
The students should point to s.

T: Point to where the smallest number is in this arrow picture. (e) How do you know this number (point to e) is the smallest?

S: Because the arrows are for - 10 and - 3.

T: I am going to draw a green arrow from the dot for the largest number to the dot for the smallest number.

Draw a green arrow from s to e.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: What could this green arrow be for?

S: - 33.

Write "- 33" in green near the arrow picture.

T: How do you know the green arrow could be for - 33?

Accept any correct explanation.

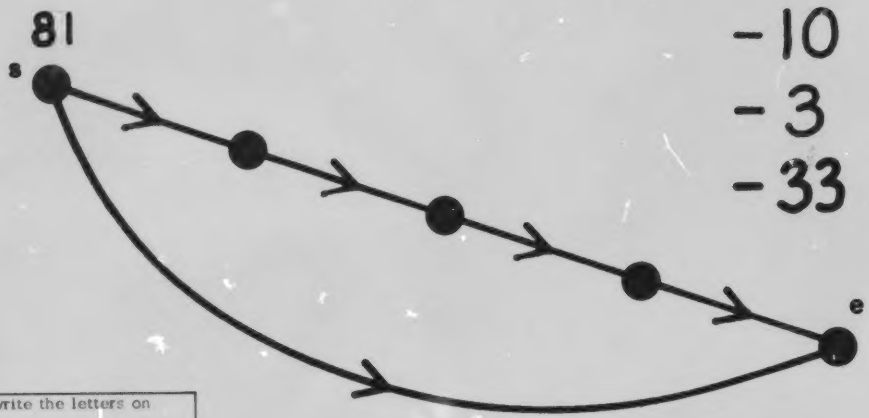
T: Let's see if we can use this arrow picture to solve some subtraction problems.

N 16

Write this problem on the board.

$$81 - 33 = ?$$

Label s "81".



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: If this number (point to s) is 81, what is this number (point to e)?

Let the students whisper their answers to you. Label the dots collectively and complete the number story. If several students have difficulty calculating "51 - 3", hold up three fingers and count backward from 51.

$$81 - 33 = 48$$

In the same manner calculate "70 - 33" and "52 - 33".

N17 Subtraction Problems #3

CAPSULE LESSON SUMMARY

Put the standard configuration for 60 on the Minicomputer ten times. Make plays replacing ten checkers on a square with one checker on the square of the same color on the next board to the left. Do a similar activity with twenty checkers on the 8-square. Use the Minicomputer to decide how much money Gina's father would have left out of $200 if he buys a television set costing $127.

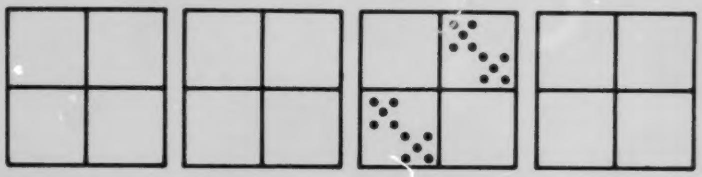
MATERIALS

Teacher: Minicomputer set
Student: Worksheets N17*, **, ***, and ****

DESCRIPTION OF LESSON

Exercise 1: Multiplying by 10

Put 10 x 60 on the Minicomputer in this configuration.



T: Which number is this?

What multiplication calculation does the Minicomputer suggest for this number? ("10 x 60")

Do we need to make some plays before we know which number this is?

Let the students react freely to these questions. Many of them already know that 10 x 60 = 600 and no longer need to make plays on the Minicomputer.

T (pointing to the checkers on the 20-square): How many checkers are on this square? (Ten)

Which play can we make with all of these checkers?

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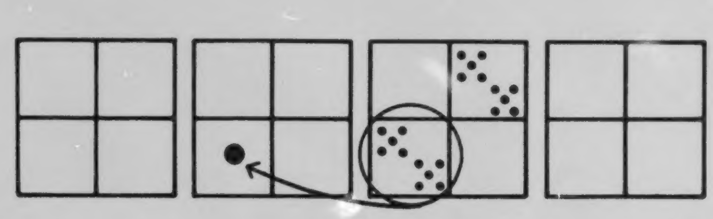
S: Take all of them off the Minicomputer and put one checker on the 200-square.

T: Ten checkers on this red square (point to the 20-square) are the same as one checker on this red square (point to the 200-square).

Invite a student to make this play.



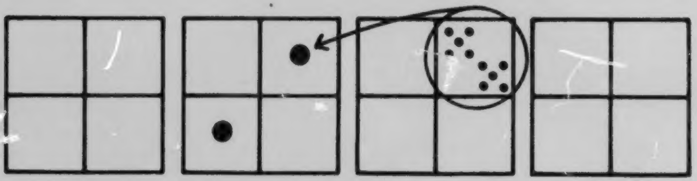
T: 10 x 20 = 200.

T (pointing to the checkers on the 40-square): How many checkers are on this square? (Ten)
Which play can we make with these ten checkers?

S: Take all of them off the Minicomputer and put one checker on the 400-square.

T: Ten checkers on this purple square (point to the 40-square) are the same as one checker on this purple square (point to the 400-square).

Invite a student to make this play.



T: 10 × 40 = 400.

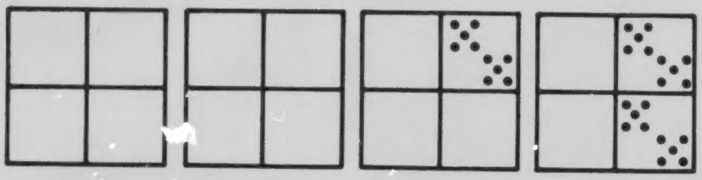
What number is 10 × 60?

S: 600.

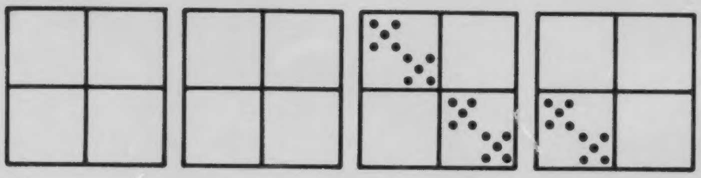
Write this number story to one side on the board.

$$10 \times 60 = 600$$

Repeat this activity with these configurations. Let students whisper the number to you before making any plays on the Minicomputer.



$= 10 \times 45$



$= 10 \times 92$

You should have these number stories on the board.

$$10 \times 60 = 600$$

$$10 \times 45 = 450$$

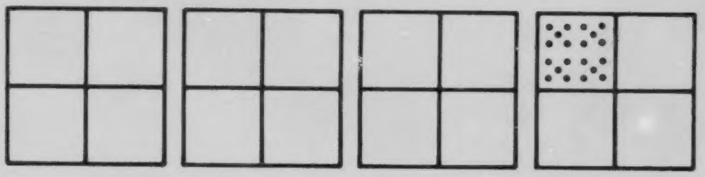
$$10 \times 92 = 920$$

T: What patterns do you see?

N 17

Allow the students to express themselves freely. Perhaps they will suggest easy ways to multiply a number (an integer) by 10; for example, simply tack on "0" thereby shifting each digit one place to the left. Do not suggest any "tricks" yourself; let your students discover their own, if not in this lesson, in a future one.

Erase the board and then put this configuration on the Minicomputer.

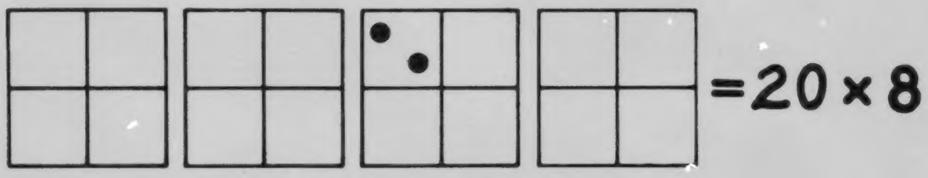


T: How many checkers are on the 8-square? (Twenty)
What multiplication calculation does the Minicomputer suggest for this number? ("20 x 8")
Which number is this? (160)

Allow several students to whisper the answer to you.

T: Remember that there are twenty checkers on this square. Which plays can we make with these checkers?

Someone might suggest removing all the checkers from the 8-square and then putting a checker on the 80-square. If so, ask "Does 20 x 8 = 80?" (No) When a student suggests making a "10 x 8 = 80" play, let the student make the play and call on another student to make a second "10 x 8 = 80" play with the remaining checkers.



Emphasize that two "10 x 8 = 80" plays were made because there were twenty checkers on the 8-square.

T: Now what do we need to calculate before we know which number is on the Minicomputer?

S: "2 x 80".

Invite students to make plays (some backward plays are needed) until the standard configuration for 160 is obtained. Conclude that 20 x 8 = 2 x 80 = 160.

Exercise 2: Subtraction Problem

Tell the following or a similar story about one of the students in your class.

T: Gina's father decides to buy a television set. He puts $200 in his wallet and goes shopping for a television set. What bills could he have in his wallet if he has exactly $200 in cash?

Encourage the students to suggest many possibilities before you continue with the lesson. A sample dialogue is given here.

S: He could have $20 bills.

T: How many $20 bills?

S: Ten.

T: 10 x 20 = 200, so ten $20 bills would be $200.

When the students exhaust their ideas about what bills make $200, continue the story.

N 17

T: Gina's father finds a television set he likes that costs $127. Does Gina's father have enough money with him to buy it? (Yes)

Write this inequality on the board.

$$127 < 200$$

T: 127 is less than 200. What calculation could we do to find out how much money Gina's father will have left after he buys the television?

S: "200 - 127".

Write "$200 - 127 = \square$" on the board and point to it as you ask:

T: Is there another way we could write a number story for this problem?

S: "$127 + \square = 200$".

If necessary, suggest such a number story yourself and add this story to those on the board.

$$127 < 200$$
$$200 - 127 = \square$$
$$127 + \square = 200$$

T: How much money will Gina's father have left after he buys the television set?

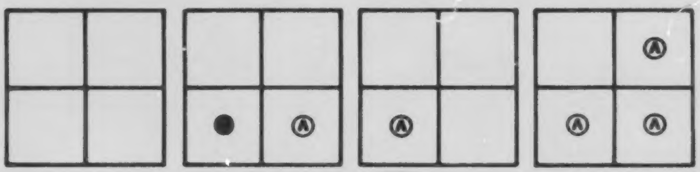
Record the students' guesses on the board. Perhaps someone will suggest using the Minicomputer.

T: How can we calculate "200 - 127" on the Minicomputer?

S: Put 200 on the Minicomputer and then take off 127.

S: Put 200 on the Minicomputer with regular checkers and then put $\widehat{127}$ on the Minicomputer with negative checkers.

If no one suggests calculating "200 + $\widehat{127}$", suggest it yourself. Invite students to put 200 and $\widehat{127}$ on the Minicomputer.



S: We need to make some backward plays.

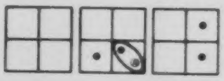
Invite students to make plays on the Minicomputer until you obtain the standard configuration for 73. Conclude that 200 - 127 = 73.

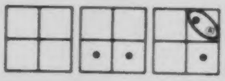
Let the students determine which guess was closest to 73.

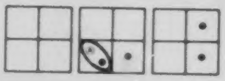
Worksheets N17*, **, ***, and **** are available for individual work.

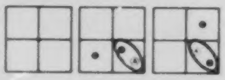
Name ______________________ **N 17 ***

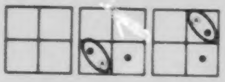
Complete


35 − 10 = 25


35 − 4 = 31

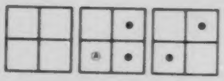

35 − 20 = 15

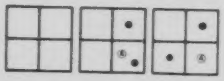

35 − 11 = 24

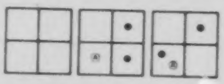

35 − 24 = 11

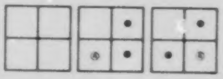
Name ______________________ **N 17 ****

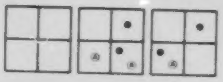
Complete


56 − 20 = 36


56 − 11 = 45

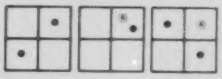

56 − 22 = 34

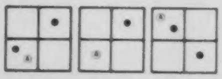

56 − 21 = 35

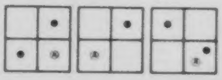

56 − 32 = 24

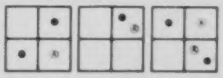
Name ______________________ **N 17 *****

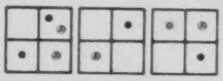
Complete


649 − 44 = 605


649 − 228 = 421

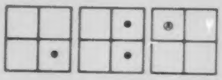

649 − 121 = 528

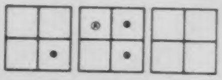

649 − 145 = 504

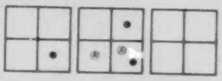

649 − 524 = 125

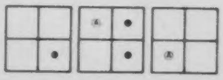
Name ______________________ **N 17 ******

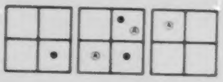
Complete.


150 − 8 = 142


150 − 80 = 70


150 − 30 = 120


150 − 82 = 68


150 − 68 = 82

N18 Return Arrows

CAPSULE LESSON SUMMARY

Ask where the largest number is and where the smallest number is in arrow roads with subtraction and addition arrows. Given the smallest number in a road, label all the dots with the help of return arrows.

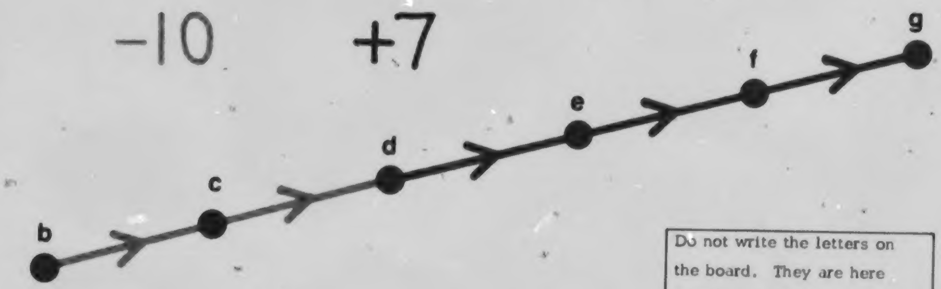
MATERIALS

Teacher: Colored chalk
Student: Paper; pencil; colored pencils

DESCRIPTION OF LESSON

Exercise 1

Draw this arrow picture on the board and ask the students to copy it on their papers.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: Where is the smallest number in this picture? (At d)
How do you know?

Students should include these ideas in their explanations:

- the numbers from b to d in the arrow road decrease because the arrows in that part of the road are for - 10; and
- the numbers from d to g in the arrow road increase because the arrows in that part of the road are for + 7.

N 18

Point to the dots indicated in parentheses as you ask the following questions.

T: How much smaller is this number (at d) than this number (at b)?

S: 20 smaller, because 10 + 10 = 20.

T: How much smaller is this number (at d) than this number (at g)?

S: 21 smaller, because 7 + 7 + 7 = 21.

T: So which number is larger: this one (at b) or this one (at g)?

S (pointing to g): This one.

T: Now I will tell you that the smallest number in this picture is 76. In your picture label the dot for 76 and then label the other dots.

Invite a student to label the dot for 76 on the board. Look quickly at the individual work to ensure that each student labeled the correct dot "76". After a few minutes, check the work collectively.

T (pointing to c): Which is this number? How do you know?

S: 86, because 86 - 10 = 76.

S: 86, because 76 + 10 = 86.

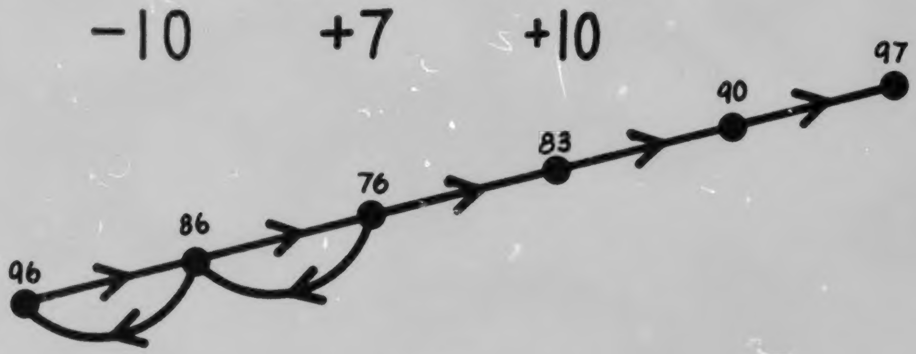
Draw a green arrow from d to c.

T: What could this arrow be for? (+10)

Trace the return arrow from c to b.

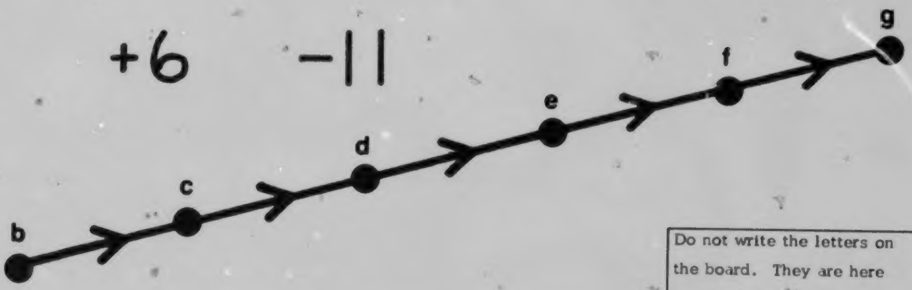
T: And this arrow . . . ? (+10)

Write "+10" in green near the picture on the board. Ask the students to draw green +10 arrows in their pictures, and then invite some students to draw these arrows in the picture on the board.



Exercise 2

Draw this arrow picture on the board and ask the students to copy it on their papers.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: Where is the largest number in this arrow picture? (At e)
How do you know?

Students should include these ideas in their explanations:

- the numbers from b to e in the arrow road increase because the arrows in that part of the road are for +6; and
- the numbers from e to g in the arrow road decrease because the arrows in that part of the road are for -11.

Point to the appropriate dots indicated in parentheses as you ask the following questions.

T: How much larger is this number (at e) than this number (at b)?

S: 18 larger, because 6 + 6 + 6 = 18.

T: How much larger is this number (at e) than this number (at g)?

S: 22 larger, because 11 + 11 = 22.

T: So which number is smaller: this one (at b) or this one (at g)?

S (pointing to g): This one.

T: How much smaller . . . ?

Perhaps one of your students will know that the number at g is 4 less than the number at b. If no one suggests this, ask the question again after the dots have been labeled.

T: Now I will tell you that the smallest number in this arrow picture is 25. In your picture, label the dot for 25 and then label the other dots.

Allow a few minutes for individual work. If several students do not know which dot to label "25", ask someone to label the dot for 25 on the board.

T (pointing to f): Which is this number? How do you know?

S: 36, because 36 - 11 = 25.

S: 36, because 25 + 11 = 36.

2 0 8

Trace an arrow from g to f (a return arrow).

T: What could this arrow be for? (+11)

Trace an arrow from f to e (a return arrow).

T: And this one . . . ? (+11)

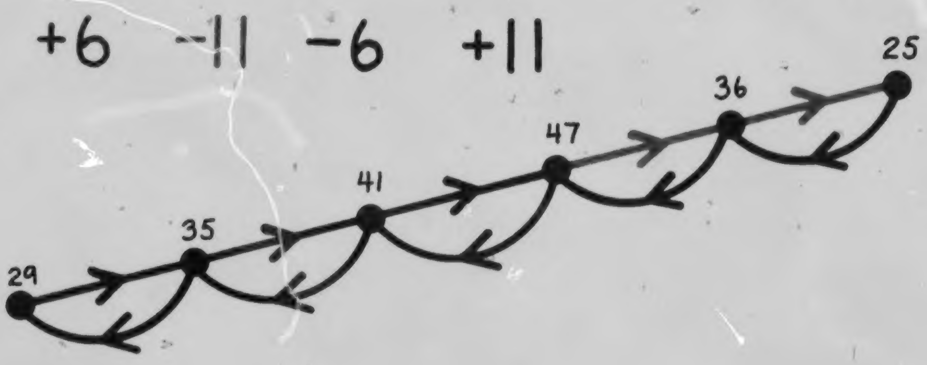
Trace an arrow from e to d (a return arrow).

T: And this one . . . ? (-6)

On the board write "+11" in green and "-6" in orange.

T: Draw all the return arrows in your arrow picture.

Let students complete the picture on the board.



Point to the dots for 25 and 29 in turn as you ask:

T: How much smaller is this number than this number? (4)

Commend any students who had answered this question correctly when none of the dots were labeled.

N19 Introduction to Decimal Numbers #1

CAPSULE LESSON SUMMARY

Through a sequence of situations involving money, support a natural extension of the Minicomputer for representing decimal numbers.

MATERIALS

Teacher: Minicomputer set[*]; colored chalk
Student : None

DESCRIPTION OF LESSON

Begin this lesson with five to ten minutes of mental arithmetic which includes halving and doubling numbers.

Display three Minicomputer boards.

Choose students from your class to star in the following stories. Substitute their names for the names which appear here.

T: Chuck and Timothy are my friends. I want to share $30 between them. I want to give them both the same amount of money. How much money should I give Chuck? How much money should I give Timothy?

Allow several students to make comments. Be sure the class understands that the $30 must be shared equally between the two children.

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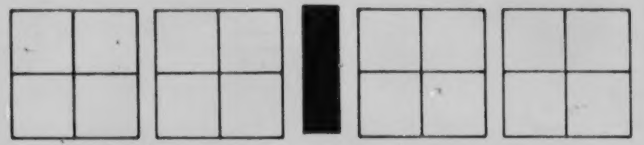
[*] This lesson requires that at times additional space be left between two of the demonstration Minicomputer boards and that a green bar be drawn in this space.



If your Minicomputers are hanging from permanently attached hooks or are displayed in some other way which fixes their position, you may need to make some adjustments before the lesson begins.

Although many students will know that $\frac{1}{2} \times 30 = 15$ without the use of the Minicomputer, the following activity of calculating "$\frac{1}{2} \times 30$" on the Minicomputer will review the process of halving on the Minicomputer. This review is in preparation for finding one-half of a number on the Minicomputer when the result is a (non-integer) decimal.

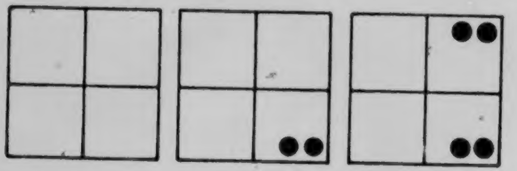
T: Can we use the Minicomputer to find which number is one-half of 30? (Yes)

Invite someone to put 30 on the Minicomputer.

T: How can we find one-half of the number on the Minicomputer?

S: Make some backward plays until all the checkers are in pairs.

Invite students to make plays. Whenever a play results in a pair of checkers on the same square, ask the student who made the play to move those checkers together into a corner of the square. Discourage students from breaking up pairs. Continue until all the checkers are in pairs.



T: Who can remove some checkers so that one-half of this number will be on the Minicomputer?

A student should remove one checker from every pair. Write "$\frac{1}{2} \times 30 = 15$" on the board. Commend any students who had given correct answers earlier.

Clear the Minicomputer and continue with another story.

T: I have two more friends. Christy and Laura are my friends. I want to share $300 between them. How much money should I give Christy? How much money should I give Laura?

Allow several students to make comments. Many students might know that $\frac{1}{2} \times 300 = 150$.

T: Can we use the Minicomputer to find which number is one-half of 300? (Yes)

Invite someone to put 300 on the Minicomputer.

T: How can we find one-half of [illegible] number on the Minicomputer?

S: Make backward plays until all the checkers are in pairs.

Invite students to make plays until all the checkers are in pairs. Ask a student to show one-half of the number on the Minicomputer and write "$\frac{1}{2} \times 300 = 150$" on the board. Commend any students who gave the correct answer earlier.

Clear the Minicomputer, display only two boards, and continue with another story.

T: I have two more friends. Jason and Lynn are my friends. I want to share $3 between Jason and Lynn. How much money should I give Jason? How much money should I give Lynn?

Let several students comment. Perhaps students will suggest that each child be given one and a half dollars.

T: How many cents is one-half dollar? (Fifty)
One and one-half dollars is one dollar and fifty cents.

Invite a student to put 3 on the Minicomputer.

T: What should we do if we want to find one-half of 3 on the Minicomputer?

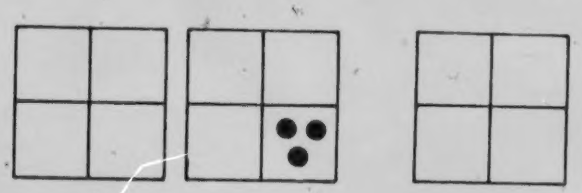
S: Make backward plays until all the checkers are in pairs.

T: Who can make a backward play?

There may be some confusion, but very likely someone will make the "2 = 1 + 1" play resulting in three checkers on the 1-square. Move two of these checkers into a corner of the square.

T: Now we have one pair of checkers but we still have one checker without a partner. What should we do now?

There will probably be even more confusion now. Be patient. Encourage students to express their ideas. In many classes where this question has been asked, at least one student suggested adding a new board to the right. If this doesn't happen, don't worry. Suggest it yourself. Place the new board to the right of the ones' board, but leave more space than usual between these boards.

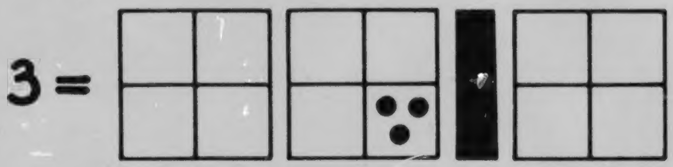


At least one of your students should object and say that now the number on the Minicomputer is not 3. Raise the question yourself, if necessary.

T: In order not to forget that this is 3, let's draw a green bar between these two boards (see the next illustration).

N 19

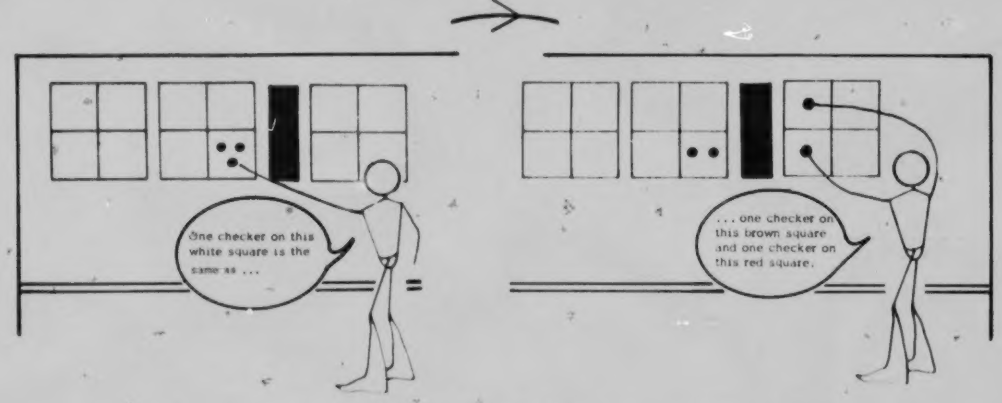
Draw a green bar[*] between the Minicomputer boards.



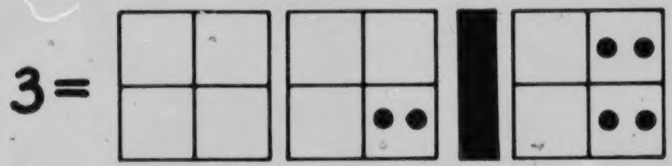
Emphasize that the green bar is there to remind us that the board to the left of the bar is still the ones' board.

T: What should we do now?

Students may still be confused and may suggest some wrong plays. It is natural for the green bar to disturb them for a short while. If this happens, make one play yourself.


One checker on this white square is the same as ...
... one checker on this brown square and one checker on this red square.

Generally students will make the next two plays without any trouble.



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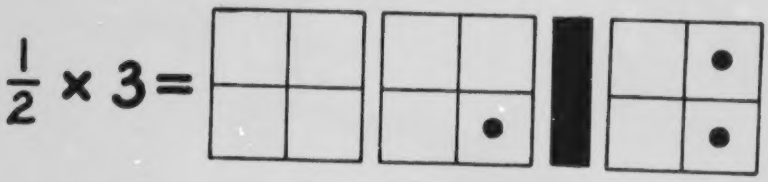
[*] The bar should be drawn with green chalk directly on your chalkboard. Do it for this lesson and all subsequent lessons involving the Minicomputer representation of decimals.

214

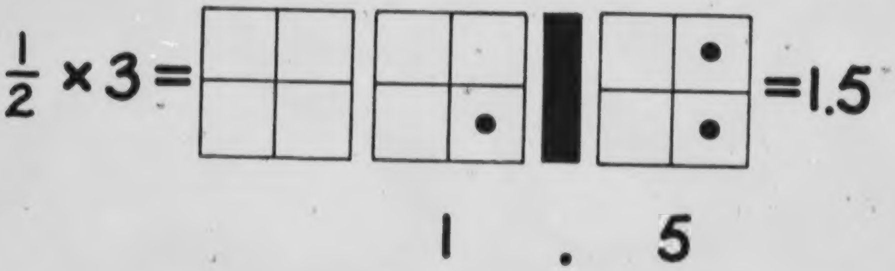
T: Who can remove some checkers so that one-half of this number will be on the Minicomputer?

The student who volunteers should remove one checker from each pair.



T: How do you suggest we write this number below the Minicomputer?

Perhaps a student will suggest using something analogous to the green bar to separate "1" and "5". If no one suggests using a decimal point to give "1.5", suggest this yourself. Many students will be acquainted with the decimal point from seeing prices in stores. Write "1.5" below and to the side of the Minicomputer.



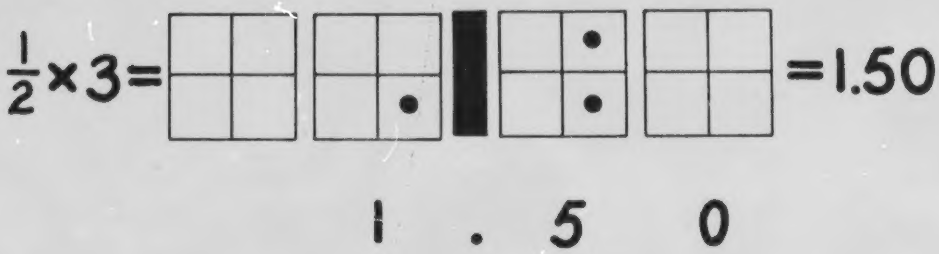
If a student suggests writing "0" below the tens' board, say that it is correct but not necessary.

T: Is this the way people usually write one and one-half dollars?

Without saying anything, add a new board to the right. Wait for the reactions of the class.

N 19

S: There are no checkers on the new board. We can write "0" below it.

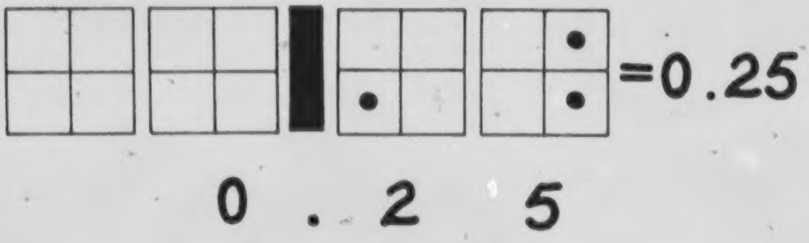


T: 1.5 (read: one point five) is the same as 1.50 (read: one point five zero).

Remove the checkers from the Minicomputer. Erase the board.

T: Who can put one quarter on the Minicomputer?
Who can write this number below the Minicomputer?

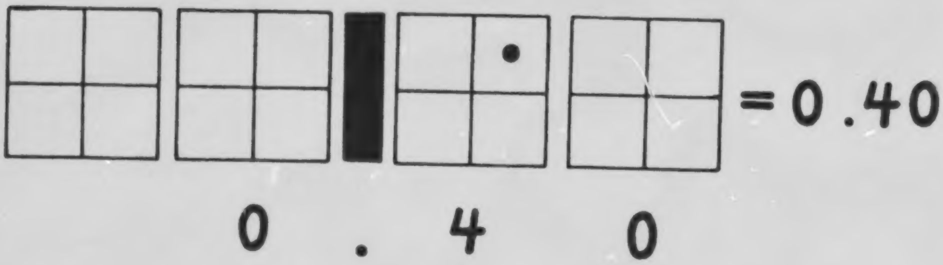
Be sure students include a zero under the board on the left and the decimal point.



T: Right. One quarter is twenty-five cents. Zero point two five.

Remove the checkers and erase the board.

T: Who can put four dimes on the Minicomputer?
Who can write this number below the Minicomputer?



T: Four dimes is forty cents and we write "0.40" (read: zero point four zero).

Point to the board directly to the left of the green bar.

T: This board is for dollars.

Point to the board directly to the right of the green bar.

T: What do you think this board is for?

S: Dimes.

Point to the board to the right of the dimes' board.

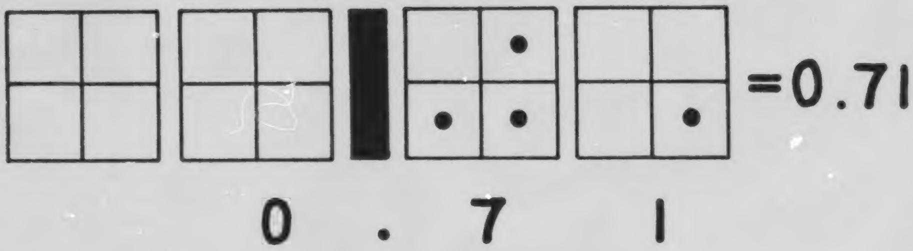
T: What do you think this board is for?

S: Pennies.

Remove the checkers and erase the board.

T: Who can put seven dimes and one penny on the Minicomputer?
Who can write this number below the Minicomputer?

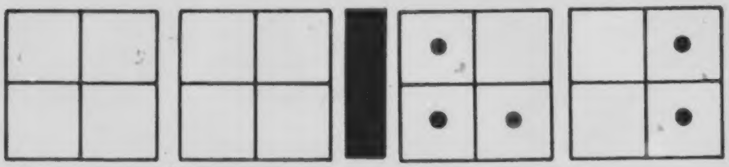
N 19



T: Seven dimes and one penny is seventy-one cents and we write "0.71" (read: zero point seven one).

Remove the checkers and erase the board.

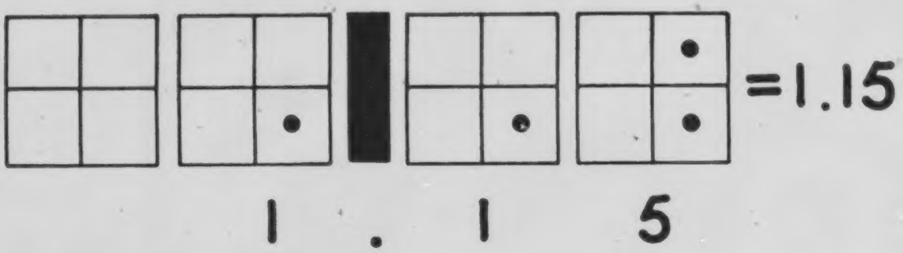
T: Who can put nine dimes on the Minicomputer with blue checkers? Who can put a quarter on the Minicomputer with red checkers?



T: Does anyone see a play we can make?

If students have difficulty verbalizing the play, say "eight dimes and two dimes is one dollar."

T: Who can write this number below the Minicomputer?



T: How much money is nine dimes and one quarter?

S: One dollar and fifteen cents.

T: We write this as "1.15" (read: one point one five).

N 20

N20 Frames of Differing Shapes

CAPSULE LESSON SUMMARY

Complete number stories which have frames of differing shapes in the same story.

MATERIALS

Teacher: None
Student: Paper; pencil

DESCRIPTION OF LESSON

Exercise 1

Write this open sentence on the board.

$$\square + \square = 20$$

T: Do you remember when we looked at sentences like this and found which numbers to put in the frames to make true number stories?

Encourage the students to comment. Remind the students that when the frames in a sentence have the same shape, they are for the same number.

T: Today we are going to look at some sentences with frames, but there will be frames of differing shapes in each sentence.

Erase the square closest to the equal sign and replace it with a triangle.

$$\square + \triangle = 20$$

T: These frames have different shapes so we do not need to put the same number in both of them. What numbers could we put in these frames to make a true number story?

As students suggest pairs of numbers, write them inside the square and triangle respectively and ask the class to determine if the number story is true. The following is a sample dialogue.

S: Put 19 in the square and 0 in the triangle.

$$\boxed{19} + \triangle{0} = 20$$

T: Is this true? (No)

Erase the numbers in the frames before asking for another possibility. Continue until three or four possible solutions have been found.

T: Let's record some of these pairs of numbers in a table.

Draw a table below the number story.

$$\square + \triangle = 20$$

□	△

T: What number could we put in the square?

Accept any number and record it in the column labeled " □ " in the table.

N 20

S: 5.

□ + △ = 20

□	△
5	

Point to the open sentence written on the board as you ask:

T: If we put 5 in the square, what number should we put in the triangle?

S: 15.

T: 5 + 15 = 20.

Record "15" in the column labeled " △ " in the table.

If someone suggests putting 10 in both the square and the triangle, be sure your class understands that this is a good solution. ("10 + 10 = 20" is true.) The numbers do not have to be the same if the frames are different, but they may be the same.

Continue asking for suggestions until your table has five pairs of numbers in it. Accept other solutions without recording them in the table. There is an infinite number of solutions to this number story. The next illustration shows five of the many solutions which could be recorded in the table.

$$\square + \triangle = 20$$

□	△
5	15
10	10
20	0
25	$\hat{5}$
$\frac{1}{2}$	$19\frac{1}{2}$

Exercise 2

Write this open sentence and chart on the board.

$$\square + \square + \triangle = 20$$

□	△

T: This number story has two squares and one triangle. The same number has to be put in both squares but a different number could be put in the triangle.

N 20

Suppose a student suggests putting 8 in each square and 4 in the triangle. Record the first suggestion inside the frames before recording it in the table.

T: Is this number story true?

S: Yes.

Record "8" and "4" in the table.

You may need to repeat the convention that the same number must be put in each square. Some students may have difficulty verbalizing their suggestions. The following is a sample dialogue.

S: 10 + 10 = 20.

T: Do you want 10 in each square and also in the triangle?

S: Yes.

Write "10" inside each frame.

$$\boxed{10} + \boxed{10} + \triangle\kern-0.9em{\scriptstyle 10} = 20$$

T: Is this number story true? (No)

S: I mean 5 goes in each square and 10 goes in the triangle.

$$\boxed{5} + \boxed{5} + \triangle\kern-0.9em{\scriptstyle 10} = 20$$

Record "5" and "10" in the table.

If the students have difficulty finding correct solutions, you could suggest a number for the square and ask them to find the number for the triangle. Encourage and help students who are trying to find solutions with negative numbers.

Continue until you have five pairs of numbers in the table. Accept other solutions without recording them in the table. There is an infinite number of solutions to this number story. Five of the many solutions are shown in this table.

$$\square + \square + \triangle = 20$$

$\square$	$\triangle$
8	4
5	10
0	20
$\hat{1}$	22
$\frac{1}{2}$	19

Exercise 3

Write this open sentence and draw this chart on the board.

$$\square + \square + \triangle + \triangle = 20$$

$\square$	$\triangle$

N 20

Continue as described in the previous activities. Remind the students that the same number goes in both squares and the same number goes in both triangles, but the number in the triangles does not have to be the same as the one in the squares. It may require several attempts before the first correct solution is found. When your table has five pairs of numbers, ask the class to look for a pattern. If necessary, read the entries in your table aloud. For example,

$$\square + \square + \triangle + \triangle = 20$$

$\square$	$\triangle$
5	5
8	2
10	0
$\hat{1}$	11
9	1

T: Do you notice a pattern to to these pairs of numbers? 5 and 5; 8 and 2; 10 and 0; $\hat{1}$ and 11; 9 and 1.

S: The numbers in each pair have a sum of 10.

If necessary, mention this yourself.

T: If 4 is in the square, what should be in the triangle? (6)
If $\hat{2}$ is in the triangle, what should be in the square? (12)

Continue with these or similar questions if your class is interested.

Exercise 4

Repeat this activity for this open sentence.

$$\square + \triangle = 1$$

After you have recorded two or three suggestions in the table, write "$\frac{1}{2}$" in a space below the square.

T: If $\frac{1}{2}$ is in the square, what number should be in the triangle? $\left(\frac{1}{2}\right)$

Continue until your table has five pairs of numbers. Five of the many solutions are shown in the table below.

$$\square + \triangle = 1$$

$\square$	$\triangle$
0	1
5	$\hat{4}$
$\frac{1}{2}$	$\frac{1}{2}$
2	$\hat{1}$
$\overset{\frown}{30}$	31

N 21

N21 Introduction to Decimal Numbers #2

CAPSULE LESSON SUMMARY
Add the cost of several items on the Minicomputer.

MATERIALS
Teacher: Minicomputer set; colored chalk
Student: Worksheets N21 *, **, ***, and ****

DESCRIPTION OF LESSON

Choose students in your class to star in these stories.

T: Gary had a birthday last week and his father took him to a toy store to buy a birthday present. Gary likes models very much, so Gary's father told him he could choose four models as his birthday present.

Allow the class to discuss what models are and what kinds of models they have. Ask the student who stars in your story to choose the kinds of models. In this dialogue, we assume Gary chooses two motorboats, a car, and an airplane. Draw a large string on the board with colored chalk. As you memtion the price of each model, draw a dot for it inside the string.

Draw a dot and label it "0.62".

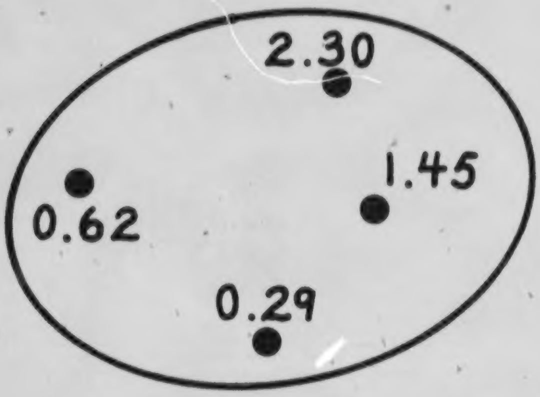
T: Gary chose a car that costs this much.
How much does the car cost? (Sixty-two cents)

Draw another dot and label it "2.30".

T: Gary chose an airplane that costs this much.
How much does the airplane cost? (Two dollars and thirty cents)

Draw two more dots and label them, "0.29" and "[illegible]".

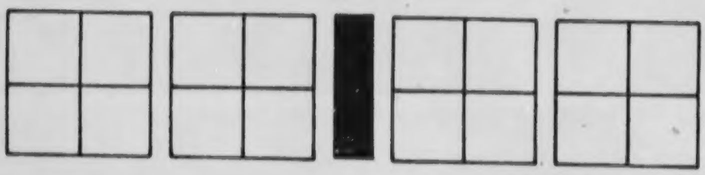
T: Gary also chose two different motorboats. What are the prices for the motorboats? (Twenty-nine cents and one dollar and forty-five cents.)



T: How much must Gary's father pay for the four models Gary chose?

Allow the students a minute to think about this question. Record several guesses on the board. If the guesses are not particularly good, you may wish to ask the class to estimate the answer and only accept guesses within an agreed upon range. For example, it should be easy for students to see that the answer is more than $3, because one of the models costs more than $2 and another costs more than $1. Also, the answer is less than $6 so you might agree to accept guesses between $3 and $6.

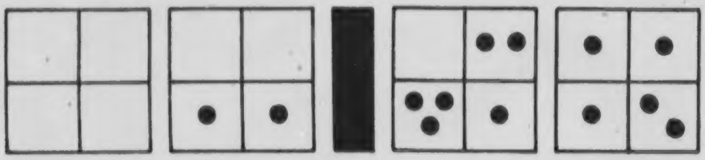
Display four Minicomputer boards. Draw a green bar between the second and third boards. Leave more space between these boards than between the other boards.



N 21

T: Let's find the price of the four models on the Minicomputer. Who can put $2.30 on the Minicomputer?

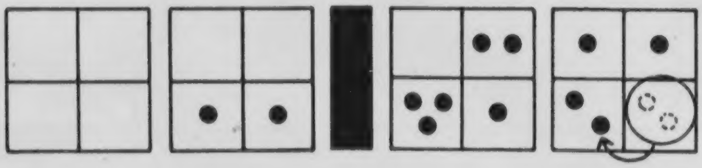
Continue until all four numbers have been put on the Minicomputer. Take advantage of any errors to explain that the number of dollars is put on the boards to the left of the green bar and the number of cents is put on the boards to the right of the green bar.



T: How can we make this number easier to read?

S: Make plays.

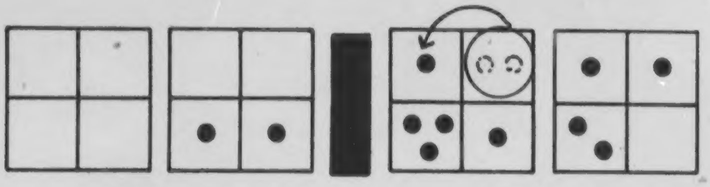
Invite students one at a time to make plays. Watch their plays closely for the green bar may cause some confusion. Help students to verbalize which plays they are making; for example,



S: 1 + 1 = 2.

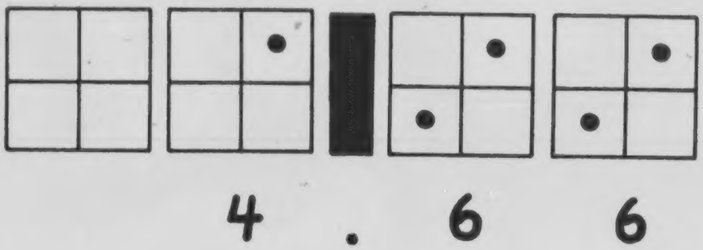
T: 1 penny + 1 penny = 2 pennies.

Sometimes a student may make a play and not say anything. In this case you should name the play.



T: 4 dimes + 4 dimes = 8 dimes.

Continue until you have this configuration on the Minicomputer. Ask someone to write the number below the Minicomputer.



T: How much did the four models cost Gary's father?

S: Four dollars and sixty-six cents.

T: Now I am going to tell you another story. Linda's mother gave her two dollar bills to go to the supermarket. Linda wants to buy a package of coconut and a package of chocolate chips. What do you think she is going to make with the coconut and chocolate chips?

Let students make a few suggestions and then continue.

T: The coconut cost 50¢ and the chocolate chips cost 75¢. Does Linda have enough money to buy both?

N 21

Write this problem on the board.

$$0.75 + 0.50 = ?$$

S: Yes, 50¢ is less than $1 and 75¢ is less than $1.

T: What coins make 50¢?

S: Two quarters.

S: Five dimes.

T: How many pennies . . . ? (Fifty)
Could we have 50¢ with only one coin? (Yes, a half-dollar)
How many nickels . . . ? (Ten)

T: What coins make 75¢?

S: Three quarters.

S: Seven dimes and a nickel.

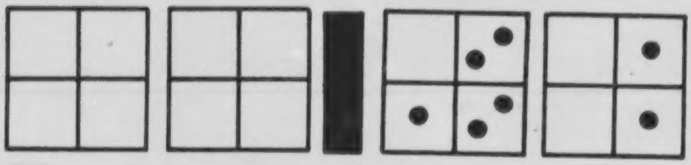
S: Seventy-five pennies.

T: How many nickels make 75¢? (Fifteen)

If this last question is difficult for your students, remind them that seven dimes and a nickel make 75¢.

T: How much is 75¢ + 50¢?

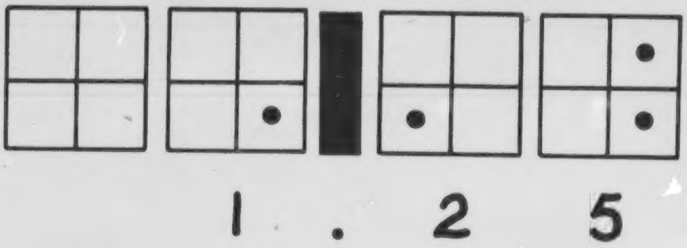
Allow several students to whisper the answer to you or show you by writing it on a piece of paper. Ask students to put 0.75 and 0.50 on the Minicomputer.



T: What number is on the Minicomputer?

S: I don't know; we need to make some plays.

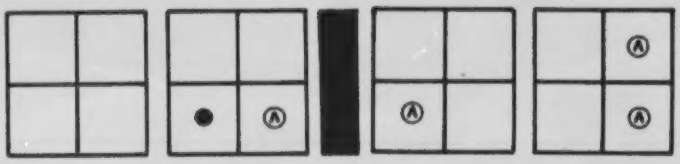
Invite students to make plays until you have this configuration, and then ask a student to write the number below the Minicomputer.



T: How much do the coconut and the chocolate chips cost? ($1.25)
Does Linda have enough money? (Yes)
How much change will Linda get back from the two dollars?

Allow several students to whisper the answer to you. Write "2.00 - 1.25 =" on the board. Remove the checkers from the Minicomputer. Ask a student to put 2 on the Minicomputer with regular checkers and another to put $\widehat{1.25}$ on the Minicomputer with negative checkers.

N 21



T: What should we do now?

S: Make some backward plays.

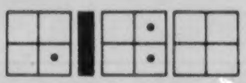
Make plays to get pairs of checkers, one positive with one negative on the same square. Remove such pairs when they are formed. Continue until 0.75 is on the Minicomputer with all positive checkers. Ask which number is on the Minicomputer and complete the number story on the board.

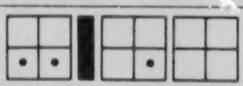
$$2.00 - 1.25 = 0.75$$

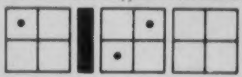
Worksheets N21 *, **, ***, and **** are available for individual work. On the worksheets, a gray bar is used to separate the dollars' board from the dimes' board.

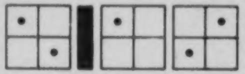
Name ____________ **N 21 ***

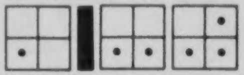
What number is on the Minicomputer ?


= 1.6


= 3.1

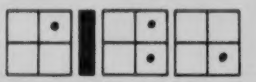

= 8.6

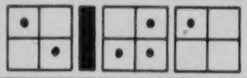

= 9.86

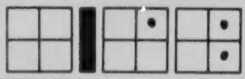

= 2.37

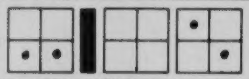
Name ____________ **N 21 ****

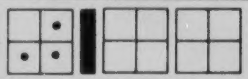
Put these numbers on the Minicomputer.

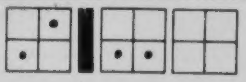
4.51 =


9.78 =


0.45 =


3.09 =


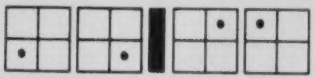
7.0 =


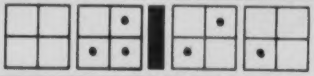
6.3 =


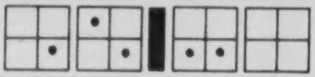
Standard configurations are given here ; however, other solutions are possible.

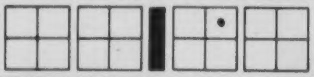
Name ____________ **N 21 *****

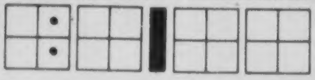
Put these numbers on the Minicomputer.

21.48 =


7.62 =


19.3 =


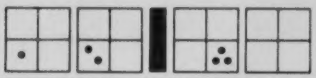
0.4 =


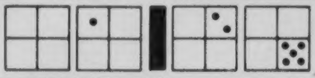
50 =


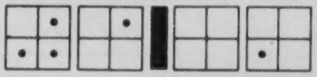
Standard configurations are given here ; however, other solutions are possible.

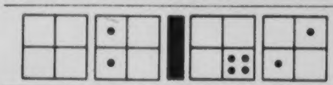
Name ____________ **N 21 ******

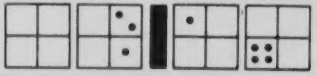
What number is on the Minicomputer ?

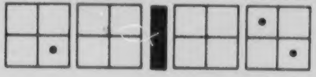

= 24.3


= 8.85


= 74.02


= 10.46


= 9.88


= 10.09

N 22

N22 Composition of Functions #2

CAPSULE LESSON SUMMARY

Draw +9 arrows in arrow roads with alternating +10 and - 1 arrows.

MATERIALS

Teacher: Colored chalk
Student: Paper; colored pencils

DESCRIPTION OF LESSON

Begin the lesson with a short mental arithmetic activity involving the functions +10, - 10, +20, - 20. A sample dialogue is given here.

T: I will choose a number. You tell me which number is 10 more than my number.

6. (16)
20. (30)
23. (33)

⋮

T: Now I am going to change the rule. You tell me which number is 10 less than my number.

10. (0)
9. ($\hat{1}$)
17. (7)

⋮

T: I am going to change the rule again. You tell me which number is 20 more than my number.

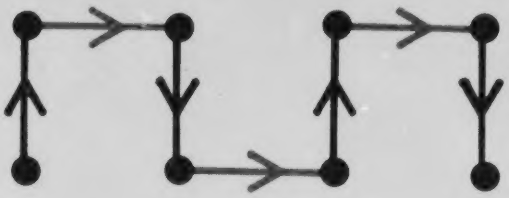
5. (25)
10. (30)
12. (32)

⋮

T: Now tell me which number is 20 less than my number.

24. (4)
50. (30)
51. (31)
⋮

Draw the following arrow picture on the board and ask the students to copy it very carefully on their papers. Instruct them to start the arrow picture close to the top edge of the paper so they will have room to draw more arrows later.



Help students who have difficulty getting started. When everyone has several arrows drawn, ask the class if they notice any patterns in this arrow picture.

S: It goes red, blue, red, blue, red, blue.

S: All the blue arrows are going across.

S: All the red arrows are going up and down.

T: Now, each of you should extend this arrow road on your paper. You may draw an arrow road any way you wish except be sure to follow my pattern: red, blue, red, blue, red, blue, and so on.

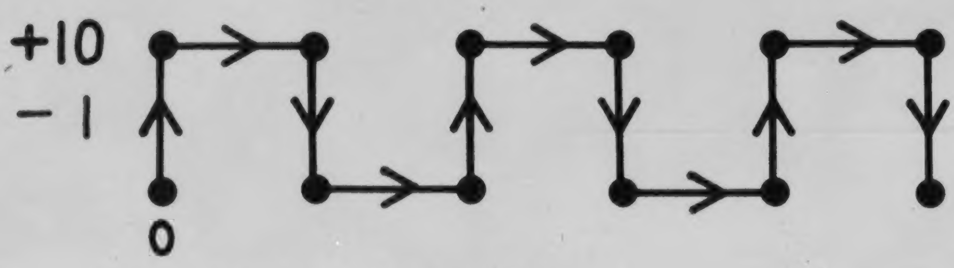
Extend the arrow picture on the board. It is not necessary that the students copy your layout as long as they are careful to alternate the colors. Encourage students to continue drawing arrows until everyone has a road with at least seven arrows.

N 22

As you ask students to record each of the following keys on their papers, record the same on the board.

T: With your red pencil, write "+10" on your paper.
With your blue pencil, write "-1" on your paper.
With your regular pencil, label the starting dot "0".

Your picture might look like this one.

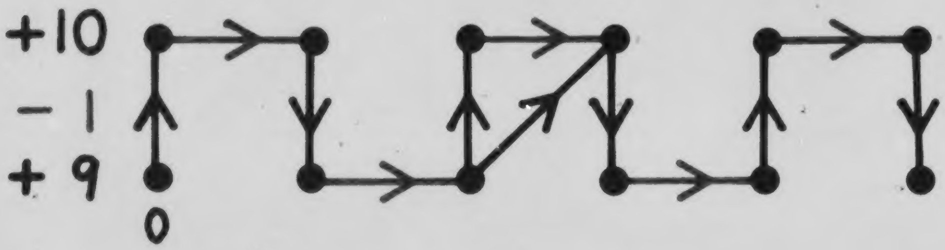


T: What are the red arrows for? (+10)
What are the blue arrows for? (-1)
Follow the red and blue arrows and label the dots on your paper.

Walk around the room and help students who are having difficulty. Encourage students who finish quickly to extend their arrow pictures. Continue until everyone has labeled at least the first eight dots.

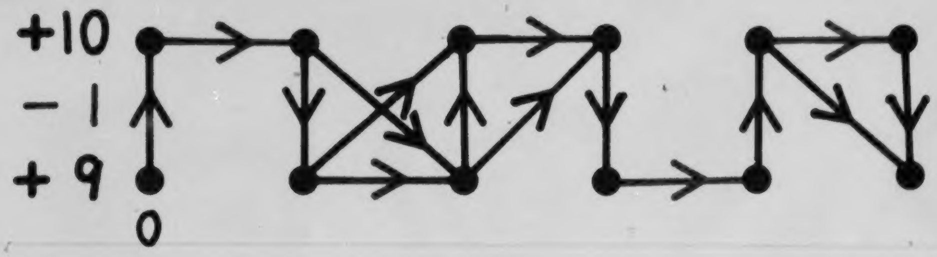
T: We are going to draw some green arrows in our arrow picture.
The green arrows will be for +9. Write "+9" on your paper with your green pencil.

Write "+9" in green near the picture. Ask someone to draw a +9 arrow on the board. Your picture might liik like this one.



T: Look at your arrow picture and draw all the +9 arrows you can.

As students finish, ask them to draw some green arrows in the arrow picture on the board. It is not necessary that all the possible green arrows be drawn for the picture on the board; for example your picture might look like this one.



T: How did we know where to draw green arrows in this picture on the board when the dots were not labeled?

Commend any student who says that a +10 arrow followed by a -1 arrow (or a -1 arrow followed by a +10 arrow) is the same as a +9 arrow.

Point to the starting dot of a red arrow with your left index finger. Trace the red arrow and then the blue arrow following it with your right index finger as you say:

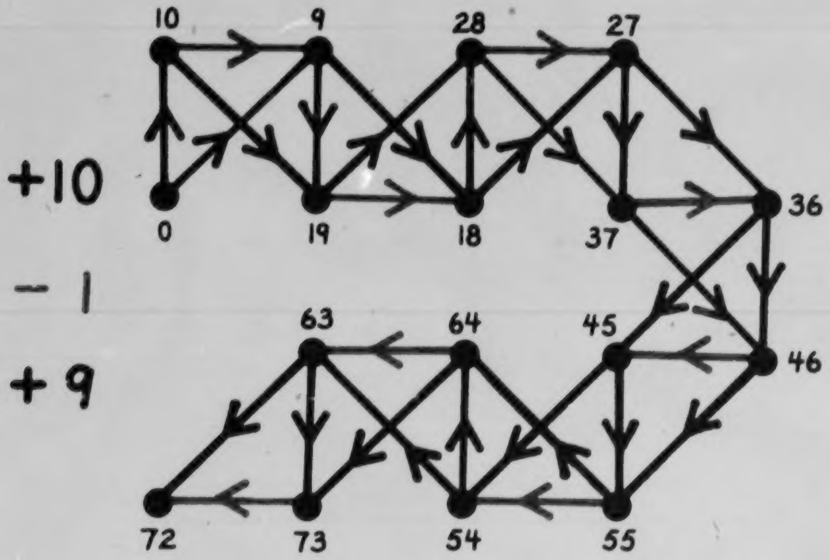
T: A +10 arrow followed by a -1 arrow is the same as a +9 arrow.

With your left index finger still on the same dot, trace the appropriate green arrow with your right index finger.

Repeat this procedure for a blue arrow followed by a red arrow to note that a -1 arrow followed by a +10 arrow is the same as a +9 arrow.

N 22

Let students draw more green arrows in their pictures for a few minutes. A completed picture is shown here.


+10
− 1
+ 9
10
9
28
27
0
19
18
37
36
63
64
45
46
72
73
54
55

End the lesson with some mental arithmetic involving +10 and +9. Use the following or any similar problems which are appropriate for the abilities of your students.

T: What number is 43 + 10? (53)
. . . 43 + 9? (52)
. . . 60 + 10? (70)
. . . 60 + 9? (69)
. . . 78 + 10? (88)
. . . 78 + 9? (87)

N23 Multiplication Problems

CAPSULE LESSON SUMMARY

Calculate "2 × 65" in a variety of ways by considering 65 as the sum of smaller numbers. Check this calculation on the Minicomputer. Use "2 × 65 = 130" to calculate "2 × 265" and "2 × 565". Do a similar activity starting with "2 × 96".

MATERIALS

Teacher: Minicomputer set
Student: Minicomputer set; Worksheets N23 *, **, ***, and ****; paper

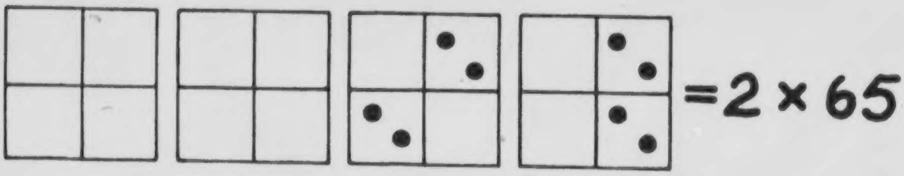
DESCRIPTION OF LESSON

Exercise 1

Display four Minicomputer boards and provide each student with four individual Minicomputer boards, checkers, and some paper.

T: Put 2 × 65 on your (individual) Minicomputer. Remember 2 × 65 is the same number as 65 + 65. Do not make any plays now.

Help students who are having difficulty putting the correct configuration on their Minicomputers. Ask a student to put 2 × 65 on the demonstration Minicomputer.



T: How can we calculate "2 × 65" without making any plays on the Minicomputer?

N 23

Many suggestions are possible; for example,

$$\begin{aligned} 2 \times 65 &= (2 \times 60) + (2 \times 5) \\ &= 120 + 10 \\ &= 130 \end{aligned}$$

or

$$\begin{aligned} 2 \times 65 &= (2 \times 40) + (2 \times 20) + (2 \times 5) \\ &= 80 + 40 + 10 \\ &= 130 \end{aligned}$$

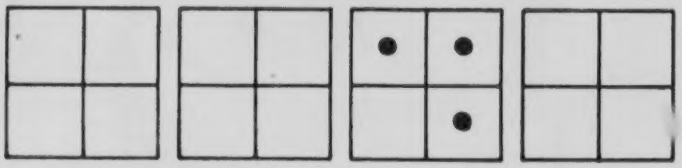
Encourage students to choose their own methods of doing the calculation and let them share their ideas with the class. Indicate on the chalkboard some of their approaches to the calculations.

Write this number story on the board.

$$2 \times 65 = 130$$

T: Now make plays on your Minicomputer.

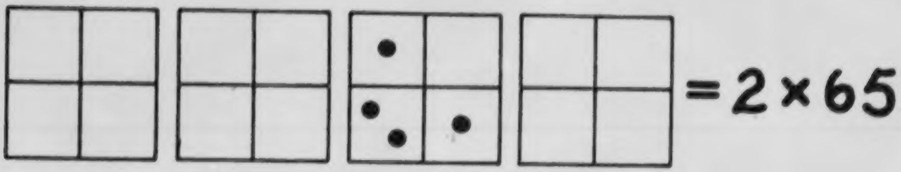
Very likely several students will make plays resulting in this configuration.



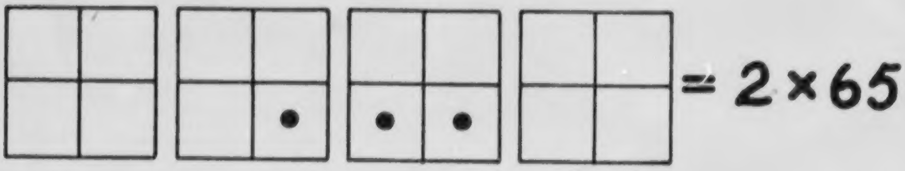
Put this configuration on the demonstration Minicomputer, and call the class' attention to the board.

T: Some of you have checkers on your Minicomputers like this. Can someone suggest a play which could be made? Remember, we want to make the number easier to read.

S: Make a "40 = 20 + 20" play . . .



and then make an "80 + 20 = 100" play . . .



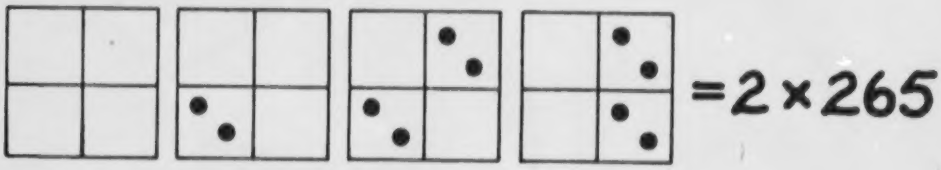
T: Making an "80 + 20 = 100" play as soon as possible helps to avoid having to make backward plays later. What number is on the Minicomputer?

S: 130.

Remove the checkers from the demonstration Minicomputer.

T: Put 2 × 265 on your (individual) Minicomputer. Remember 2 × 265 is the same number as 265 + 265. Without making any plays, can you decide which number 2 × 265 is?

Check that students put the correct configuration on the Minicomputer. Ask a student to put 2 × 265 on the demonstration Minicomputer.



N 23

T: We already know that 2 × 65 = 130. Does this help us to calculate "2 × 265"?

S: Yes, because 2 × 65 = 130; 2 × 200 = 400; so all we need to do is add "130 + 400".

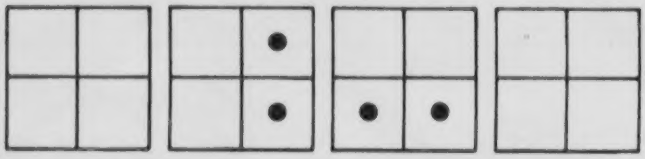
Write the problem on the board and ask a student to do the addition.

$$\begin{array}{r} 130 \\ +400 \\ \hline 530 \end{array}$$

If no one suggests this calculation, ask these questions and write each number story on the board.

T: 2 × 65 = 130.
2 × 200 = . . . ? (400)
2 × 265 = . . . ? (530)

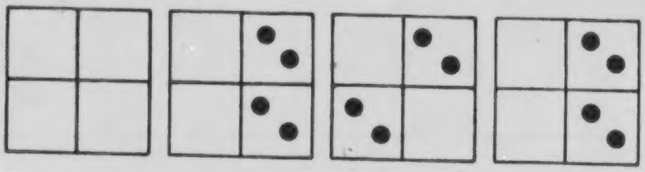
Accept other explanations but emphasize that this method (described above) is and easy way to solve this problem. Ask the students to make plays on their Minicomputers. You may need to help some students if they reach a position where a "40 = 20 + 20" play is needed. Students should arrive at the standard configuration for 530.



$= 2 \times 265 = 530$

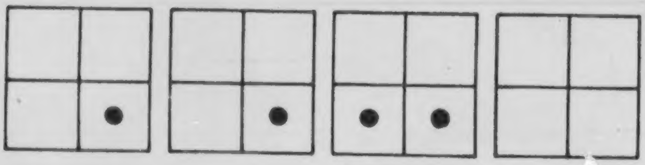
Invite someone to show a sequence of plays on the demonstration Minicomputer which will lead to the configuration shown above.

Repeat this activity to calculate "2 × 565".



$= 2 \times 565$

$$2 \times 65 = 130$$
$$2 \times 500 = 1{,}000$$
$$2 \times 565 = 1{,}130$$



$= 2 \times 565$

$= 1{,}130$

Remove the checkers from the Minicomputer and erase the board.

Exercise 2

Write these problems on the board.

$$2 \times 96 =$$
$$2 \times 196 =$$
$$2 \times 396 =$$
$$2 \times 596 =$$

T: Copy these problems on your paper. You may use your Minicomputer, if you wish, to do the calculations.

Quickly check each student's answer to the first problem. If students calculate "2 × 96" incorrectly, it may cause them to get all the answers wrong. As necessary, help students make the plays needed to calculate "2 × 96" on their individual Minicomputers.

When some students have solved all the problems, ask a student to put 2 × 96 on the demonstration Minicomputer. Make the plays collectively and complete the number story.

$$2 \times 96 = 192$$

T: We know 2 × 96 = 192. What number is 2 × 196? (392)
How do you know?

S: 2 × 96 = 192; 2 × 100 = 200; so 2 × 196 = 392.

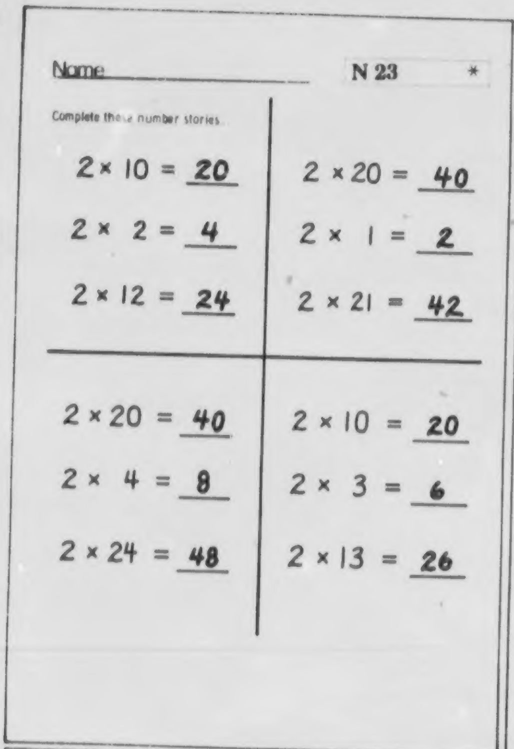
Complete the number story and continue until all the number stories are complete.

$$\begin{aligned} 2 \times 96 &= 192 \\ 2 \times 196 &= 392 \\ 2 \times 396 &= 792 \\ 2 \times 596 &= 1{,}192 \end{aligned}$$

If your class is interested, you may wish to continue by doing the following or similar calculations.

3 × 25	4 × 35
3 × 225	4 × 235
3 × 325	4 × 535
3 × 425	4 × 1,035

Worksheets N23 *, **, ***, and **** are available for individual work.


Name

N 23 *

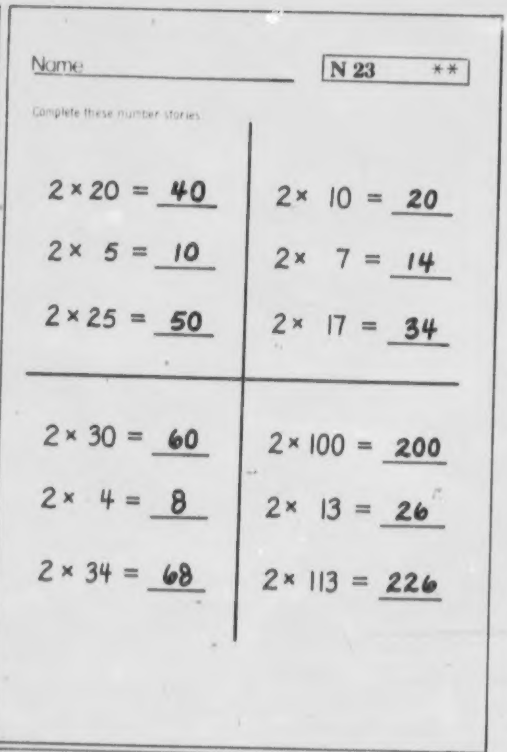
Complete these number stories

2 × 10 = 20
2 × 2 = 4
2 × 12 = 24

2 × 20 = 40
2 × 1 = 2
2 × 21 = 42

2 × 20 = 40
2 × 4 = 8
2 × 24 = 48

2 × 10 = 20
2 × 3 = 6
2 × 13 = 26


Name

N 23 **

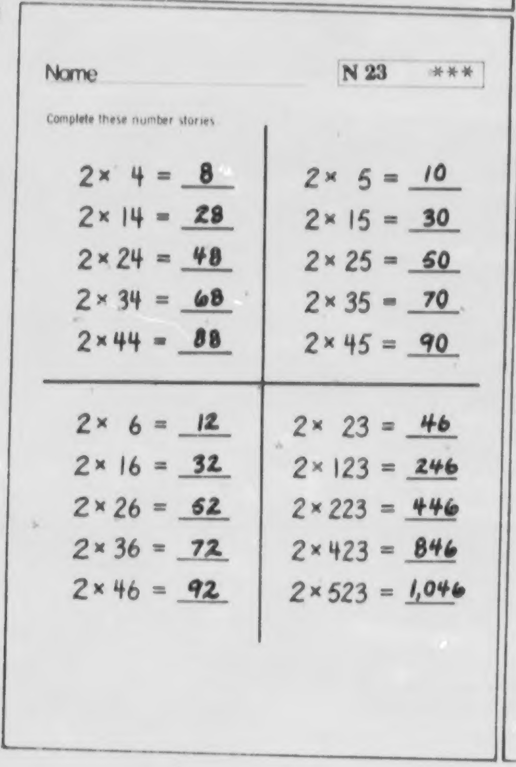
Complete these number stories

2 × 20 = 40
2 × 5 = 10
2 × 25 = 50

2 × 10 = 20
2 × 7 = 14
2 × 17 = 34

2 × 30 = 60
2 × 4 = 8
2 × 34 = 68

2 × 100 = 200
2 × 13 = 26
2 × 113 = 226


Name

N 23 ***

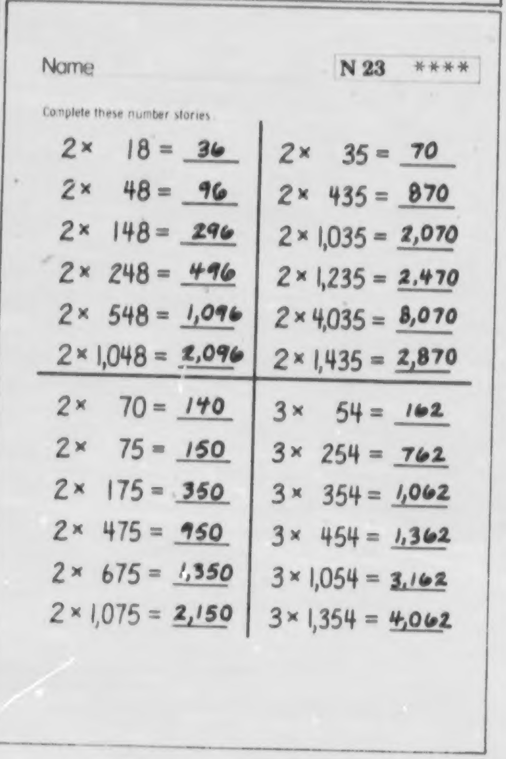
Complete these number stories

2 × 4 = 8
2 × 14 = 28
2 × 24 = 48
2 × 34 = 68
2 × 44 = 88

2 × 5 = 10
2 × 15 = 30
2 × 25 = 50
2 × 35 = 70
2 × 45 = 90

2 × 6 = 12
2 × 16 = 32
2 × 26 = 52
2 × 36 = 72
2 × 46 = 92

2 × 23 = 46
2 × 123 = 246
2 × 223 = 446
2 × 423 = 846
2 × 523 = 1,046


Name

N 23 ****

Complete these number stories

2 × 18 = 36
2 × 48 = 96
2 × 148 = 296
2 × 248 = 496
2 × 548 = 1,096
2 × 1,048 = 2,096

2 × 35 = 70
2 × 435 = 870
2 × 1,035 = 2,070
2 × 1,235 = 2,470
2 × 4,035 = 8,070
2 × 1,435 = 2,870

2 × 70 = 140
2 × 75 = 150
2 × 175 = 350
2 × 475 = 950
2 × 675 = 1,350
2 × 1,075 = 2,150

3 × 54 = 162
3 × 254 = 762
3 × 354 = 1,062
3 × 454 = 1,362
3 × 1,054 = 3,162
3 × 1,354 = 4,062

N 24

N24 The Function 10 ×

CAPSULE LESSON SUMMARY

Multiply a number by 10 on the Minicomputer by moving each checker in the configuration from one square to the square of the same color on the next board to its left. Use this idea to help label the dots in 10 × arrow roads. Build roads with exactly one 10 × arrow and three +10 arrows. Notice how the order in which the arrows appear affects the ending number of the road. Decide which such road has the largest ending number and which has the smallest.

MATERIALS

Teacher:	Minicomputer set; colored chalk
Student:	Paper; colored pencils

DESCRIPTION OF LESSON

Exercise 1

Begin the lesson with mental arithmetic involving 10 ×. Display four Minicomputer boards. Put a checker on the 2-square.

T: What number is on the Minicomputer? (2)
What number is 10 × 2? (20)

Move the checker from the 2-square to the 20-square as you say, "10 × 2 = 20."

T: What number is 10 × 20? (200)

Move the checker from the 20-square to the 200-square as you say, "10 × 20 = 200."

T: What number is 10 × 200? (2,000)

Move the checker from the 200-square to the 2,000-square as you say, "10 × 200 = 2,000."

T: What number is 10 × 7? (70)

Write "10 × 7 = 70" on the board.

T: What number is 10 × 70? (700)

Write "10 × 70 = 700" on the board.

T: What number is 10 × 700? (7,000)

Write "10 × 700 = 7,000" on the board.

You should have this list of number stories on the board.

$$\begin{aligned} 10 \times 7 &= 70 \\ 10 \times 70 &= 700 \\ 10 \times 700 &= 7{,}000 \end{aligned}$$

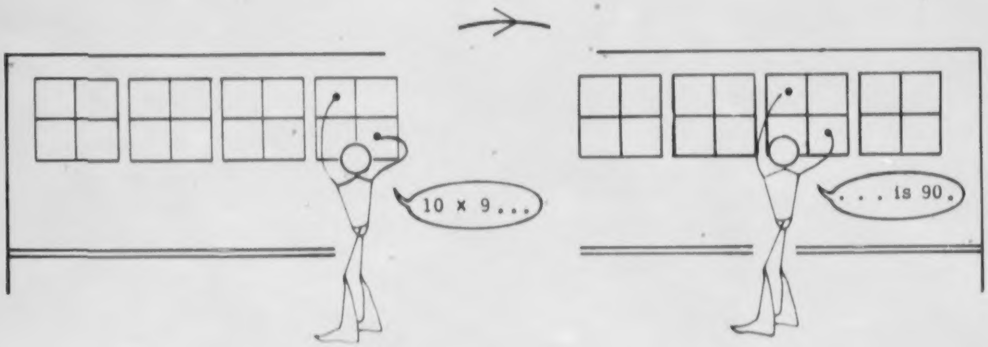
Ask the students if they notice any patterns in this list of number stories.

Clear the Minicomputer and put a checker on the 8-square and another checker on the 1-square.

T: What number is on the Minicomputer? (9)
What number is 10 × 9? (90)

N 24

Move the checkers simultaneously to the tens' board: the checker from the 8-square to the 80-square and the checker from the 1-square to the 10-square.



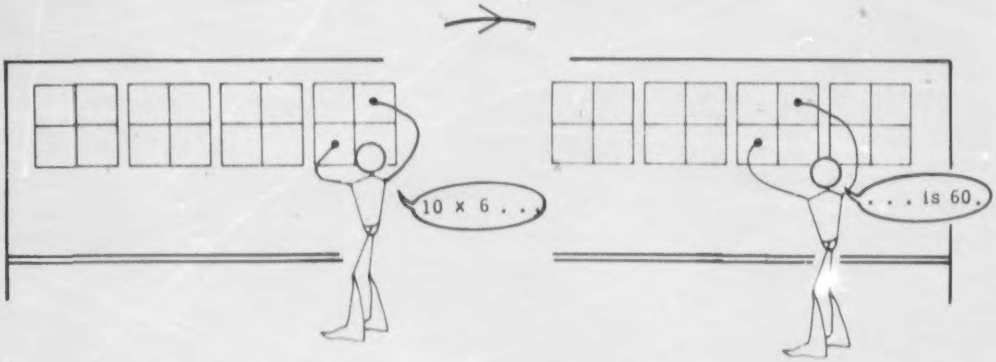
Continue this activity with "10 × 90" and "10 × 900".

Exercise 2

Ask a student to put 6 on the Minicomputer.

T: What number is 10 × 6? (60)
Who can move these checkers to put 10 × 6 on the Minicomputer?

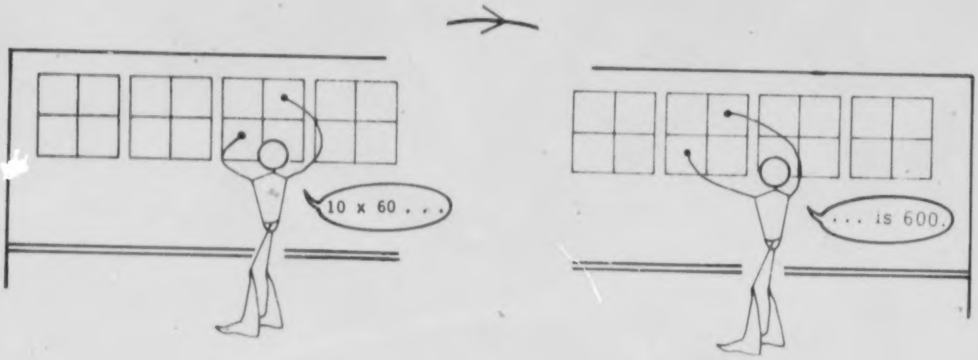
The volunteer should move the checkers from the ones' board to the tens' board. Then repeat the move yourself, moving the checkers simultaneously.



T: What number is on the Minicomputer now? (60)

Who can move these checkers to put 10 × 60 on the Minicomputer?

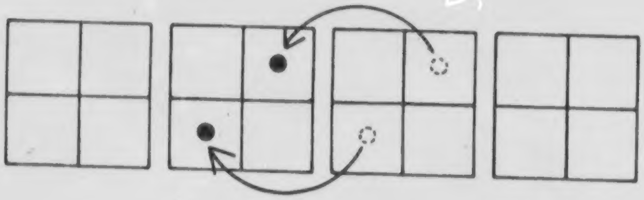
Accept any correct method and then repeat the move from the tens' board to the hundreds' board yourself, moving the checkers simultaneously.



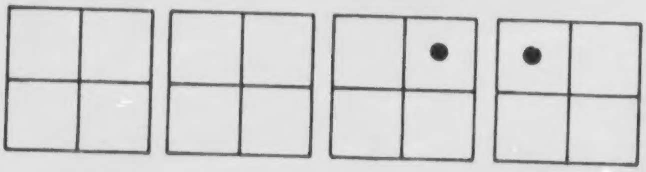
Avoid verbalizing a rule for the students to follow.

T: What number is 10 × 600? (6,000)

Who can move these checkers to put 10 × 600 on the Minicomputer?



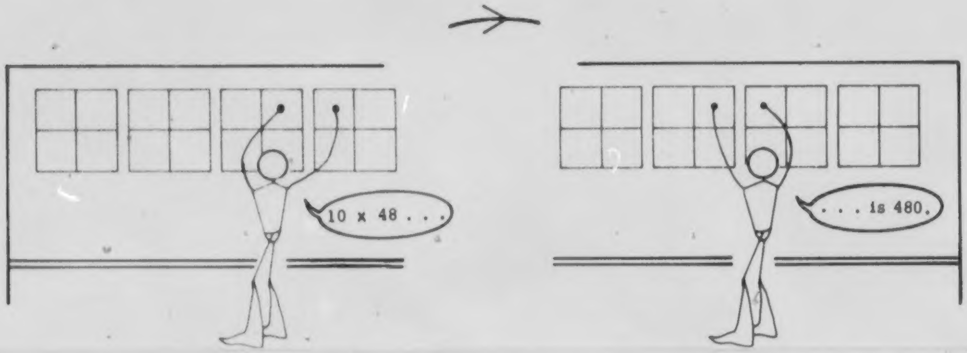
Ask a student to put 48 on the Minicomputer. If necessary, make plays until the standard configuration for 48 is obtained.



N 24

T: How could we move these checkers to put 10 × 48 on the Minicomputer?

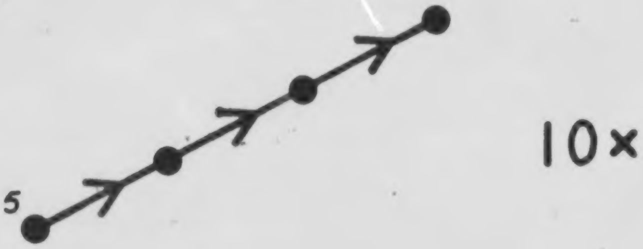
Commend anyone who moves the checker from the 40-square to the 400-square and the checker from the 8-square to the 80-square. Demonstrate this move yourself.



Ask a student to put 10 × 480 on the Minicomputer. Conclude that 10 × 480 = 4,800.

Exercise 3

Draw this arrow picture on the board leaving space to extend it later.



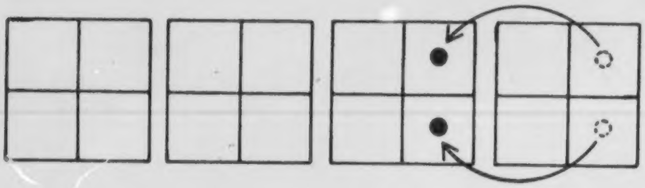
Ask the students to copy this arrow picture and label the dots. Encourage students who finish quickly to extend the arrow picture. Allow enough time for all the students to at least copy the arrow picture. Trace the arrow starting at 5.

T: What are the red arrows for? (10 x)

What number is 10 x 5? (50)

Ask a student to put 5 on the Minicomputer.

T: Who can move these checkers to put 10 x 5 on the Minicomputer?



T (tracing the arrow starting at 5): 10 x 5 = 50.

Label the second dot from the left "50".

T (tracing the arrow starting at 50): 10 x 50 = . . . ? (500)

Who can move the checkers so that 10 x 50 is on the Minicomputer?

Continue this activity until all the dots are labeled.

T: Do you see a pattern?

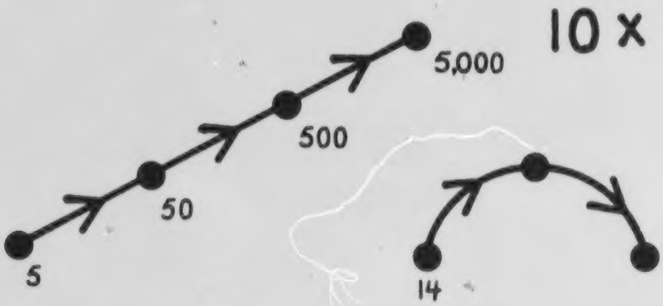
S: Each number has one more zero at the end of it than the number before it.

N 24

T: If we drew another red arrow, what would the next number be? (50,000)

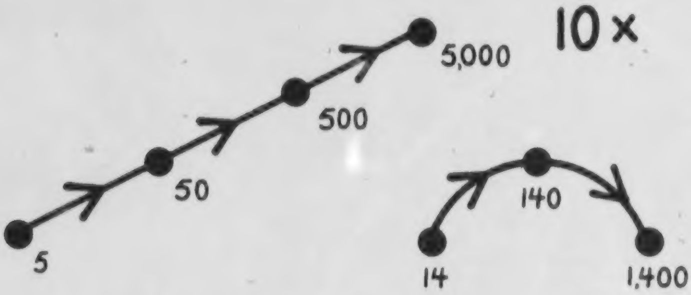
And after 50,000 . . . ? (500,000)

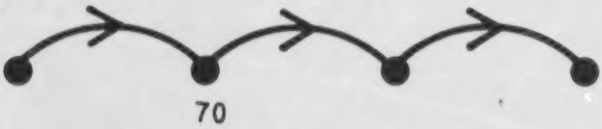
Draw another arrow road in your 10× arrow picture.


10×
5,000
500
50
5
14

Continue as before. Ask the students to work individually; then label the dots collectively.

Draw another arrow road in your 10× arrow picture.


10×
5,000
500
50
5
140
14
1,400



Ask the students to work individually. When several students have labeled all the dots, ask a student to label the dots on the board. Ask again if they see a pattern.

Exercise 4

Ask the students to turn their papers over and draw arrow roads which each have exactly one red arrow and three blue arrows. Emphasize that in each road they can draw the arrows in any order.

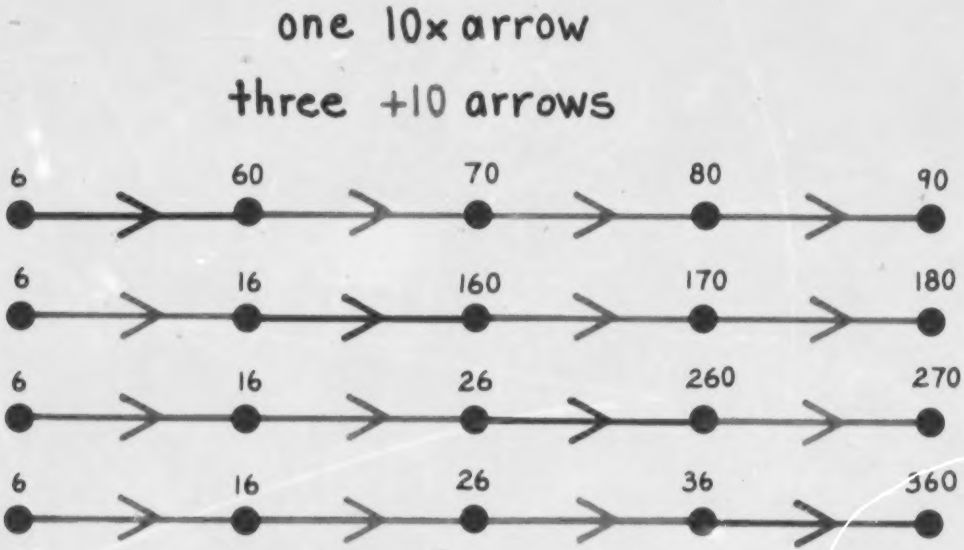
T: The red arrow is for 10x. Use exactly one red arrow in each arrow road.

Write "one **10x** arrow" on the board.

T: The blue arrows are for +10. Use exactly three blue arrows in each arrow road.

Write "three +10 arrows" on the board. When most of the students have drawn at least one arrow road, instruct students to label the starting dot of each of their arrow roads "6" and then to label the other dots.

Allow enough time so that each student can label all the dots on at least one arrow road. There are four possible arrow roads.



N 24

Suppose a student named "Jerry" drew at least two different arrow roads. This is a sample dialogue.

T: Jerry, were the ending numbers of your roads the same?

S: No.

T: Tell us one of the ending numbers.

S: 180.

T: Does anyone else have a road which ends at 180?

S: Yes.

T: Jerry, describe your arrow road which ends at 180.

S: A blue arrow; then a red arrow; then a blue; and then another blue.

Students may comment that they have arrow roads like Jerry's.

T: Does anyone have a road with an ending number larger than 180?

S: I have a road which ends at 270.

S: I have one which ends at 360.

T: Describe the road ending at 360.

S: Blue; blue; blue; red.

T: Can we build a road with one red and three blue arrows which ends at a number larger than 360? (No)

Let the students discuss the situation freely. Perhaps someone will comment that there are only four places in the road to put the red arrow and putting it at the end gives the largest ending number.

T: Does anyone have a road with an ending number smaller than 180?

S: I have one which ends at 90. The arrows are red; blue; blue; blue.

T: Can we build a road with one red and three blue arrows which ends at a number smaller than 90? (No)

Again let the students discuss the situation freely. Perhaps someone will comment that putting the red arrow first gives the smallest ending number.

N 25

N25 Addition Algorithm #3

CAPSULE LESSON SUMMARY

Use the addition algorithm to solve addition problems with two, three, and four digit numbers.

MATERIALS

Teacher: None

Student: Paper; pencil; ADDITION PROBLEMS booklet

DESCRIPTION OF LESSON

Tell the following story or a similar one to the class. Choose two of your students to be the stars of this story.

T: Oscar and Heather volunteered to bake cookies for the school picnic. Oscar baked 437 cookies and Heather baked 283 cookies. Which student prepared more cookies for the picnic?

S: Oscar; 437 is more than 283.

Write "437 > 283" on the board.

T: How many cookies will there be at the school picnic?

S: We need to add 437 and 283.

Write this problem on the board.

$$\begin{array}{r} 437 \\ +283 \\ \hline \end{array}$$

Ask the students to solve this problem on their papers. Give individual help to students who have difficulty doing the addition.

T: First, we add the numbers in the ones' column.

Ask for a volunteer to do the addition at the board and to explain what is being done.

S: 7 + 3 = 10. Write "1" above the tens' column and "0" below the ones' column.

$$\begin{array}{cccc} & & 1 & \\ & 4 & 3 & 7 \\ + & 2 & 8 & 3 \\ \hline & & & 0 \end{array}$$

T: Seven ones plus three ones is ten ones, or 10. We record "1" above the tens' column for 10 and "0" below the ones' column. Next, we add the numbers in the tens' column.

Ask for a volunteer to do the addition at the board and to explain.

S: 1 + 3 + 8 = 12. Write "2" below the tens' column and "1" above the hundreds' column.

$$\begin{array}{cccc} & 1 & 1 & \\ & 4 & 3 & 7 \\ + & 2 & 8 & 3 \\ \hline & & 2 & 0 \end{array}$$

T: One ten plus three tens plus eight tens is twelve tens, or 120. We record "2" below the tens' column for 20 and "1" above the hundreds' column for 100. Next, we add the numbers in the hundreds' column.

N 25

Ask for a volunteer to do the addition at the board and to explain.

S: 1 + 4 + 2 = 7. Write "7" below the hundreds' column.

$$\begin{array}{r} \scriptstyle 1\ 1 \\ 437 \\ +283 \\ \hline 720 \end{array}$$

T: One hundred plus four hundreds plus two hundreds is seven hundreds. We record "7" below the hundreds' column.

Conclude that 437 + 283 = 720. Proceed in the same way to solve these problems.

$$\begin{array}{r} 724 \\ +876 \\ \hline \end{array} \qquad \begin{array}{r} 625 \\ 236 \\ +137 \\ \hline \end{array}$$

Write the following problems on the board and ask the students to solve them on their papers. Allow a few minutes for individual work and then solve the problems collectively at the board.

$$\begin{array}{r} 217 \\ 353 \\ +224 \\ \hline \end{array} \qquad \begin{array}{r} 75 \\ 142 \\ +\ 36 \\ \hline \end{array} \qquad \begin{array}{r} 3{,}156 \\ 2{,}326 \\ +2{,}406 \\ \hline \end{array}$$

Distribute the ADDITION PROBLEMS booklets and instruct students to continue working independently for 10 to 15 minutes. At the end of the class period, collect the booklets for your review. (This is the last lesson using this booklet.)

N26 Number Friends

CAPSULE LESSON SUMMARY

Find pairs of numbers whose sum is 20, pairs of numbers whose sum is 10, and pairs of numbers whose sum is 100.

MATERIALS

Teacher: Colored chalk
Student: Paper ; pencil ; colored pencils

DESCRIPTION OF LESSON

Exercise 1: 20-friends

Write this open sentence on the board.

$$\square + \triangle = 20$$

Remind the students that since the frames have different shapes, the numbers in the square and in the triangle may be different or they may be the same.

T: What number could we put in the square and what number could we put in the triangle to get a true number story? There are many answers.

When you have received several correct answers, explain to the class that since there is more than one correct answer you will record some of the answers in a table.

N 26

Draw the following table on the board under the open sentence.

□ + △ = 20

□	△

Record students' suggestions in the table. When your table has five pairs of numbers in it, accept other solutions without recording them. Ask whether a negative number could go in the square or the triangle , and record at least one solution where one of the numbers in the pair is negative.

□ + △ = 20

□	△
5	15
10	10
20	0
21	$\hat{1}$
$\widehat{80}$	100

T: Do you know another way to say that these pairs of numbers have a sum of 20?[*]

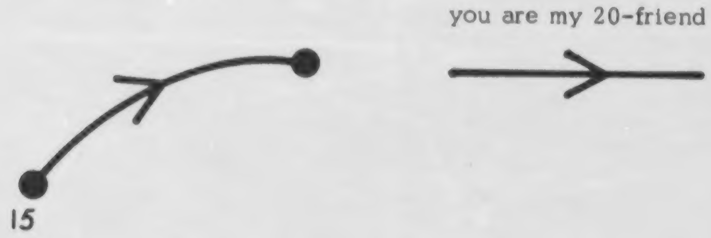
Perhaps a veteran student will recall that these numbers are 20-friends.

T: This table shows us some numbers which are 20-friends. What does it mean to say that two numbers are 20-friends?

S: The sum of the two numbers is 20.

You will need to provide this information yourself to an entry class.

Draw this arrow picture on the board.



T: Which number does 15 point to when the numbers are playing the 20-friends game?

S: 5.

T: 15 + 5 = 20, so 5 is 15's 20-friend.

Label the dot for 5. Emphasize that two numbers are 20-friends if and only if their sum is 20.

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[*] Veteran students will be familiar with number friends from lessons in CSMP Mathematics for the First Grade.

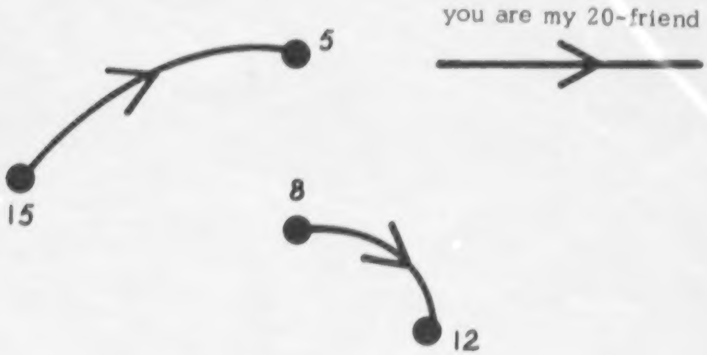
N 26

T: What are some other numbers which are 20-friends?

Accept students' suggestions and draw appropriate dots and arrows on the board. A possible dialogue is given here.

S: 8 and 12 are 20-friends.

T: 8 + 12 = 20. I'll show that 12 is 8's 20-friend in our arrow picture.



S: Also, 12 can point to 8.

Draw a red arrow from 12 to 8.

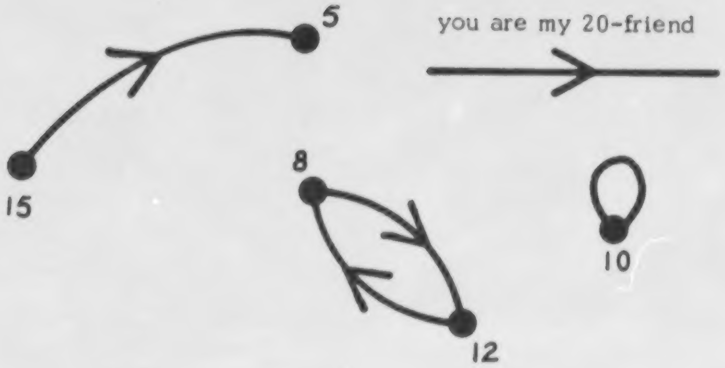
S: 10 + 10 = 20.

Draw and label a dot for 10 on the board.

T: Who does 10 point to when the numbers are playing the 20-friend game?

S: 10 points to 10. We need to draw a loop.

If no one suggests including 10 in the arrow picture, include 10 yourself.

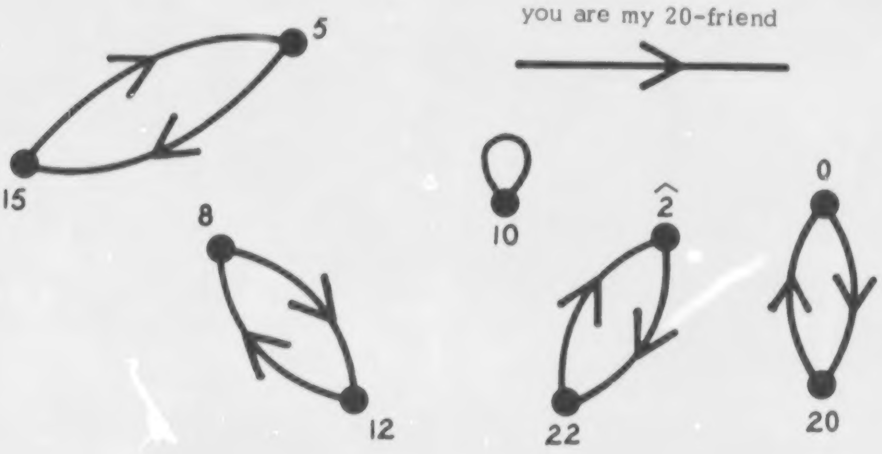

you are my 20-friend
5
15
8
12
10

T: Let's show some of the negative numbers playing the 20-friends game.

S: $\widehat{2}$ can point to 22.

Draw an arrow from $\widehat{2}$ to 22 on the board.

Continue this activity as long as you feel it is appropriate. If no one suggests drawing return arrows, suggest it yourself and ask for volunteers to draw the missing return arrows. Your final picture will look similar to this one.

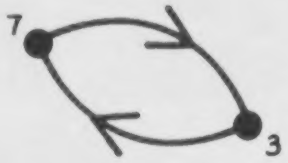

you are my 20-friend
5
15
8
12
10
$\widehat{2}$
22
0
20

Erase the board before going on to Exercise 2.

N 26

Exercise 2: 10-friends

Draw this arrow picture on the board.

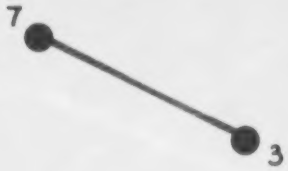


T: What could these blue arrows be for?

S: "You are my 10-friend".

Erase the arrows and draw a cord between 7 and 3. Write the key on the board.

you are my 10-friend



T: Instead of drawing two arrows, we draw a cord between 7 and 3. We can go in either direction on a cord.

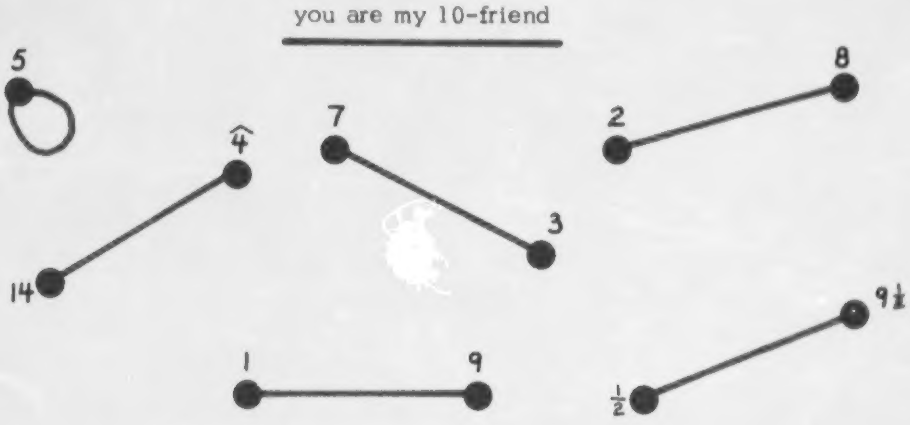
Trace the cord from 7 to 3 as you say:

T: 7 can say to 3, "You are my 10-friend."

Trace the cord from 3 to 7 as you say:

T: 3 can say to 7, "You are my 10-friend."

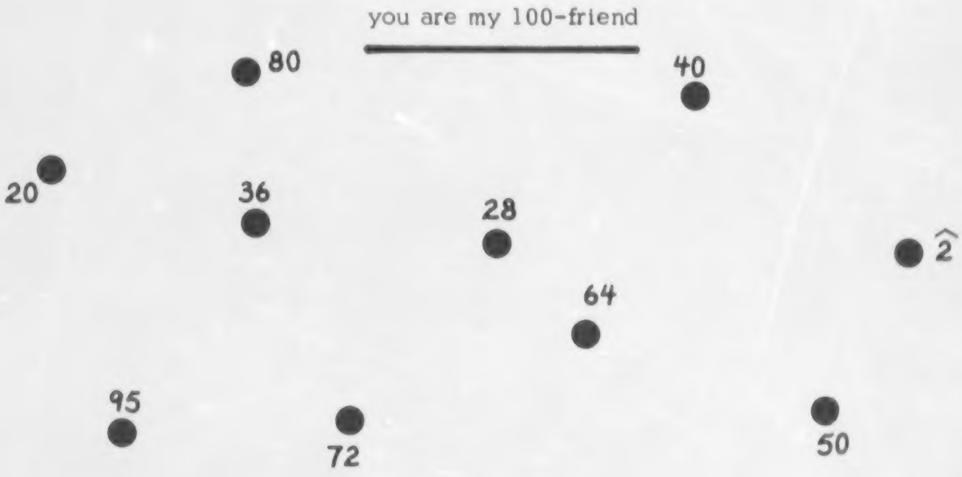
Ask students to suggest other number pairs which are 10-friends and include several of them in your cord picture. Encourage finding pairs in which one of the numbers is negative. Perhaps someone will suggest some non-integers which are 10-friends. A possible cord picture is shown below.


you are my 10-friend
5
$\hat{4}$
7
2
8
3
14
$9\frac{1}{2}$
1
9
$\frac{1}{2}$

Erase the board before going on to Exercise 3.

Exercise 3: 100-friends

Draw the following picture on the board and ask the students to copy it.


you are my 100-friend
80
40
20
36
28
$\hat{2}$
64
95
72
50

N 26

T: These numbers are playing the 100-friends game. Draw cords on your paper to show which of these numbers are 100-friends.

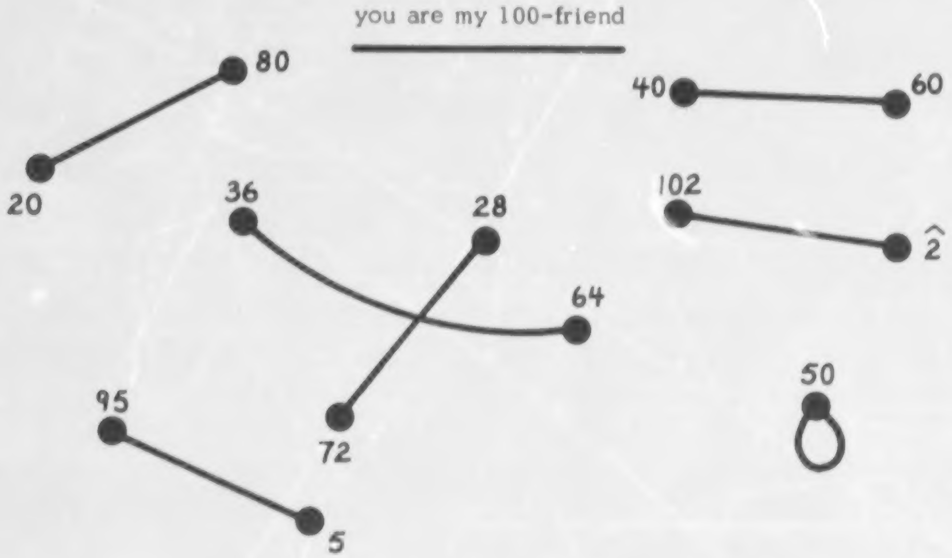
Walk around the room and observe students' work. When you note a correctly drawn cord, ask the student to draw it on the board. Announce the findings; for example,

T: We can draw a cord between 20 and 80 because 20 + 80 = 100.

When at least three cords are drawn on the board ask:

T: Are there any of these numbers whose 100-friend is not in this picture? (Yes)
Try to find the 100-friend of each of the numbers in this picture.
You will need to put some more numbers in the picture.

Continue inviting students to help complete the picture on the board.



For students who finish quickly during the individual work, add other numbers to their pictures. Challenge better students with numbers larger than 1,000.

N27 The Functions 3x and 1/3x

CAPSULE LESSON SUMMARY

Write number stories about rectangular arrays of dots. Consider a configuration on the Minicomputer in which the checkers are grouped in threes and in which all three checkers in each triple are on the same square. Note what one-third of the number on the Minicomputer is and then make plays to find the number. Record the situation in an arrow picture.

MATERIALS

Teacher: Minicomputer set; colored chalk
Student : Paper; pencil

DESCRIPTION OF LESSON

Begin the class with mental arithmetic activities involving the functions 3x and $\frac{1}{3}x$. These are some suggestions.

$3 \times 10 =$

$\frac{1}{3} \times 30 =$

$3 \times 5 =$

$\frac{1}{3} \times 15 =$

$3 \times 12 =$

$\frac{1}{3} \times 36 =$

$3 \times 4 =$

$\frac{1}{3} \times 12 =$

$3 \times 14 =$

$\frac{1}{3} \times 42 =$

$3 \times 100 =$

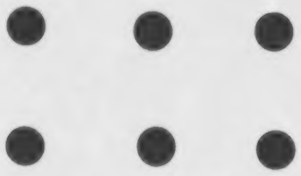
$\frac{1}{3} \times 300 =$

This is also a good opportunity to review multiplying the whole numbers from 0 to 10 by 3. Occasionally ask students to explain their methods of doing these calculations.

N 27

Exercise 1

Draw this dot picture on the board.



T: What number stories does this dot picture suggest?

Record many of the students' number stories on the board.

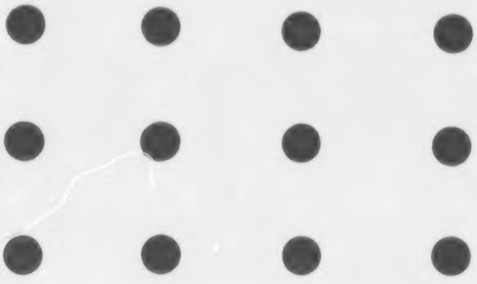
$$3+3=6 \qquad 2\times 3=6$$

$$2+2+2=6 \qquad 3\times 2=6$$

$$\frac{1}{2}\times 6=3 \qquad \frac{1}{3}\times 6=2$$

Do not insist that the students find all of these number stories. If no one observes "$\frac{1}{2} \times 6 = 3$" and "$\frac{1}{3} \times 6 = 2$", suggest these number stories yourself.

Erase the board and draw this dot picture.



Instruct the students to copy this dot picture and to write many number stories about this picture on their papers. Observe students' work and occasionally ask a student to write a number story on the board. Commend students who have written either of the following number stories on their papers. If necessary, suggest these number stories yourself.

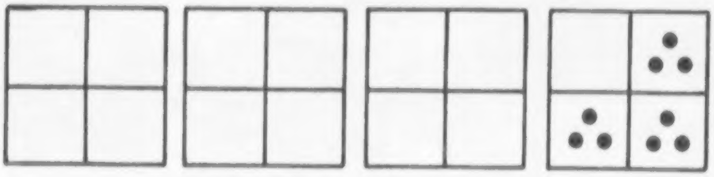
$$\frac{1}{3} \times 12 = 4 \qquad \frac{1}{4} \times 12 = 3$$

Perhaps some students will record the following number stories.

$$\frac{1}{2} \times 12 = 6 \qquad \frac{1}{6} \times 12 = 2 \qquad \frac{1}{12} \times 12 = 1$$

Exercise 2

Put this configuration on the Minicomputer.



T: What can you tell me about this number?

Let students express their ideas. They probably will tell you that 3 × 7 is on the Minicomputer. Emphasize that:

$$\begin{aligned} 7 = 4 + 2 + 1, \text{ so } 3 \times 7 &= (3 \times 4) + (3 \times 2) + (3 \times 1) \\ &= 12 + 6 + 3 \\ &= 21 \end{aligned}$$

T: What is one-third of 21?

S: 7.

N 27

Write these number stories on the board.

$$3 \times 7 = 21$$

$$\frac{1}{3} \times 21 = 7$$

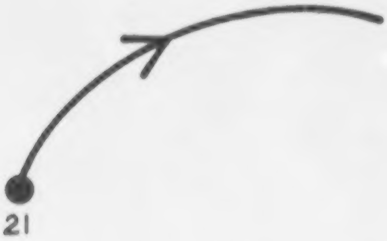
T: Let's show these number stories in an arrow picture. What kind of arrows should we use?

S: $3\times$ and $\frac{1}{3}\times$ (read: one-third of) arrows.

Draw a large dot on the board near the Minicomputer as you say:

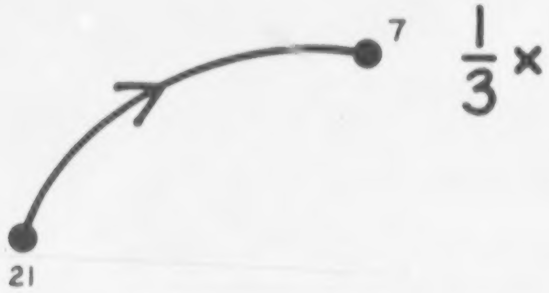
T: This dot is for the number on the Minicomputer.

Label the dot "21" and then draw a red arrow starting from that dot.



T (drawing the ending dot of the red arrow): This dot is for one-third of that number.

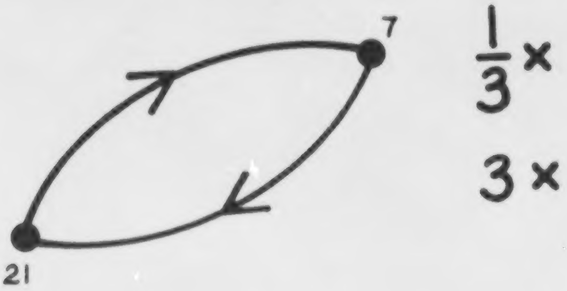
Label the ending dot "7" and then write "$\frac{1}{3}\times$" near the arrow picture.



T (tracing an arrow from 7 to 21): If I drew an arrow here, what could it be for?

S: 3 ×.

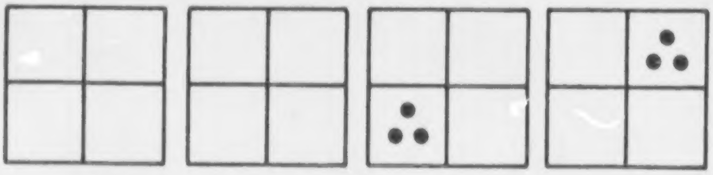
Invite someone to draw a blue arrow for 3 ×.



T: What number stories are told by this arrow picture?

S: "$\frac{1}{3} \times 21 = 7$" and "$3 \times 7 = 21$".

Repeat this activity with this configuration.



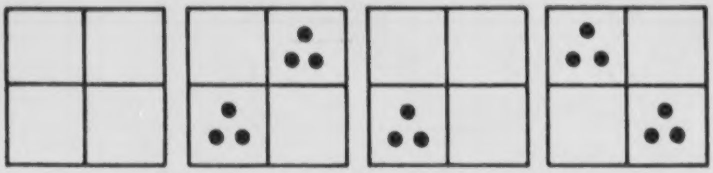
Emphasize that:

$$\begin{aligned} 3 \times 24 &= (3 \times 20) + (3 \times 4) \\ &= 60 + 12 \\ &= 72 \end{aligned}$$

$$\frac{1}{3} \times 72 = 24$$

N 27

Erase the dot labels and then put this configuration on the Minicomputer.



T (pointing to the dot on the left in the arrow picture): This dot is for the number on the Minicomputer.

Trace the red arrow.

T (pointing to the dot on the right): What is one-third of this number?

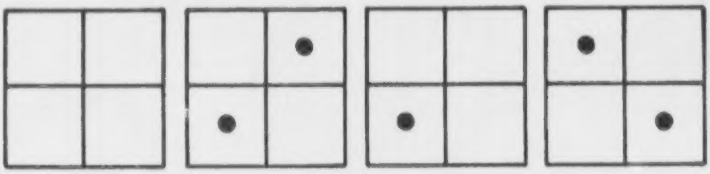
S: 629.

T: How were you able to calculate one-third of this number so quickly?

S: The checkers are in groups of three.

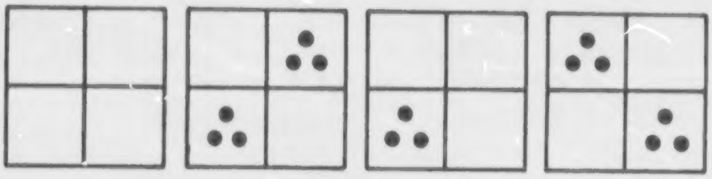
S: I looked at one checker in each group of three.

Invite a student to remove some checkers so that one-third of this number is on the Minicomputer. Emphasize that the checkers were in groups of three and that the student is leaving only one checker of each group of three on the Minicomputer.



T: Now we easily see that one-third of the number is 629.

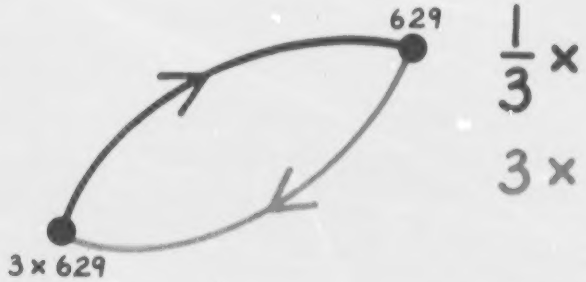
Label the dot on the right "629" and then put the original configuration back on the Minicomputer.



Trace the blue arrow.

T (pointing to the dot on the left in the arrow picture): So the number on the Minicomputer is 3 × 629.

Label that dot "3 × 629".



T: The number on the Minicomputer is 3 × 629. Look carefully at the Minicomputer. Is 3 × 629 more than 1,000? (Yes)
How do you know?

S: There are three checkers on the 400-square.

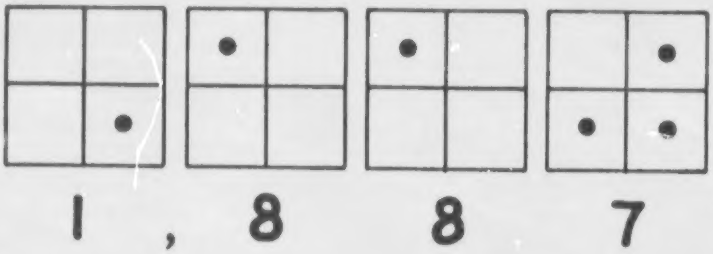
S: Two checkers on the 400-square and one checker on the 200-square is 1,000 and there are more checkers.

T: Is 3 × 629 more than 2,000?

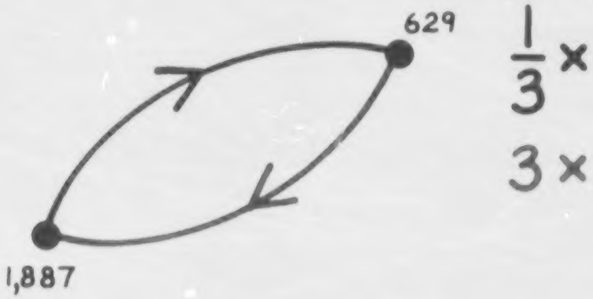
Students probably will disagree.

N 27

Ask the students to predict what number 3 x 629 is and to write their predictions on their papers. Then invite students to make plays on the Minicomputer until the standard configuration is obtained. Ask a student to write the number below the Minicomputer.



Conclude that 3 x 629 = 1,887 and label the dot on the left "1,887" (erase "3 x 629" near that dot).



T: What number stories are told by this arrow picture?

S: "$\frac{1}{3}$ x 1,887 = 629" and "3 x 629 = 1,887".

T: What addition problem did we solve?

S: "629 + 629 + 629".

Write the problem on the board and invite students to do the addition.

$$\begin{array}{r} 629 \\ 629 \\ +629 \\ \hline \end{array}$$

N28 Subtraction with -10 Arrows

CAPSULE LESSON SUMMARY

Use an arrow road having three -10 arrows to present a way to subtract 30 from a number. Solve subtraction problems by building arrow roads with arrows that compose to give the appropriate subtraction function.

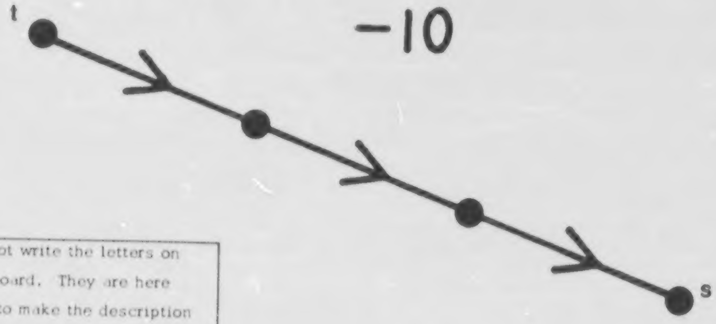
MATERIALS

Teacher: Colored chalk
Student: Colored pencils; paper; pencil

DESCRIPTION OF LESSON

Exercise 1

Draw this arrow picture on the board.



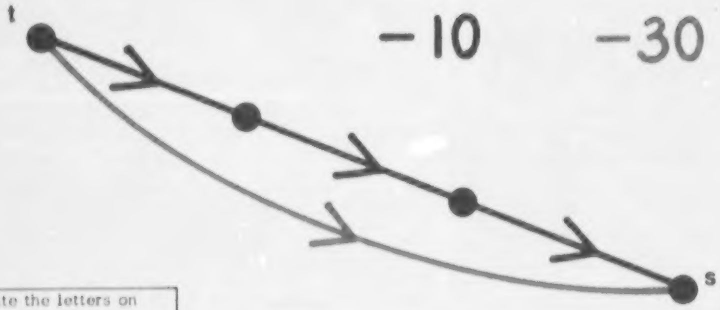
Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: Where is the largest number in this arrow picture? (At t)
Where is the smallest number in this arrow picture? (At s)
How much smaller is this number (point to s) than this number (point to t)? (30 smaller)

Draw a blue arrow from t to s.

T: What could this arrow be for? (-30)

Write "-30" near the arrow picture.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: If the largest number (point to t) were 80, what would the smallest number (point to s) be?

S: 50.

Record "80 - 30 = 50" on the board.

Repeat this activity several times by considering at t 65, 153, and 29 or any other numbers which are appropriate for the abilities of your students. Erase the board.

Exercise 2

Write this subtraction problem on the board.

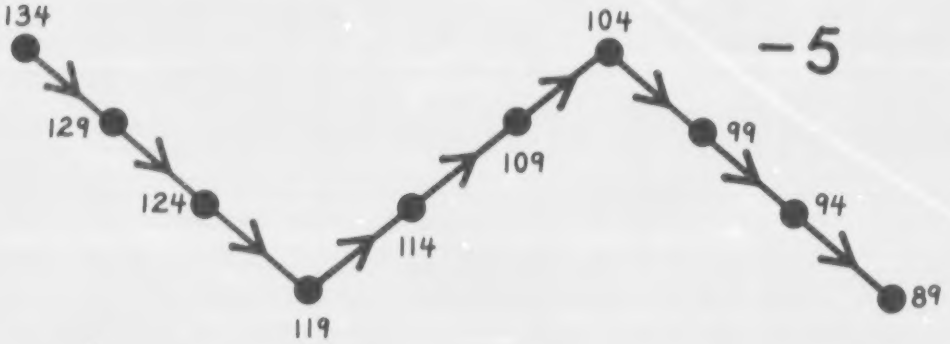
134 - 45

T: What kind of arrow picture could we draw to help us do this calculation?

S: We could start an arrow road at 134 and draw four -10 arrows and one -5 arrow.

S: We could start an arrow road at 134 and draw nine - 5 arrows.

Discuss the appropriateness of any arrow roads that are suggested. Then, provide paper and colored pencils and ask the students to use only - 5 arrows to calculate the answer. Encourage the students to look for a pattern when they are labeling the dots in their pictures. You may need to remind some students that they should draw nine - 5 arrows. Ask students who finish quickly to use a different arrow picture to calculate the answer. Invite one student to draw the - 5 arrow picture on the board and discuss the pattern in the final digits of the numbers in this arrow picture.



Complete the number story on the board.

$$134 - 45 = 89$$

Repeat the activity with this subtraction problem. Allow students to decide on their own which arrow road to use to do this calculation.

$$130 - 52$$

Ask the students who finish quickly to put their arrow pictures on the board.

N 29

N29 Sharing Three Ways

CAPSULE LESSON SUMMARY

Decide how Mr. McDonald could evenly share his animals among his three children.

MATERIALS

Teacher: Colored chalk; Minicomputer set
Student: Paper; pencil

DESCRIPTION OF LESSON

Tell the following or a similar story to your class.

T: Mr. McDonald is a farmer who has decided to retire and take a long trip around the world. He has decided to give all the animals on his farm to his three children. He wants to give the same number of horses to each child. He also wants each of his children to receive exactly the same number of chickens, pigs, goats, turkeys, and so on. One-third of the animals of each type will be given to each child.

T: Mr. McDonald has six dogs. He wants to give one-third of the dogs to each of his children. How many dogs will each child receive? (Two)
How do you know?

Accept any correct reasoning, but emphasize that $3 \times 2 = 6$ and $\frac{1}{3} \times 6 = 2$.

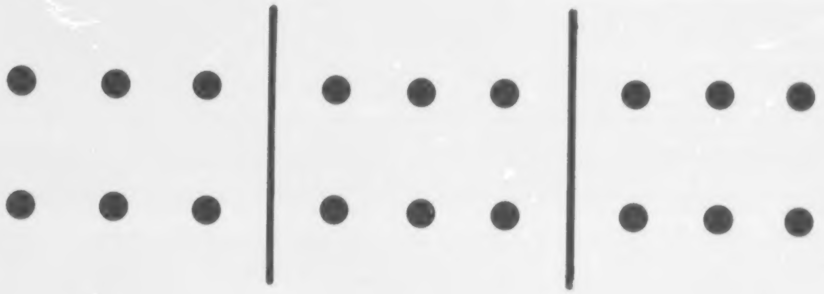
T: Mr. McDonald has nine cats. How many cats will he give to each child? (Three)
How do you know?

Accept any correct reasoning, but emphasize that $3 \times 3 = 9$ and $\frac{1}{3} \times 9 = 3$.

T: Mr. McDonald also has eighteen horses which he wants to share between his children. I want you to show me on your paper how you decided how many horses Mr. McDonald should give to each child. Either draw a picture which shows how many horses each child will receive or write a number story which shows how you solved this problem.

Explain to a student who is having difficulty getting started that one method of sharing the horses is to give one horse to each child, and then to give another horse to each child, and so on until all eighteen horses have been given away. Then draw three strings on the student's paper and indicate that eighteen dots or marks can be put in the strings one at a time using that same method.

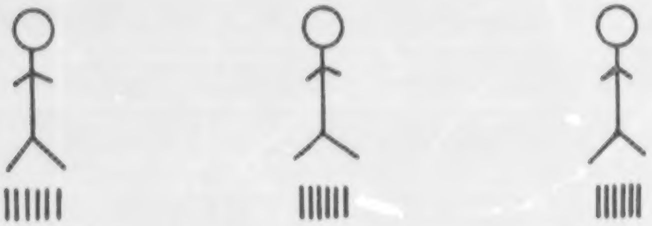
When many of the students know how many horses each of Mr. McDonald's children will receive, discuss a few of the students' pictures or number stories with the class. Some possible pictures and number stories are shown below and on the next page.



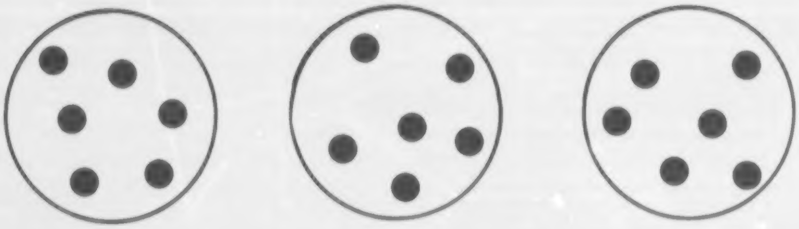
$$3 \times 6 = 18$$

$$\frac{1}{3} \times 18 = 6$$

N 29



$$6+6+6=18$$



Conclude that each child will receive six horses.

T: Mr. McDonald has 36 pigs. How many pigs will he give to each of his children? Write the answer on your paper.

Allow the students a few minutes to work on this problem. Walk around the room and look at many of their answers.

T: Will each of Mr. McDonald's children get more or less than ten pigs? (More) How do you know?

S: $3 \times 10 = 30$; so if each child got ten pigs, there would still be six pigs left to share.

T: $3 \times 10 = 30$, so $\frac{1}{3} \times 30 = \ldots$?

S: 10.

Write these number stories on the board.

$$\frac{1}{3} \times 30 = 10$$
$$\frac{1}{3} \times 6 = ?$$

T: There would be six more pigs left to share. How many more pigs should each child receive?

S: Two.

T: $3 \times 2 = 6$, so $\frac{1}{3} \times 6 = 2$.

Complete the second number story and write a third story on the board.

$$\frac{1}{3} \times 30 = 10$$
$$\frac{1}{3} \times 6 = 2$$
$$\frac{1}{3} \times 36 = ?$$

T: If $\frac{1}{3} \times 30 = 10$ and $\frac{1}{3} \times 6 = 2$, what number is $\frac{1}{3} \times 36$?

S: 12.

T: How do you know?

S: I just added "10 + 2".

T: $\frac{1}{3} \times 30 = 10$ and $\frac{1}{3} \times 6 = 2$, so $\frac{1}{3} \times 36 = 12$.

N 29

Complete the number story on the board.

$$\frac{1}{3} \times 30 = 10$$

$$\frac{1}{3} \times 6 = 2$$

$$\frac{1}{3} \times 36 = 12$$

Conclude that each of Mr. McDonald's children will receive twelve pigs.

T: Mr. McDonald also has 45 cows. How should he share 45 cows between his three children? Write your answer on your paper.

Allow the students a few minutes to work on this problem. Walk around the room and look at many of their answers.

T: Would each of Mr. McDonald's children get more or less than ten cows? (More)
Would each of them get more or less than twenty cows? (Less)
How do you know?

S: 2 × 20 = 40, so 3 × 20 = 60. Mr. McDonald only has 45 cows.

T: Each child will get between ten and twenty cows.

Display three Minicomputer boards and ask a student to put 45 on the Minicomputer.

T: Do you remember how to calculate one-half of a number on the Minicomputer?

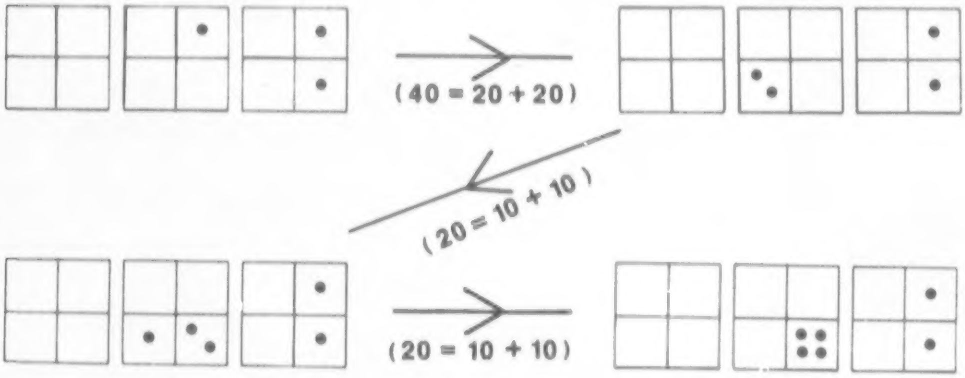
S: We start by making plays until all the checkers are in pairs.

T: How could we calculate one-third of a number on the Minicomputer?

S: We could make plays until all the checkers are in groups of three.

T: We need to make plays until all the checkers are in groups of three and the checkers in each group are on the same square. Who can make a play to help do this?

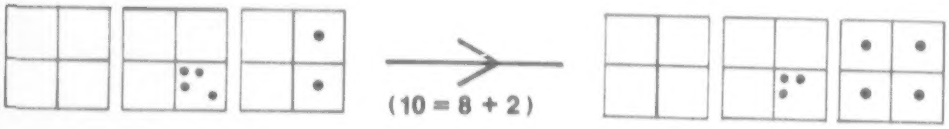
A possible sequence of plays and dialogue is given here.



T: Now we can form a group of three checkers on the same square. We don't need to make any more plays with these checkers.

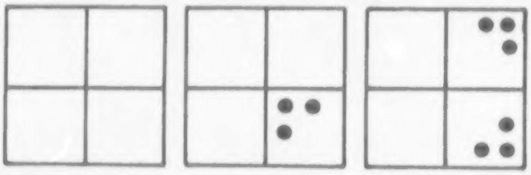
Push three checkers into one corner of the 10-square.

T: Who can make a play with the extra checker on the 10-square?



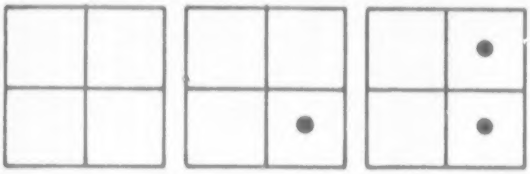
N 29

Continue until all the checkers are in groups of three. Perhaps you will have this configuration on the Minicomputer.



T: Now we have all the checkers in groups of three. Who can remove some checkers so that one-third of this number will be on the Minicomputer?

The volunteer should leave one checker of each group of three on the Minicomputer. Depending upon which configuration is on the Minicomputer, it may or may not be necessary to make some plays to obtain the standard configuration for 15. Write "$\frac{1}{3} \times 45 = 15$" to one side of the Minicomputer.



$$= \frac{1}{3} \times 45 = 15$$

Conclude that each of McDonald's children will receive fifteen cows.

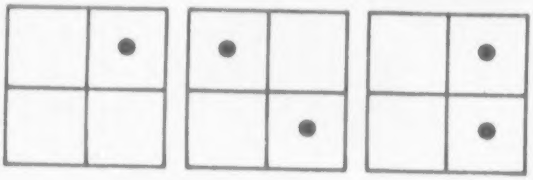
T: Mr. McDonald also has 495 turkeys.
Will each child receive more or less than 100 turkeys? (More)
. . . more or less than 200 turkeys? (Less)
Each of the children will receive more than 100 turkeys but less than 200 turkeys.
How many turkeys will each child receive? What number is $\frac{1}{3} \times 495$?

Record many of the students' predictions on the board.

T: How can we determine exactly how many turkeys each child will receive?

S: We can calculate "$\frac{1}{3}$ x 495" on the Minicomputer.

Ask a student to put 495 on the Minicomputer.

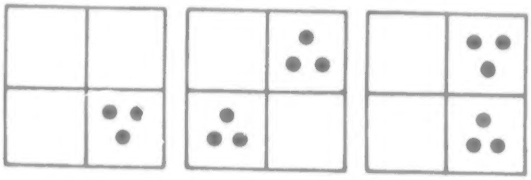


T: How can we calculate one-third of this number?

S: We need to make some backward plays until all the checkers are in groups of three.

Invite students to make plays on the Minicomputer. During the activity, feel free to suggest a play which the class might be overlooking. Whenever there is a group of three checkers on the same square, push them into a corner of that square. Discourage students from making a play with any of the checkers already in a group of three, but otherwise do not stop students from making inefficient plays.

Continue until all the checkers are in groups of three and the checkers in each group are on the same square. Perhaps you will have this configuration on the Minicomputer.



N 29

Ask someone to remove some checkers so that one-third of this number will be on the Minicomputer. Conclude that $\frac{1}{3} \times 495 = 165$, so each child will receive 165 turkeys.

You may wish to discuss with your class some other animals Mr. McDonald might have on his farm.

N30 Common Multiples #1

CAPSULE LESSON SUMMARY

Find non-negative common multiples of 3 and 5 by taking consecutive + 3 jumps starting at 0 and consecutive + 5 jumps starting at 0 on the number line and looking at common landing points.

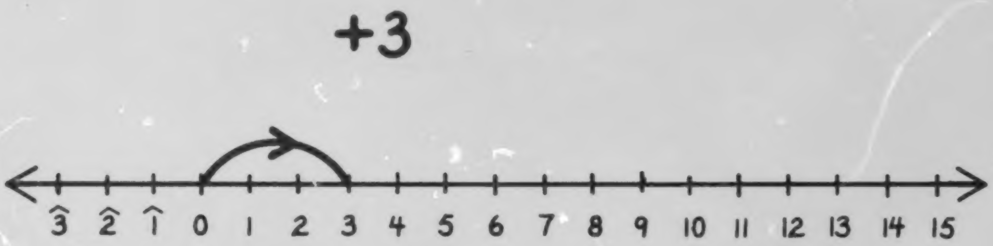
MATERIALS

Teacher: Colored chalk

Student: Worksheet N30; pencil; colored pencils

DESCRIPTION OF LESSON

Draw this picture on the board.


+3
$\hat{3}$ $\hat{2}$ $\hat{1}$ 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

T: If we start at 0 and make + 3 jumps, what are some of the numbers we will land on?

Encourage the students to make several suggestions including numbers which are more than 15. Do not draw any more arrows on the board. You may need to remind the students that the number line goes on and on in both directions.

A sample dialogue is given here.

S: We would land on 6.

T: How many + 3 jumps would we make to go from 0 to 6?

S: Two.

T: 2 x 3 = 6. What are some other numbers we would land on?

S: We would land on 15.

T: How many +3 jumps would we make to go from 0 to 15?

S: Five.

T: 5 x 3 = 15 so we would meet 15. What is a number larger than 15 which we would land on?

S: 36.

T: If we continue making +3 jumps, would we land on 36?

The class may be uncertain or may disagree.

T: How many +3 jumps would we make to go from 0 to 30?

S: Ten.

T: 10 x 3 = 30. How many +3 jumps would we make to go from 30 to 36?

S: Two; 30 + 3 = 33 and 33 + 3 = 36.

T: How many +3 jumps would we make to go from 0 to 36?

S: Twelve.

T: 12 x 3 = 36. What kind of numbers do we land on if we start at 0 and make +3 jumps?

S: Multiples of 3.

Continue asking for numbers which would be landed on; that is, for multiples of 3.

S: 60.

S: 63.

S: 103.

T: Is 103 a multiple of 3?

The class may be unable to agree.

T: Does everyone agree that 30 is a multiple of 3? (Yes)
10 × 3 = 30.
If we start at 0 and make +3 jumps, would we meet 60? (Yes)
How many +3 jumps would we make to go from 30 to 60? (Ten)
Ten +3 jumps between 0 and 30 and ten more +3 jumps between 30 and 60.
Is 60 a multiple of 3? (Yes)
Is 90 . . . ? (Yes)
If we start at 90 and make +3 jumps, what is the next number we would meet? (93)
. . . and then? (96, 99, 102, 105)
Would we ever meet 103?

S: We already skipped 103, so 103 can't be a multiple of 3.

T: Is 100 a multiple of 3?

S: No, we skipped 100 also.

N 30

T: Are there any negative numbers which are multiples of 3? (Yes) Name one of them.

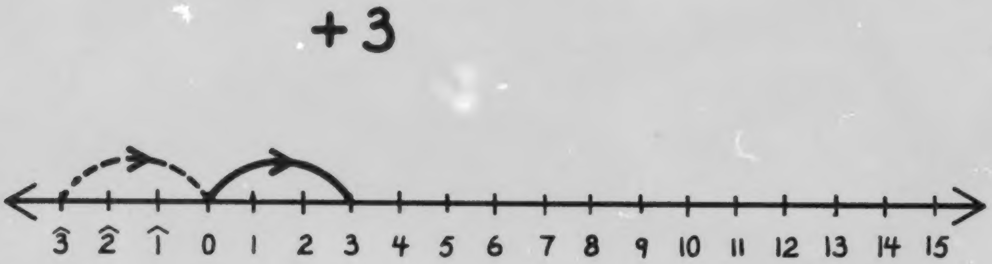
S: $\hat{3}$.

T: How many +3 jumps would we make to go from $\hat{3}$ to 0?

S: One.

T: $\hat{3} + 3 = 0$.

Trace a +3 arrow from "$\hat{3}$" to "0" on the number line.

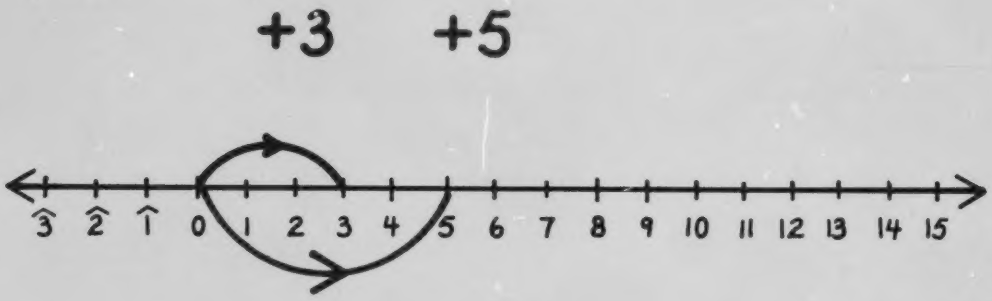

+3
$\hat{3}$ $\hat{2}$ $\hat{1}$ 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

T: Are there any other negative numbers which are multiples of 3?

Encourage the students to suggest several negative numbers which are multiples of 3.

NOTE: You can determine easily if a number is a multiple of 3 by first summing its digits. If the sum of the digits is a multiple of 3, then the number is also a multiple of 3. For example, since 7 + 2 = 9 is a multiple of 3, 72 is also a multiple of 3. We know that 200 is not a multiple of 3, because 2 + 0 + 0 = 2 is not a multiple of 3. This test is for your information only.

Write " +5" on the board and draw a blue arrow from 0 to 5.



T: If we start at 0 and make +5 jumps, what are some of the numbers we would land on?

Encourage the students to make many suggestions as you did for the +3 jumps. After several multiples of 5 have been suggested, ask the students if they observe any patterns. If necessary, mention yourself that all the multiples of 5 end with 0 or 5. You need not spend as much time soliciting multiples of 5 as you did multiples of 3.

Hold up a copy of Worksheet N30 as you say:

T: We can use this number line to show some of the numbers which are multiples of 5. We will start at 0 and make +5 jumps with arrows. Write "+5" in blue at the top of your worksheet.

Point to "0" on the number line on the board.

T: If we start at 0 and make a +5 jump, where will we land?

S: On 5.

T: Draw a blue arrow from 0 to 5 on your worksheet.

Likewise draw a blue arrow on the board.

N 30

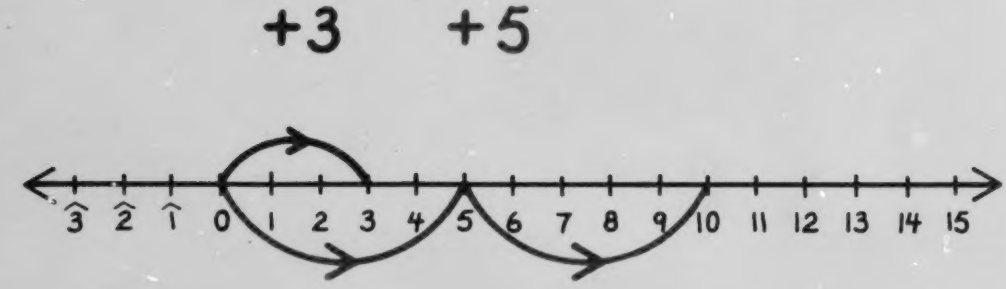
T: Is 5 a multiple of 5?

S: Yes.

T: Start at 5 and draw another + 5 arrow on your worksheet.
5 + 5 = . . . ?

S: 10.

Draw a blue arrow from 5 to 10 on the board.



T: Is 10 a multiple of 5?

S: Yes.

Ask the students to continue the + 5 arrow road on their worksheets. Indicate that when they reach 15 on the top number line, they can start another + 5 arrow at 15 on the second number line. Allow the students to work independently until several students reach 45. You may need to help some students who have difficulty drawing + 5 arrows. The reverse side of the worksheet is a copy of the front side in case a student needs to start over.

T: What are some of the numbers on this worksheet which are multiples of 5?

The class should observe that all the numbers landed on are multiples of 5.

T: We are going to use this same number line to show some of the numbers which are multiples of 3. We will start at 0 and make +3 jumps with arrows. Write "+3" in red at the top of your worksheet.

Point to "0" on the number line on the board.

T: If we start at 0 and make a +3 jump, where will we land?

S: On 3.

T: Draw a red arrow from 0 to 3 on your worksheet.

Trace this arrow on the board.

T: Is 3 a multiple of 3?

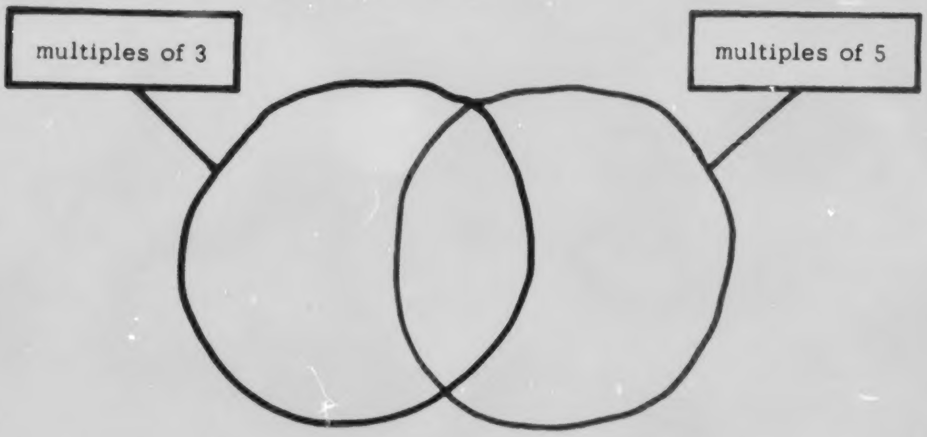
S: Yes.

T: If we start at 3 and draw another +3 arrow, where will we land?

S: On 6.

Instruct students to continue drawing +3 arrows on their worksheets. Allow them to work independently until several students reach 45.

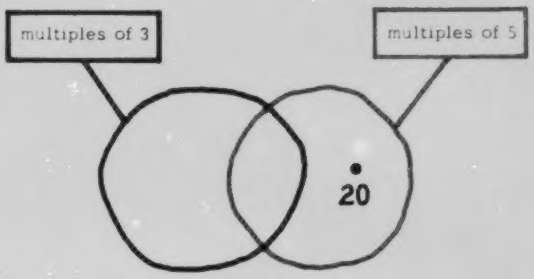
Draw this string picture on the board.



T: I would like to put some of the numbers on this worksheet in this string picture.

Invite students to put numbers in this string picture. Numbers should be put in all four regions of the picture. Frequently ask the students to explain how they know where a particular number goes in the string picture. A possible dialogue is given here.

A student puts 20 in the string picture.

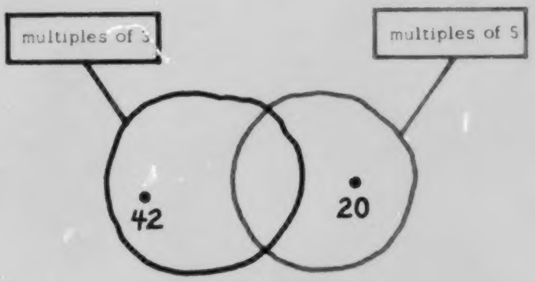


T: How do you know that 20 is a multiple of 5 but not a multiple of 3?

S: A blue arrow ends at 20 but none of the red arrows end at 20.

T: We know that 20 is a multiple of 5, because it is on the + 5 arrow road which starts at 0. We also know that 20 is not a multiple of 3, because it is not on the + 3 arrow road which starts at 0.

A student puts 42 in the string picture.



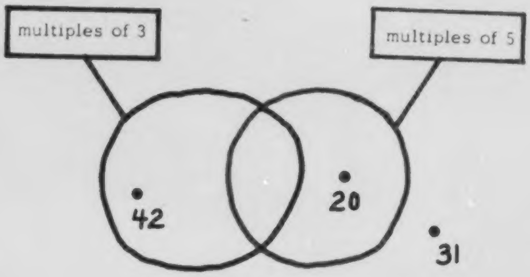
T: How do you know that 42 is a multiple of 3 but not a multiple of 5?

S: 42 is on the red arrow road but not on the blue arrow road.

S: 42 can't be a multiple of 5 because the last digit is 2.

T: Are there any numbers on your worksheet which go outside both the red string and the blue string?

A student puts 31 in the string picture.


multiples of 3
multiples of 5
42
20
31

T: Are you sure that 31 isn't a multiple of 3?

S: We know 30 is a multiple of 3. 30 + 3 = 33, so 31 can't be a multiple of 3.

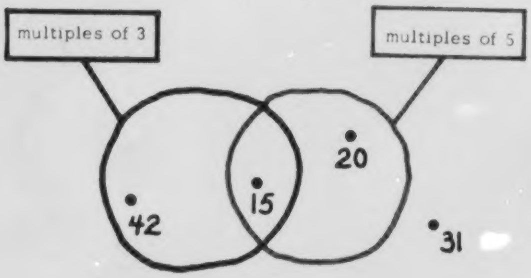
S: 31 isn't on the +3 arrow road.

T: How do you know 31 isn't a multiple of 5?

S: It doesn't end in 5 or 0.

T: Are there any numbers which will be inside both the red and the blue strings, that is, in the middle?

A student puts 15 in the string picture.


multiples of 3
multiples of 5
42
15
20
31

N 30

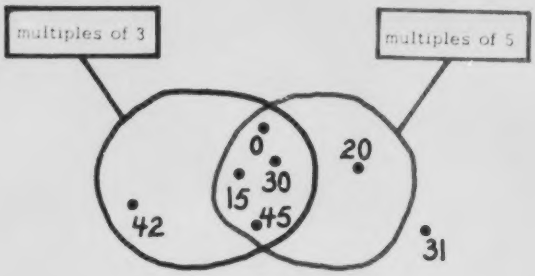
T: Is 15 a multiple of 3 and also a multiple of 5? How do you know?

S: Yes, 15 is on both arrow roads.

T: How many + 3 jumps do we make to go from 0 to 15? (Five)
$5 \times 3 = 15$.
How many + 5 jumps do we make to go from 0 to 15? (Three)
$3 \times 5 = 15$.
Are there any other numbers on this worksheet which are both multiples of 3 and multiples of 5? (Yes)

Record the common multiples of 3 and 5 which the students suggest in the string picture. If no one suggests that 0 is a common multiple of 3 and 5, suggest it yourself.

Perhaps your string picture will look like this one.



T: Look at the numbers which are multiples of 3 and multiples of 5. Do you see any patterns?

Allow the students to express themselves. If necessary, point out that $0 + 15 = 15$, $15 + 15 = 30$, and $30 + 15 = 45$.

S: They get larger by 15.

T: The common multiples of 3 and 5 are the numbers which we would meet if we start at zero and count by 15's. Are there any numbers which aren't on this worksheet which are also common multiples of 3 and 5?

Encourage the students to suggest several common multiples of 3 and 5, including some which are negative numbers. A possible dialogue is given here.

S: 60 is a multiple of 5 and a multiple of 3.

T: How do you know?

S: 60 ends in 0, so 60 is a multiple of 5. 60 is a multiple of 3, because 30 is a multiple of 3 and $30 + 30 = 60$.

S: $45 + 15 = 60$.

T: 45 and 15 are both common multiples of 5 and 3, so 60 is a common multiple of 5 and 3. Are there any negative numbers which are common multiples of 3 and 5?

S: Yes; $\widehat{15}$ because $\widehat{15} + 15 = 0$.

Perhaps one of your students will suggest that the numbers in the intersection of the two strings are the multiples of 15. Commend this student, but do not suggest this yourself.

If time remains, play a game similar to that described in Exercise 1 of Lesson L9 Multiples #1, and use two strings rather than one.

N 31

N31 Addition Problems #1

CAPSULE LESSON SUMMARY

Solve addition problems which arise in story situations and which involve decimal numbers.

MATERIALS

Teacher: Colored chalk; Minicomputer set

Student: Paper; pencil; Worksheets N31*, **, ***, and ****

DESCRIPTION OF LESSON

Exercise 1

Tell the following or a similar story choosing one of your students to be the star.

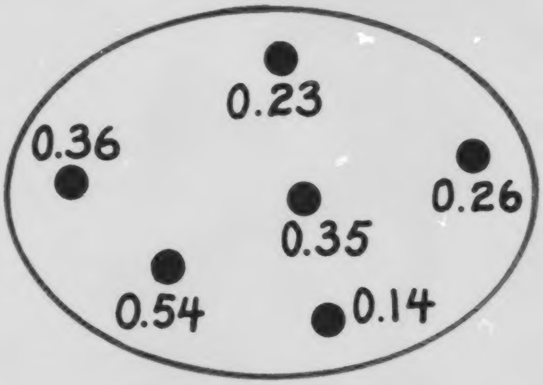
T: Karen has a half-dollar to spend in a flower shop. She wants to buy some plants for her garden. She especially likes six of the plants at the shop. I will put some numbers inside a string to show the prices of these plants.

On the board draw a large string and a dot in it. Label the dot "0.54".

T: One of the plants Karen would like to buy costs this much. How much does this plant cost?

S: 54 cents.

Continue in this manner until you have this string picture on the board.



T: After thinking awhile, Karen finds that she can buy two of the six plants and spend exactly 50¢ for them. Which two plants are they?

Let the students react to this problem and listen to their suggestions. Ask for some guesses and then record them on the board. For example, your class may suggest the following pairs.

0.35 and 0.14
0.25 and 0.36
0.26 and 0.36
0.25 and 0.26
0.14 and 0.36
0.25 and 0.14
0.23 and 0.26

Through class discussion try to eliminate the less accurate guesses. A sample dialogue is given here.

T: How can we be sure that the two plants don't cost 25¢ and 14¢?

S: 25¢ + 25¢ = 50¢, so 25¢ + 14¢ is less than 50¢.

Erase this pair of prices from your list. Continue to eliminate pairs until only three or four very close guesses remain; for example,

0.35 and 0.14
0.25 and 0.26
0.14 and 0.36
0.23 and 0.26

N 31

Ask the students to write on their papers the prices of the two plants Karen can buy and spend exactly 50¢ for them.

T: Which two of these prices have a sum of 50 cents?

The following dialogue assumes someone suggests 36¢ and 14¢.

T: How can we be sure that 36¢ + 14¢ = 50¢?

Several of your students may be able to calculate "36 + 14" mentally and can explain this process to their classmates.

S: 30 + 10 = 40; 6 + 4 = 10; 40 + 10 = 50; so 36 + 14 = 50.

S: 36 + 10 = 46 and 46 + 4 = 50.

T: This is one way we could write this addition problem.

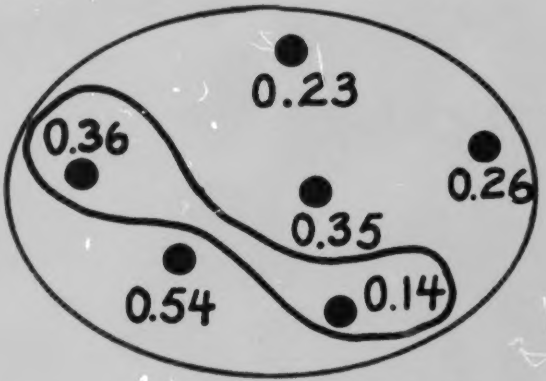
$$\begin{array}{r} 0.36 \\ +0.14 \\ \hline \end{array}$$

T: I have lined up the decimal points to be sure that the dollars get added together, that the dimes get added together, and that the pennies get added together.

Invite someone to do the problem at the board. If necessary, write the decimal point and "0" to the left of it in the answer yourself.

$$\begin{array}{r} \overset{1}{0.36} \\ +0.14 \\ \hline 0.50 \end{array}$$

Conclude that Karen can buy the plants which cost 36¢ and 14¢ and spend exactly 50¢ for them. Draw a red string around "0.36" and "0.14".



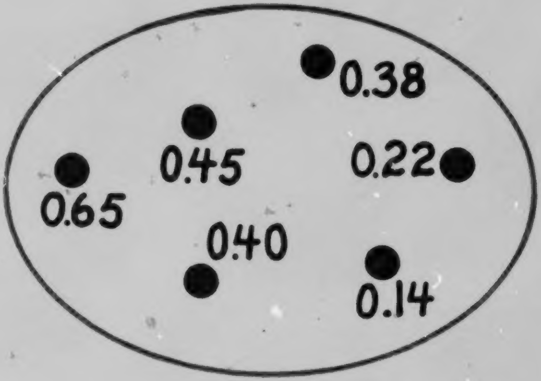
Erase the board before going on to the next exercise.

Exercise 2

Choose one of your students to be the star of this story.

T: Edward likes minature kites. His uncle gives him one dollar to spend in a special shop for kites. Edward finds six kites which he would especially like to buy.

Allow the students to briefly discuss kites. Draw a large string on the board and ask students to read the price of each kite as you put the corresponding number inside the string.



N 31

T: After some hesitation, Edward chooses three kites and spends the whole dollar. Which three kites are they?

Record the students' guesses on the board. Through class discussion try to eliminate the less accurate guesses. For example if someone suggests: 0.40; 0.45; 0.38, your class should be able to estimate that the sum of these three numbers is more than 1.00. Ask the students to write on their papers the prices of the three kites Edward bought.

T: Which three of these prices have a sum of one dollar?

Suppose, for example, someone suggests 14¢, 45¢, and 38¢. Write this addition problem on the board.

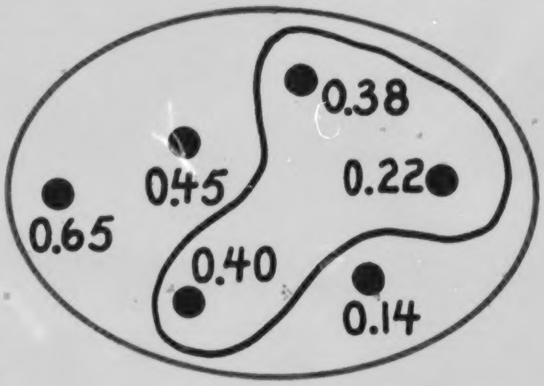
$$\begin{array}{r} 0.14 \\ 0.45 \\ +\,0.38 \\ \hline \end{array}$$

Remind the class that the dollars are on the left of the decimal point and the dimes and the pennies are on the right of the decimal point. Invite a student to do the problem at the board.

$$\begin{array}{r} {}^{1} \\ 0.14 \\ 0.45 \\ +\,0.38 \\ \hline 0.97 \end{array}$$

Conclude that Edward didn't buy these three kites, because together they cost less than one dollar. Cross off this possibility but don't erase it. This will give you a record of the combinations which you have already checked so you can avoid doing the same calculation twice. Continue until your class concludes that Edward bought the kites with prices $0.40, $0.22, and $0.38. If your class has a great deal of difficulty solving this problem, tell them that one of the kites Edward bought cost 40¢ and ask them for the other two prices.

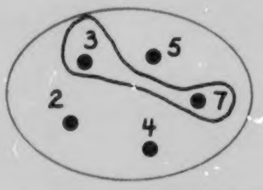
When your class has found the three prices of the kites Edward bought, draw a red string around "0.38", "0.22", and "0.40".



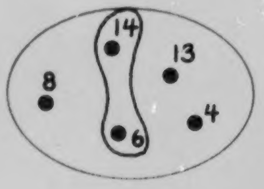
Worksheets N31*, **, ***, and **** are available for individual work.

Name ______________________ **N 31 ***

Janet buys two different candies and spends exactly 10¢.
Draw a red string around the prices of these two candies.

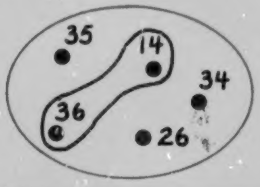


Mike buys two different balls and spends exactly 20¢.
Draw a red string around the prices of these two balls.

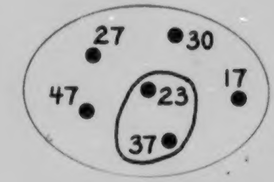


Name ______________________ **N 31 ****

Kenny buys two different cards and spends exactly 50¢.
Draw a red string around the prices of these two cards.

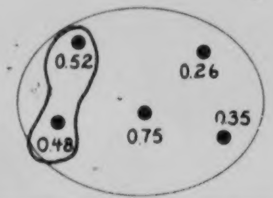

35
14
34
36
26

Lisa buys two different toys and spends exactly 60¢.
Draw a red string around the prices of these two toys.

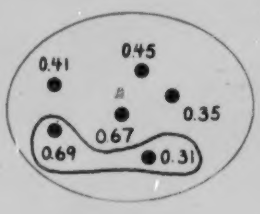

27
30
17
47
23
37

Name ______________________ **N 31 *****

Robin buys two different books and spends exactly $1.
Draw a red string around the prices of these two books.

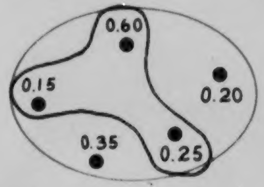

0.52
0.26
0.35
0.75
0.48

Roberto buys two different rings and spends exactly $1.
Draw a red string around the prices of these two rings.

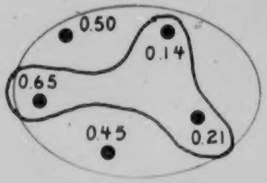

0.41
0.45
0.35
0.67
0.69
0.31

Name ______________________ **N 31 ******

Daniel buys three different pieces of fruit and spends exactly $1.
Draw a red string around the prices of these three pieces of fruit.


0.60
0.15
0.20
0.35
0.25

Sandra buys three different flowers and spends exactly $1.
Draw a red string around the prices of these three flowers.


0.50
0.14
0.65
0.45
0.21

N32 Roads #3

CAPSULE LESSON SUMMARY

Consider several arrow roads which start at the same number and which use only +100, +10, and +1 arrows. Ask which road has the largest ending number and which has the smallest. Label the dots in the four roads. Explore the question of whether or not the ending number of any of the roads would change if the order of the arrows were changed. Find the largest and the smallest ending number of an arrow road with exactly four arrows.

MATERIALS

Teacher: Colored chalk
Student: Worksheets N32* and **; paper; colored pencils

DESCRIPTION OF LESSON

Begin the lesson with some mental arithmetic on adding 10 and 100. Ask a student to start at 0 and count by tens. Ask other students to start at a different whole number less than 10 and count by tens.

T: I am going to say a number between 0 and 100. The person I point to should say the number which is 10 more than my number.

T: 5. S: 15.

20. S: 30.

22. S: 32.

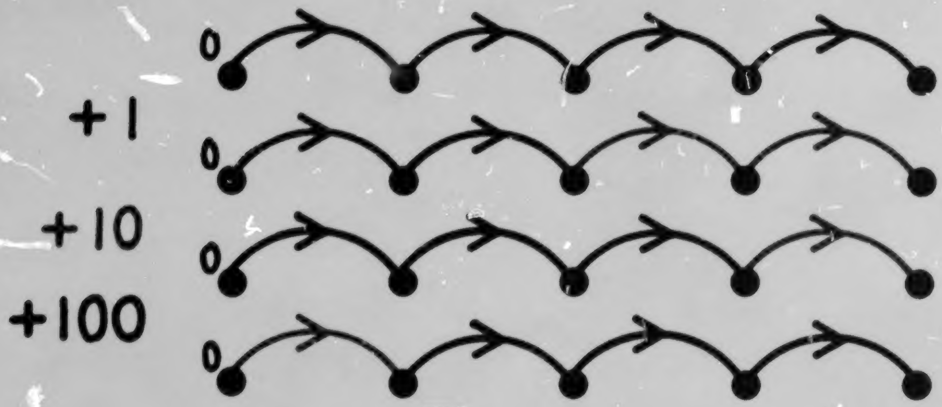
24. S: 34.

34. S: 44.

Continue this activity until most of the students have participated. Choose numbers which fit the abilities of your students. Vary the activity by asking students to add 100 to the numbers you say. You could make the exercise a little harder by asking students to add 20.

N 32

Draw these arrow roads on the board.



T: Look at the arrow roads on the board. How are they alike?

S: They all have four arrows.

S: They all start at 0.

T: Which of these arrow roads has the largest ending number?

S: The third one.

T: Why?

S: It has the most +100 arrows.

T: Which of these arrow roads has the smallest ending number?

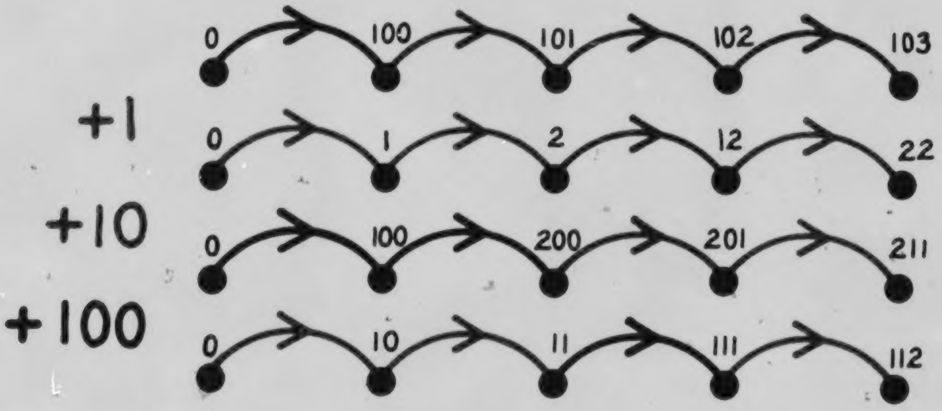
S: The second one.

T: Why?

S: It doesn't have any +100 arrows and each of the other arrow roads do.

Emphasize that even though all the roads have four arrows and all start at 0, having different combinations of arrows in this case gives different ending numbers. Ask the students to copy the arrow roads on their papers and label the dots. Help individual students get started as necessary. Encourage the students who finish quickly to draw other arrow roads which start at 0, have exactly four arrows, and use only +1, +10, and +100 arrows. Ask these students to find the largest and the smallest possible ending numbers. If students draw arrow roads with ending numbers 4 or 400, ask that those roads be drawn on a side board.

Invite students to label the dots in the original arrow roads.



Again note with your students that the third of the original arrow roads has the largest ending number and that the second arrow road has the smallest ending number.

T: Would the ending number of any of these roads change if we changed the order of the arrows in that road?

Encourage your students to discuss this problem. If some of the students doubt that the ending number would remain unchanged, quickly calculate the ending number of the first arrow road if one of the +1 arrows preceded the +100 arrow. Conclude that the ending number would still be 103. You may wish to repeat this process changing the order of the arrows in the second arrow road.

T: Some of you have drawn some more arrow roads with exactly four of these arrows. Did you get any ending numbers which are different from the ones on the board?

A sample dialogue is given here.

S: 301.

T: 301. What kind of arrows do you think are in Brad's arrow road?

S: Three +100 arrows and one +1 arrow.

T: Brad, does your road have three +100 arrows and one +1 arrow?

S: Yes.

Continue this activity for several of the students' ending numbers.

T: What is the largest possible ending number if we build a road starting at 0 with exactly four arrows and use only +1, +10, and +100 arrows?

The following is a sample dialogue.

S: 310.

T: How could we build a four arrow road which goes from 0 to 310 and which uses only +100, +10, and +1 arrows?

S: Three +100 arrows and one +10 arrows.

Confirm that the student who suggested 310 had drawn (or was thinking of) three +100 arrows and one +10 arrows.

T: Does anyone have an ending number which is more than 310?

S: I do; I have one ending at 320.

T: Is 320 more than 310?

S: Yes.

T: Which of these arrows would we need to go from 0 to 320?

S: Three +100 arrows and two +10 arrows.

T: How many arrows would that be?

S: Five.

T: We are trying to find the largest possible ending number of a + 1, +10, and +100 arrow road with exactly four arrows starting at 0.

Check the road of the student who suggested 320. It is probable that this road has five arrows or doesn't start at 0. Explain that if you start at 0 it is possible to reach 320 using +100, +10, and +1 arrows but not with only four of them.

T: Does anyone have an ending number which is more than 310?

S: I do; I have one which ends at 400.

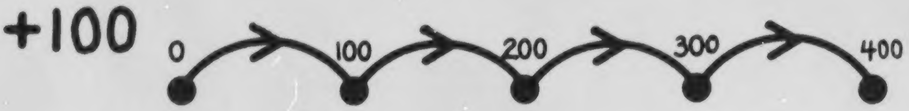
If no one has found the road which ends at 400, tell the students that 400 is a possible ending number and ask them to build the road.

T: Which of these arrows would we need to go from 0 to 400 with exactly four arrows?

S: Four +100 arrows.

N 32

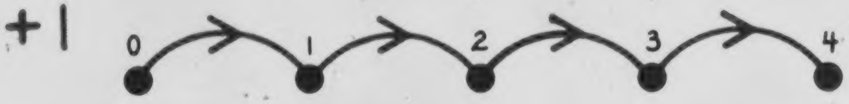
If you have this arrow road on one of your side boards, point it out to your class. If it is not on display, draw it quickly yourself.



Ask if it is possible to have an ending number that is more than 400. Encourage the students to comment and then conclude that 400 is the largest possible ending number.

T: What is the smallest possible ending number if we start at 0 and draw an arrow road with exactly four of these arrows?

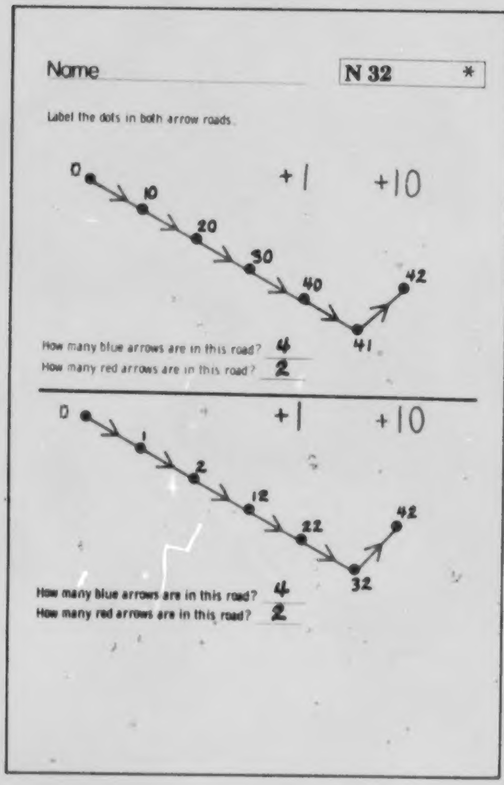
Proceed as you did before to find the largest possible ending number. If necessary, draw this arrow road on the board yourself.



Conclude that 4 is the smallest possible ending number.

Worksheets N32* and ** are available for individual work. If some students finish quickly, challenge them to build other roads such as the following:

- a road from 0 to 134 using +1, +10, and +100 arrows;
- a road from 98 to 452 using +1, +10, and +100 arrows.



Name

N 32 *

Label the dots in both arrow roads.

+1
+10
0
10
20
30
40
41
42

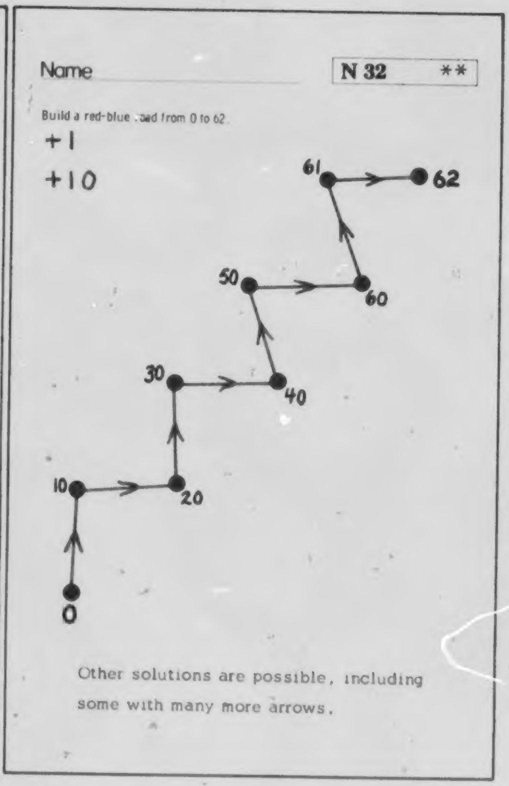
How many blue arrows are in this road? 4

How many red arrows are in this road? 2

+1
+10
0
1
2
12
22
32
42

How many blue arrows are in this road? 4

How many red arrows are in this road? 2



Name

N 32 **

Build a red-blue road from 0 to 62.

+1
+10
0
10
20
30
40
50
60
61
62

Other solutions are possible, including some with many more arrows.

313

N 33

N33 Introduction to Decimal Numbers #3

CAPSULE LESSON SUMMARY

$78 is the amount of prize money being offered in a stamp club contest; if there are two winners, calculate on the Minicomputer how much money each should receive. Calculate how much each winner would receive when there are three winners. Decide how to divide an increased prize of $100 among three winners.

MATERIALS

Teacher: Minicomputer set; colored chalk
Student: None

DESCRIPTION OF LESSON

Begin this lesson with a few minutes of mental arithmetic involving "one-half of", "one-third of", and one-fourth of".

T: What number is $\frac{1}{2} \times 10$? (5)

What number is $\frac{1}{2} \times 20$? (10)

What number is $\frac{1}{2} \times 50$? (25)

What number is $\frac{1}{2} \times 70$? (35)

What number is $\frac{1}{2} \times 100$? (50)

What number is $\frac{1}{3} \times 30$? (10)

What number is $\frac{1}{3} \times 60$? (20)

What number is $\frac{1}{3} \times 90$? (30)

What number is $\frac{1}{3} \times 15$? (5)

What number is $\frac{1}{3} \times 75$? (25)

What number is $\frac{1}{4} \times 40$? (10)

What number is $\frac{1}{4} \times 20$? (5)

What number is $\frac{1}{4} \times 100$? (25)

3.1

Exercise 1

T: A stamp club decides to have a contest for the students in a local elementary school. The stamp club announces that it will give a prize of $78 in cash to the student with the best stamp collection. How could the club award the $78 in cash? Which bills or coins could they use?

Encourage your class to suggest many ways to have $78 in cash and make corrections as necessary. Some of the ways to have $78 in cash are:

- seventy-eight $1-bills;
- three $20-bills, three $5-bills, three $1-bills;
- seven $10-bills, eight $1-bills;
- one hundred fifty-six half-dollars;
- seven hundred eighty dimes.

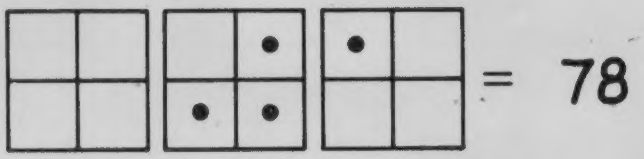
T: Many students are excited by this contest and begin collecting stamps from all over the world. The day for the contest judging arrives and almost every student in the entire school has entered a stamp collection. There are so many stamp collections that the judges can't decide which collection is the best. Hours go by and the judges still can't decide. Then someone suggests that they choose two best collections and give half of the $78 prize money to each of the winners.

How much would each of the winners receive if the judges choose two best collections?

Record students' guesses on the board.

T: How can we calculate "$\frac{1}{2}$ x 78"?

Perhaps someone will suggest using the Minicomputer; if necessary, suggest this yourself. Ask a student to put 78 on the Minicomputer.

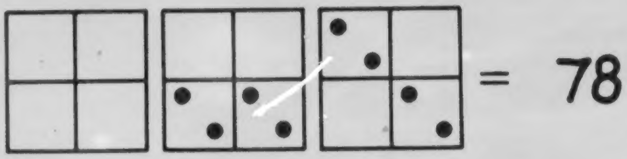


T: How can we calculate "$\frac{1}{2} \times 78$" on the Minicomputer?

S: We need to make some backward plays so we can put the checkers in pairs.

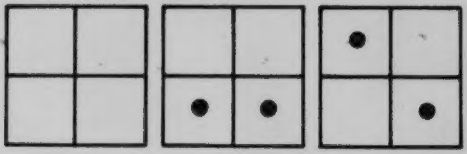
T: We need to make plays until all the checkers are in pairs and both checkers in each pair are on the same square.

Invite students to make plays on the Minicomputer. Whenever there is a pair of checkers on the same square, push those checkers into a corner of the square and mention that you do not want to make any more plays with those checkers. Discourage any students who start to break up a pair. Continue until all the checkers are in pairs. Perhaps you will have this configuration on the Minicomputer.



T: Who can remove some checkers so that one-half of this number will be on the Minicomputer?

The student who volunteers should remove one checker from each pair of checkers. Depending upon which configuration was on the Minicomputer, the class may or may not now need to make a few plays to obtain the standard configuration for 39. Write " = $\frac{1}{2}$ x 78 = 39" to one side of the Minicomputer.



Conclude that if the judges choose two winners and give each half of the prize money, then each winner will receive $39. Determine which student had the closest guess.

Exercise 2

T: After a long while, the judges finally announce that they have chosen three winners who will share the prize money. Everyone is glad that the contest is finally over and that the winners will be announced soon. How much money will each of the three winners receive?

Record the students' guesses on the board.

T: How can we determine exactly how much money each winner will receive?

S: We could calculate "$\frac{1}{3}$ x 78" on the Minicomputer.

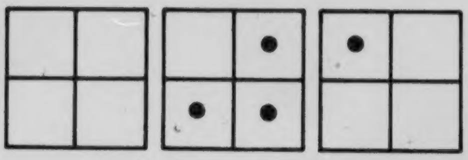
T: We already have found that $\frac{1}{2}$ x 78 = 39. Is $\frac{1}{3}$ x 78 more or less than 39? Why?

S: Less, because there is one more person to share in the prize money, so each winner will get less money.

N 33

T: We know that $\frac{1}{3} \times 78$ is less than 39. Are there any of these guesses (point to the list on the board) we can erase because they couldn't be right?

Allow the students to tell you to erase any guess which is not less than 39. Ask a student to put 78 on the Minicomputer.



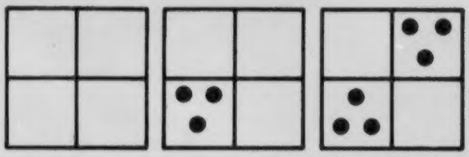
T: How can we calculate "$\frac{1}{3} \times 78$" on the Minicomputer?

S: Since we are sharing three ways, we need groups of three.

S: When we take one-half of a number, we need pairs of checkers. When we take one-third of a number, we need groups of three checkers.

T: We need to make plays until all the checkers are in groups of three checkers and all the checkers in a group are on the same square.

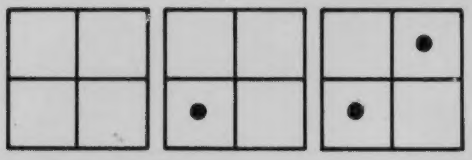
Ask for volunteers to make plays on the Minicomputer. Whenever there is a group of three checkers on the same square, push those checkers into a corner of the square and mention that you do not want to make any more plays with those checkers. Discourage any students who start to break up a group of three. Continue until all the checkers are in groups of three. Perhaps you will have this configuration on the Minicomputer.



3 8

T: Who can remove some checkers so that one-third of this number will be on the Minicomputer?

The student who volunteers should leave one checker in each group of three on the Minicomputer. Depending upon which configuration was on the Minicomputer, the class may or may not now need to make a few plays to obtain the standard configuration for 26. Write " = $\frac{1}{3}$ x 78 = 26" to one side of the Minicomputer.



$$= \frac{1}{3} \times 78 = 26$$

Conclude that each of the winners will receive $26.

Exercise 3

T: At the awards ceremony, the judges announce that since they have chosen three winners instead of one, they have decided to increase the amount of prize money to $100. Each winner will receive one-third of the $100. Will each winner receive more or less than $26?

S: More, because 100 is more than 78.

T: How much money will each of the three winners receive?

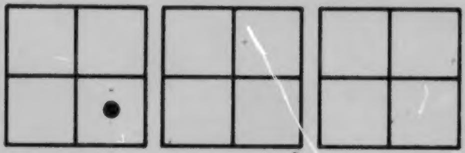
Record the students' guesses on the board. Do not record any guess which is less than 26.

T: How can we calculate "$\frac{1}{3}$ x 100" on the Minicomputer?

S: We need to put 100 on the Minicomputer and then make plays until all the checkers are in groups of three.

N 33

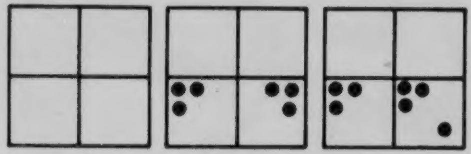
Ask a student to put 100 on the Minicomputer.



Invite students to make plays on the Minicomputer. Since there are many plays to be made, you may wish to allow each student to make several plays at a time. Whenever there is a group of three checkers on the same square, push these checkers into a corner of the square.

Before a jump is made from the tens' board to the ones' board, perhaps a student will notice that all the checkers are in groups of three except for one extra checker on the 10-square. Otherwise, mention this yourself.

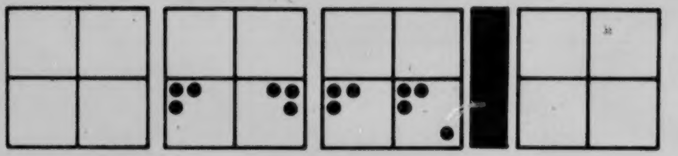
Continue this activity until all the checkers are in groups of three except for one extra checker on the 1-square. Perhaps you will have this configuration on the Minicomputer.



T: All the checkers are in groups of three except for this extra checker on the 1-square. This checker is for $1 which needs to be shared. What should we do now?

Allow the students to suggest solutions. Whether or not it is suggested, place a Minicomputer board to the right of the ones' board leaving extra space between them.

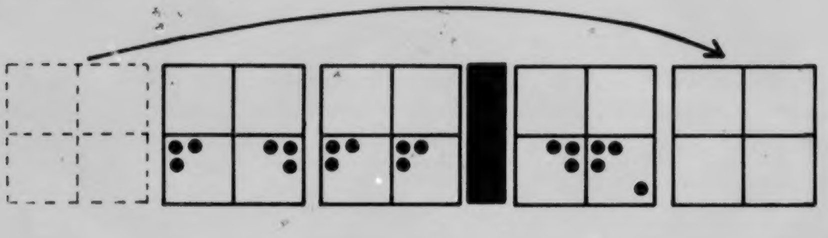
If no one mentions drawing the green bar between the two boards on the right, ask if you have changed the number. Then suggest the green bar yourself.



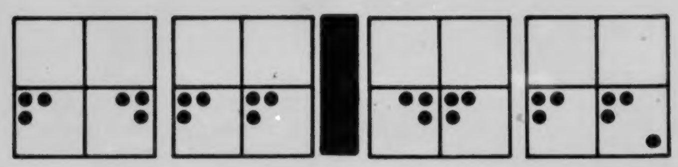
T: The green bar will remind us that the Minicomputer boards on the left are for dollars and the boards on the right are for dimes and for pennies.

Invite students to make plays. Continue until all the checkers are in groups of three except for one extra checker on the white square of the dimes' board. Some students may foresee that to continue making plays onto the pennies' board would result in the same situation only with the extra checker on the white square of the pennies' board.

Move a Minicomputer board to the right of the dimes' board.



Continue inviting students to make plays until you have a group of three checkers except for one extra checker on the white square of the pennies' board.



T: What would happen if I put another Minicomputer board on the right?

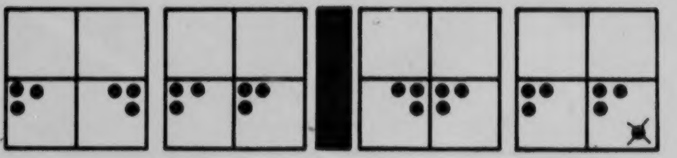
S: We could make more plays, but we would still end up with an extra checker on a white square.

T: No matter how many boards we put up, there will always be a checker that won't be in a group of three.

T (pointing to the extra checker): A checker on this white square is for what?

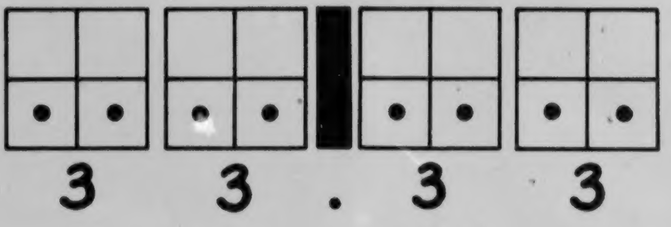
S: One penny.

T: The judges didn't know any way to share one penny (remove the extra checker) . . .



. . . so they decided to simply keep the one penny.

Call on a volunteer to remove some checkers so that one-third of the number will be on the Minicomputer. The student should remove two checkers from each group of three. Invite another student to write the number below the Minicomputer.



T: How much money will each of the winners receive?

S: $33.33.

T: If the judges give each winner $33.33, how much money will this be altogether?

Write this problem on the board.

$$\begin{array}{r} 33.33 \\ 33.33 \\ +\,33.33 \\ \hline \end{array}$$

Call on a student to solve this problem at the board. The class will discover that $3 \times 33.33 = 99.99$.

Write this number story on the board . . .

$$100 = (3 \times 33.33) + 0.01$$

as you summarize that each winner received $33.33 and that the judges kept a penny.

N 34

N34 Composition of Functions #3

CAPSULE LESSON SUMMARY

Decide which of two arrow roads starting at the same number has the larger ending number. Build a road from 0 to 50 using only 2 × arrows and + 5 arrows. Shorten it, if possible.

MATERIALS

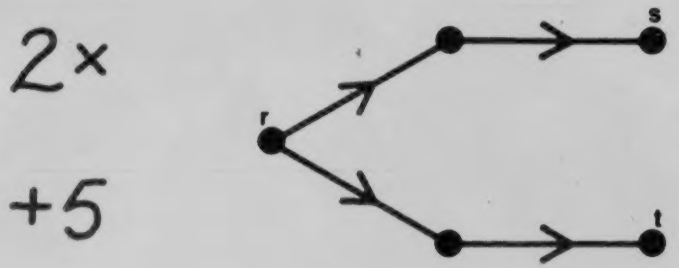
Teacher: Colored chalk

Student: Worksheets N34*, **, ***, and ****; pencil

DESCRIPTION OF LESSON

Exercise 1

Draw this arrow picture on the board.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

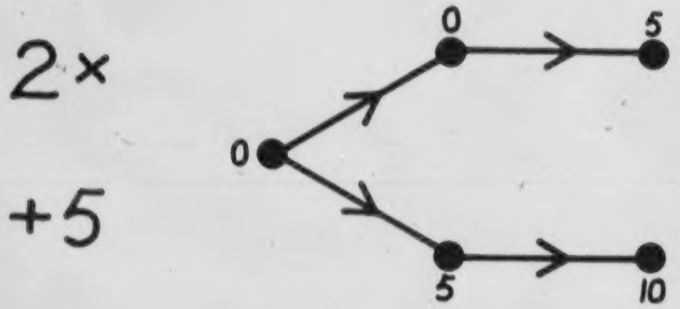
T: All the dots in this arrow picture are for whole numbers. Where is the smallest number? (At r) How do you know?

S: If we follow the red and blue arrows, the numbers get larger.

T: This dot (point to r) is the starting dot of two arrow roads. One road has a red arrow and then a blue arrow. The other road has a blue arrow and then a red arrow. Where is the largest number in this arrow picture?

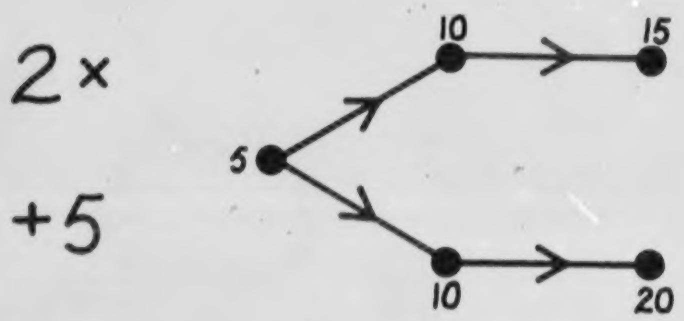
Encourage the students to express themselves. Some students may believe initially that s and t are for the same number. In this case, you should suggest that they try to find a way to check if they are correct. Very likely students will want to choose a number for r and then to label the other dots accordingly. This method should lead the class to decide that the largest number is at t. Insist that you do not depend on only one example of a labeling. Let several students suggest numbers to start with at r. Each time ask where the largest number is and then ask how much larger t is than s.

NOTE: Perhaps one of your students will label r "0" and obtain this arrow picture.



In this case, you should point out that there are two dots for 0 in this arrow picture. Mention that if r were for 0, then the red arrow starting at 0 would be a loop because 2 x 0 = 0. Conclude that r is not for 0.

A similar situation occurs if r is labeled "5".



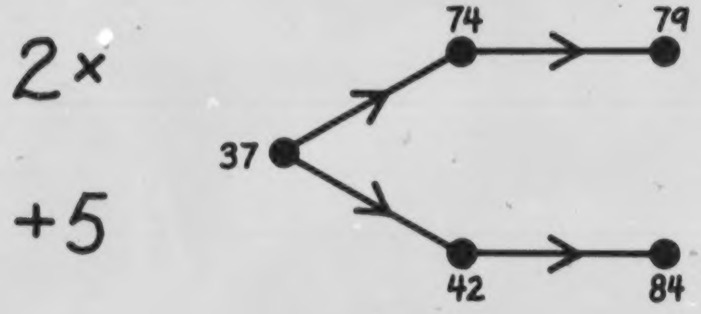
N 34

If r were 5, then both the red arrow and the blue arrow starting at r would end at the same dot because 2 × 5 = 10 and 5 + 5 = 10.

T: Do you think that this number (point to t) is always the largest? Let's start with another number and see if a blue arrow followed by a red arrow gives us a larger ending number than a red arrow followed by a blue arrow.

Label r "37".

Ask students to label the other dots. When all the dots are labeled, conclude that the largest number is 84.



T: How much larger is 84 than 79?

S: 5 larger.

If appropriate, count with your class from 79 to 84 and verify that 79 + 5 = 84.

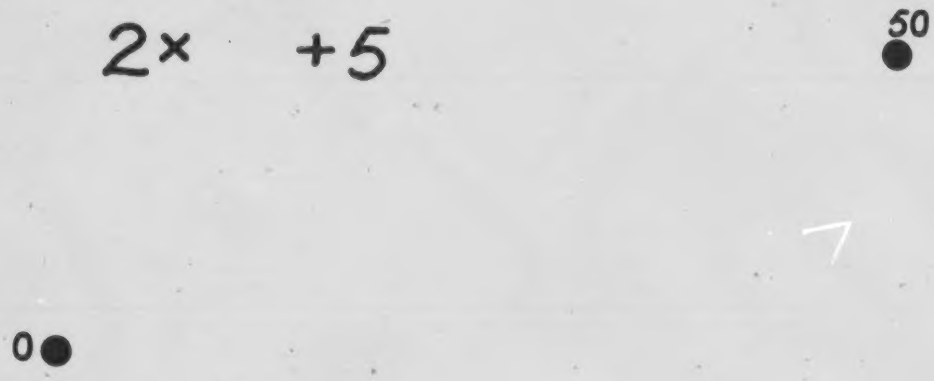
Point to the dot for 37 with your left forefinger. Trace the appropriate arrows with your right forefinger as you speak.

T: If we start at 37 and follow a blue arrow first and then a red arrow, we end at a larger number than if we follow a red arrow first and then a blue arrow.

Repeat this activity one or two more times until the students are confident that t will always be for the largest number and that the number at t is always 5 more than the number at s. Choose starting numbers which are appropriate for the abilities of your students.

Exercise 2

Draw this picture on the board.



T: Will we be able to build a road between 0 and 50 using only 2 × and + 5 arrows?

S: We can build a road between 0 and 50 using just + 5 arrows.

T: If we were to build the road using all + 5 arrows, how many would we need?

S: Ten.

T: Could we build a shorter road?

S: Yes.

T: Let's try to build one. Should we start at 0 or at 50?

N 34

S: At 0, because if we start at 50 the numbers we meet will be larger than 50 and we will never get to 0.

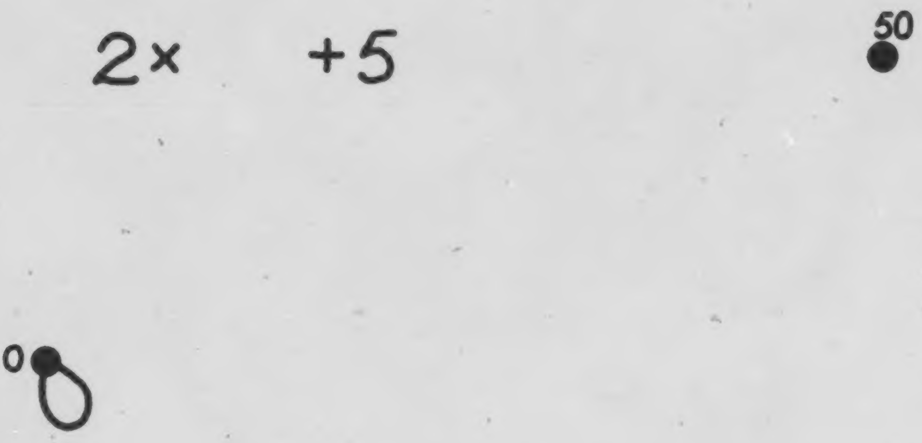
T: Shall we use a 2 x or a + 5 arrow first?

Accept a student's suggestion. A possible dialogue is given here.

S: Use a red arrow first.

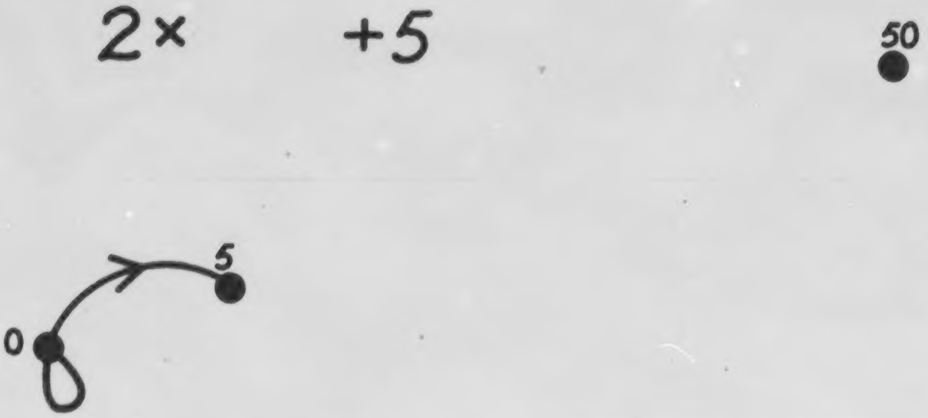
T: The red arrows are for 2 x. How shall I draw this arrow?

S: 2 x 0 = 0. You need to draw a loop.

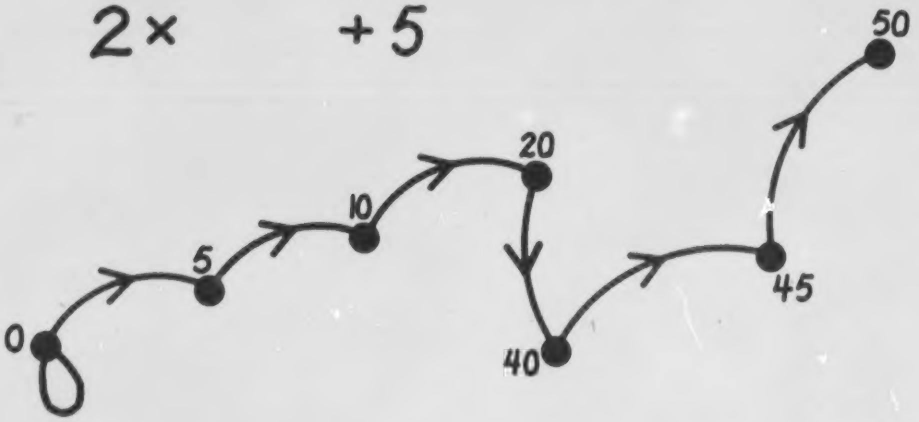


T: What kind of arrow shall we use now?

S: A blue arrow; 0 + 5 = 5.



Continue this activity until the road from 0 to 50 is complete. Ask the class whether or not they think a shorter road from 0 to 50 can be built. A sample dialogue is given here based on one possible road.



T :	This road starts with a loop and then goes : blue, red, red red, blue, blue. How many arrows are in this road? (Seven) Could we build a shorter red-blue road from 0 to 50?
S :	We don't need the loop. It doesn't help us get to 50.

Erase the loop.

N 34

T: How many arrows are in this road now? (Six)
Could we build an even shorter road from 0 to 50?

Encourage the students to offer suggestions. Perhaps no one will see a way to shorten this road. In such a case, start tracing the road backward; stop at each dot asking if there is a shorter road between the corresponding number and 50 until the road is shortened, if possible. (See p.269 for an illustration of the shortest road possible.)

Point to the dots for 40 and 50.

T: Is there a shorter road from 40 to 50?

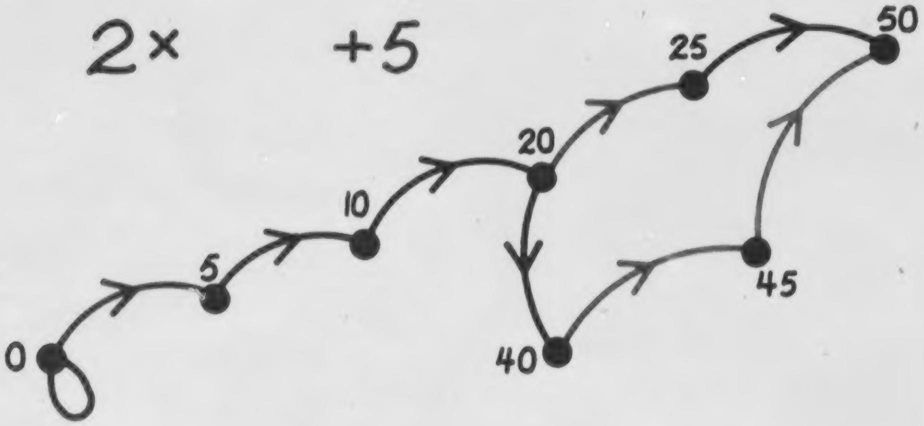
S: No, we can't use any red arrows because 2 x 40 = 80.

Point to the dots for 20 and 50.

T: Is there a shorter road between 20 and 50?

S: We can go from 20 to 50 with only two arrows; 20 + 5 = 25 and 2 x 25 = 50.

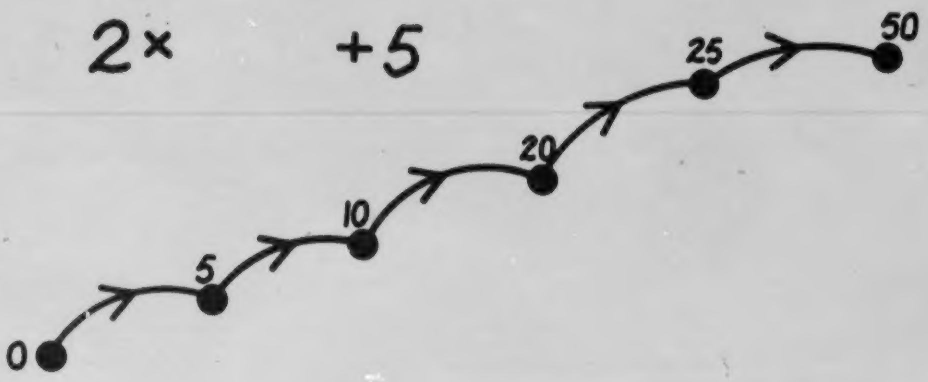
Draw these arrows in your arrow picture.



Point to the dot for 20 with your left forefinger and trace the appropriate arrows with your right forefinger as you speak.

T: We can go from 20 to 50 with a blue arrow followed by a red arrow or with a red arrow followed by two blue arrows.

Erase the longer road from 20 to 50.



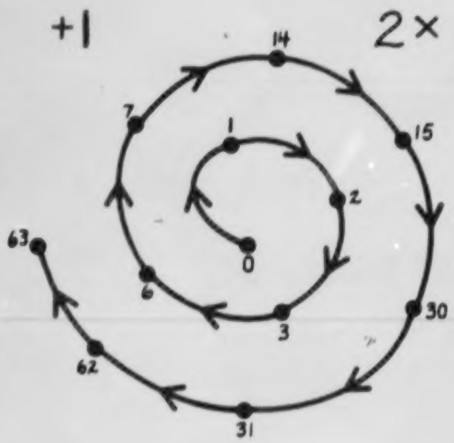
T: How many arrows are in this road now? (Five)
Could we build a shorter road from 0 to 50?

Allow the students to offer their opinions and then tell them that you can't build a red-blue road from 0 to 50 with less than five arrows.

Worksheets N34*, **, ***, and **** are available for individual work.

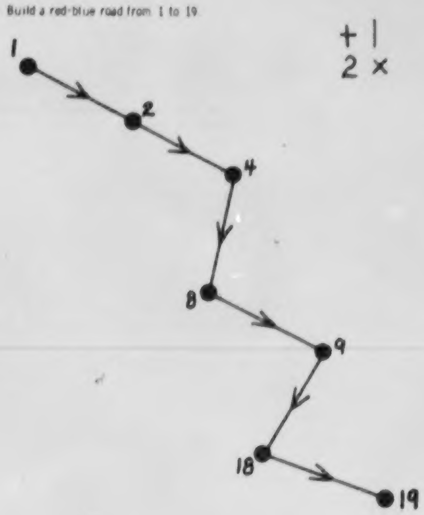
Name ______________ **N 34** *

Label the dots.


+1
2×
0
1
2
3
6
7
14
15
30
31
62
63

Name ______________ **N 34** **

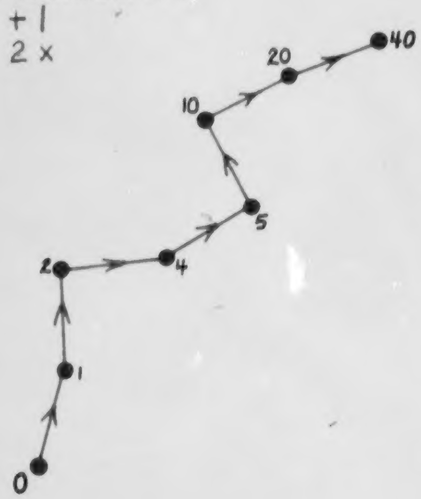
Build a red-blue road from 1 to 19.


+1
2×
1
2
4
8
9
18
19

This road has the fewest possible number of arrows. Other solutions are possible.

Name ______________ **N 34** ***

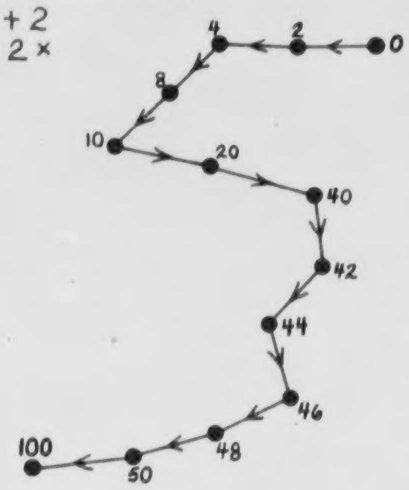
Build a red-blue road from 0 to 40.


+1
2×
0
1
2
4
5
10
20
40

This road has the fewest possible number of arrows. Other solutions are possible.

Name ______________ **N 34** ****

Build a red-blue road from 0 to 100.


+2
2×
0
2
4
8
10
20
40
42
44
46
48
50
100

This road has the fewest possible number of arrows. Other solutions are possible.

N35 Addition Problems #2

CAPSULE LESSON SUMMARY

Solve addition problems which arise from story situations and which involve decimal numbers. Play the Number Line Game with decimal numbers.

MATERIALS

Teacher: Colored chalk
Student: Paper; pencil

DESCRIPTION OF LESSON

Exercise 1

Tell the following or a similar story which is appropriate for your class. Choose one of your students to be the star of this story.

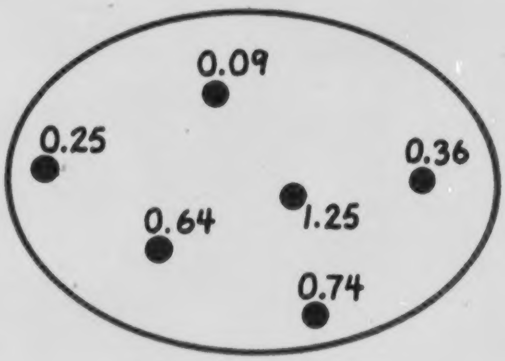
T: Colin has one dollar to spend in a whistle shop. He especially likes six of the whistles at this shop. I will put some numbers inside a string to show the prices of these whistles.

Draw a large string on the board. Inside the string, draw and label a dot "0.09".

T: One of the whistles Colin would like to buy costs this much (point to "0.09").

S: 9 cents.

Continue in this manner until you have this string picture on the board.



333

T: Which of these whistles is the most expensive?

S: The whistle that costs $1.25.

T: Which whistle is the cheapest?

S: The one that costs 9¢.

T: Are there any of the six whistles which Colin can't afford to buy?

S: He can't buy the whistle which costs $1.25 because he only has $1.00.

T: After thinking awhile, Colin found that he could buy two whistles and spend all of his money. Which two whistles are they?

Follow even incorrect choices as indicated in this possible dialogue.

S: He could buy the whistles which cost 25¢ and 74¢.

T: Does 25¢ + 74¢ = $1.00?

Ask the students to add 25 and 74.

S: 25 + 74 = 99.

T: How did you calculate the answer?

Record some of the students' methods of calculation on the board.

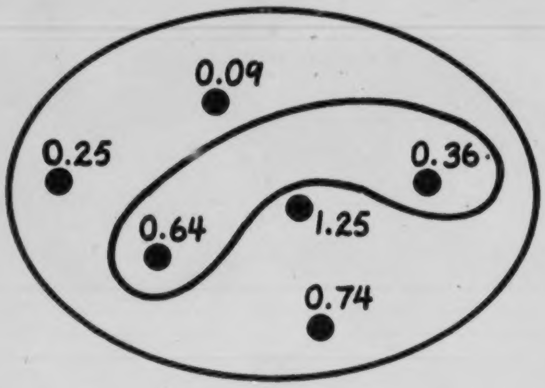
- Some students might write an addition problem and then use the addition algorithm.

$$\begin{array}{r} 25 \\ +74 \\ \hline 99 \end{array} \quad \text{or} \quad \begin{array}{r} 0.25 \\ +0.74 \\ \hline 0.99 \end{array}$$

- Some students might calculate the answer mentally; for example,

25 + 75 = 100	(75 is 1 more than 74)	100 - 1 = 99
20 + 74 = 94	(25 is 5 more than 20)	94 + 5 = 99
20 + 70 = 90	(25 is 5 more than 20 74 is 4 more than 70)	90 + 5 + 4 = 99

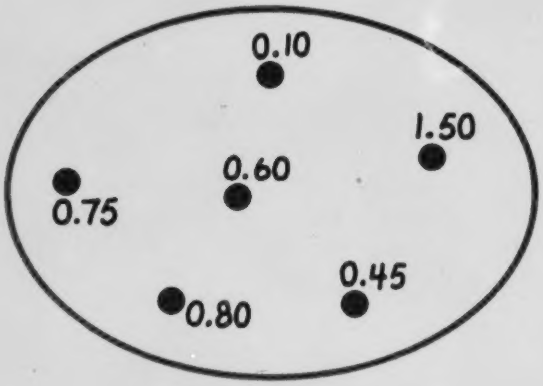
Decide that 25¢ and 74¢ are not the prices of the whistles; then ask for another suggestion. Continue this activity until your class concludes that Colin could buy the whistles which cost 64¢ and 36¢ and spend exactly one dollar for them. Draw a red string around "0.64" and "0.36".



Exercise 2

Present another story with one of your students as the star.

T: Laura likes to collect bicycle decals. She has found six decals which she would especially like to buy.



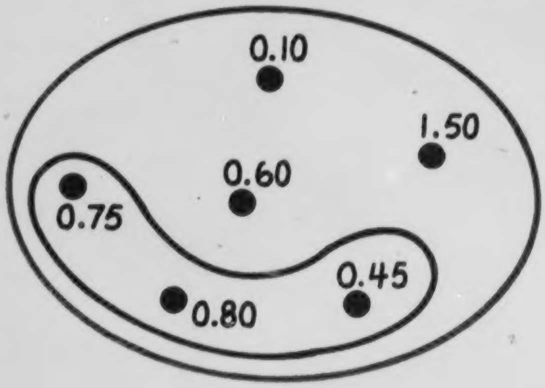
N 35

T: What is the price of the most expensive decal? ($1.50)
What is the price of the cheapest decal? (10¢)
After thinking very carefully, Laura bought three different decals and spent exactly $2. Which three decals did Laura buy?

Ask the students to write the prices of the three decals on their papers. As you observe students' work, you may want to encourage individual students to try other combinations of prices. If your class has difficulty discovering the correct combinations of prices, tell them that one of the decals Laura bought cost 80¢. Continue until someone suggests 0.80, 0.75, and 0.45. Check with the class that the sum of these numbers is 2.00 by writing this addition problem on the board and asking someone to do the calculation.

$$\begin{array}{r} 0.80 \\ 0.75 \\ +\ 0.45 \\ \hline 2.00 \end{array}$$

Draw a red string around "0.75", "0.80", and "0.45".

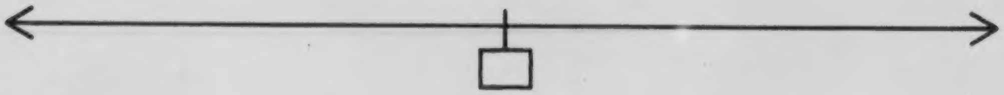


Erase the board.

Exercise 3: The Number Line Game

T: Now we are going to play the Number Line Game. I am thinking of a number between 0 and 10. I'll put a mark for it on the number line. The number is hidden behind the box.

Draw a line on the board and position a box for the secret number near the center of your picture.



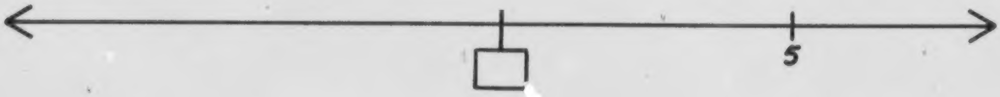
T: Guess what my secret number is.

If someone suggests a number which is not between 0 and 10, do not write anything. Just remind the students that their guesses should be between 0 and 10 and ask someone else to guess.

This is a description of a possible game in which the secret number is 3.25.

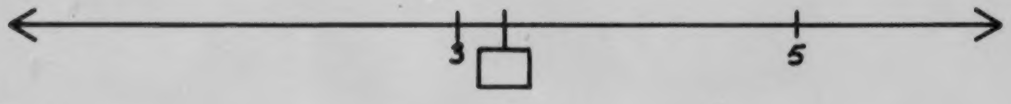
S: 5.

T: I'll show you where 5 is on this number line. It might not be in exactly the right place, but I'll try to make it as close as possible.



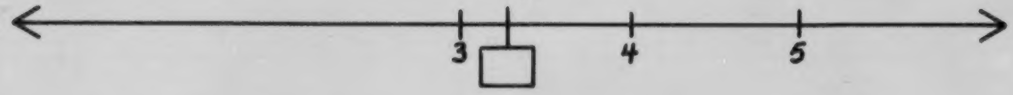
T: 5 is more than my secret number.

S: 3.



T: 3 is less than my secret number.

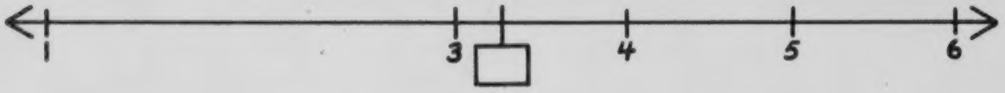
S: 4.



T: 4 is more than my number.

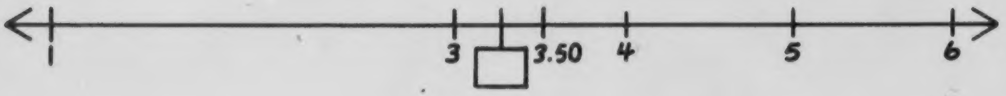
At this point, students may be confused and suggest several whole numbers before a number between 3 and 4 is suggested. Be patient and record these numbers on your number line as long as they are between 0 and 10. You may wish to remind the students that the secret number is between 3 and 4.

Perhaps your number line will look like this one.



S: $3\frac{1}{2}$.

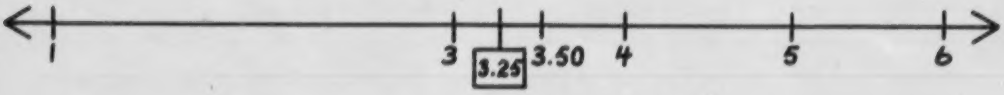
T: Another name for $3\frac{1}{2}$ is "3.50" (read "three point five zero"). I'll show you where 3.50 is on this number line.



T: 3.50 is more than my secret number.

S: $3\frac{1}{4}$ (three and a quarter).

T: How much money is one-fourth of a dollar? (25¢)
Another name for $3\frac{1}{4}$ is "3.25".



T: 3.25 is my secret number.

Play the game several times. Use other secret numbers such as 9.50, 7.10, and 6.80.

N 36

N36 Common Multiples #2

CAPSULE LESSON SUMMARY

Find non-negative common multiples of 6 and 4 by taking consecutive +6 jumps starting at 0 and +4 jumps starting at 0 on the number line and looking at common landing points.

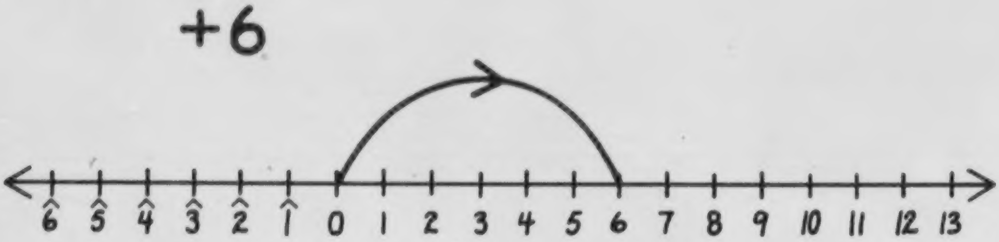
MATERIALS

Teacher: Colored chalk

Student: Worksheets N36 (no stars), *, and **; pencil; colored pencils

DESCRIPTION OF LESSON

Draw this picture on the board.



T: If we start at 0 and make +6 jumps, what are some of the numbers we would land on?

Encourage the students to make several suggestions. If necessary, solicit numbers which are more than 15 but do not draw any more arrows on the board. You may need to remind the students that the number line continues in both directions.

A sample dialogue is given here.

S: We would meet 12.

T: How many +6 jumps would we make to go from 0 to 12?

S: Two.

T: 2 × 6 = 12. What is another number we would land on?

S: 18.

T: How many +6 jumps would we make to go from 0 to 18?

S: Three.

T: 3 × 6 = 18. What is another number we would land on?

S: 30.

T: If we continue drawing +6 arrows, would we meet 30?

S: 5 × 6 = 30, so we would meet 30 after five jumps.

T: What kind of numbers do we meet if we start at 0 and make +6 jumps?

S: Multiples of 6.

Continue asking for numbers on which you would land on, that is, for multiples of 6.

S: 60.

S: 66.

Perhaps someone will suggest a number which is not a multiple of 6; for example,

S: 100.

N 36

T: Is 100 a multiple of 6?

The class may be in disagreement.

T: Does everyone agree that 60 is a multiple of 6? (Yes; 10 x 6 = 60)
Let's start at 60 and count by sixes. (66, 72, 78, 84, 90)
Since 90 is a multiple of 6, can 100 be a multiple of 6?

S: 90 + 6 = 96 and 96 + 6 = 102, so 100 is not a multiple of 6.

T: What are some large numbers which are multiples of 6?

S: 102.

S: 600.

S: 6,000,000.

S: 666.

T: Are there any negative numbers which are multiples of 6? (Yes)
Name some of them. ($\hat{6}$, $\widehat{12}$, $\widehat{18}$, and so on)
How many +6 jumps would we make to go from $\hat{6}$ to 0? (One)
$\hat{6} + 6 = 0$.

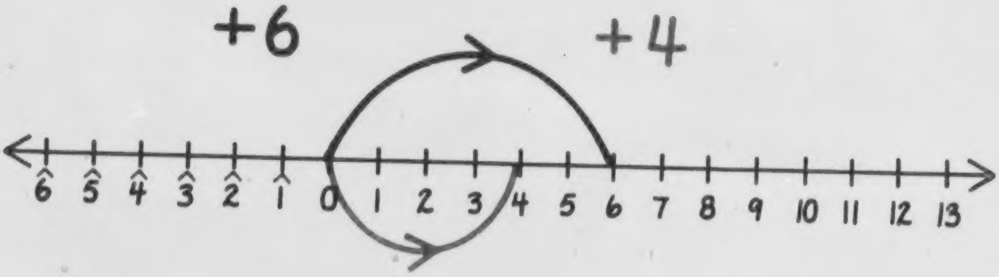
Trace an arrow from " $\hat{6}$ " to " 0 " on the number line.

NOTE: For your information only, one way to check that a number is a multiple of 6 is to check that it is both a multiple of 2 and a multiple of 3; for example,

- 72 is a multiple of 2 (an even number) and 72 is a multiple of 3 (7 + 2 = 9 and 9 is a multiple of 3); therefore, 72 is a multiple of 6.

- 200 is a multiple of 2, but 200 is not a multiple of 3 (2 + 0 + 0 = 2 and 2 is not a multiple of 3); therefore, 200 is not a multiple of 6.

Add the key "+ 4" to your picture and draw a blue arrow from "0" to "4" on the number line.



T: If we start at 0 and make + 4 jumps, what are some of the numbers we would meet?

Encourage the students to make several suggestions as you did for the multiples of 6.

NOTE: The following observation is for your information only. Since 100 is a multiple of 4, we know that an integer is a multiple of 4 if the number determined by its last two digits is a multiple of 4; for example,

- 308 is a multiple of 4 because 8 is a multiple of 4;
- 510 is not a multiple of 4 because 10 is not a multiple of 4.

T: Are there any numbers which are common multiples of 6 and 4?

Commend students who suggest numbers which are common multiples of 4 and 6, and let them explain if possible. Then provide Worksheet N36 (no stars).

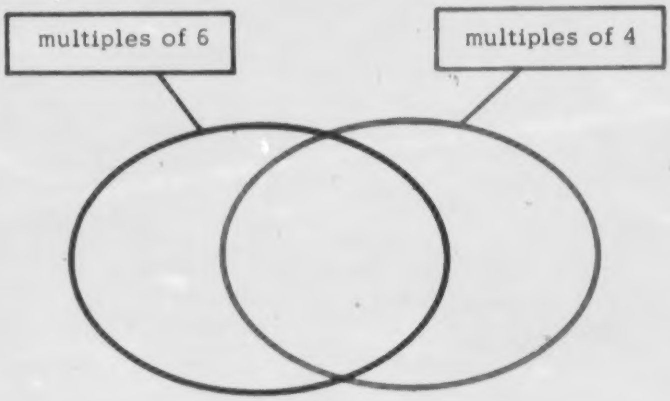
N 36

T: We can find the common multiples of 4 and 6 by starting at 0 and drawing + 4 and + 6 arrow roads on the number line. On this worksheet, start at 0 and draw a + 4 arrow road; then start at 0 again and draw a + 6 arrow road on the same number line.

Ask the students to write "**+6**" and "**+4**" at the top of the worksheet before they begin drawing arrows.

Allow the students to work independently until four or five students have finished. Students who have difficulty may wish to begin again on the reverse side of the worksheet.

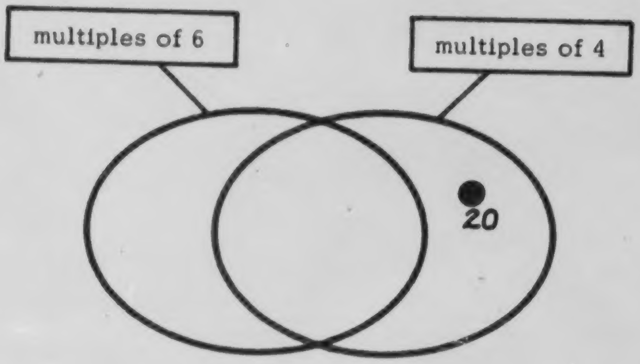
Draw this string picture on the board.



T: I would like to put some of the numbers on the worksheet in this string picture.

Invite students to put numbers in this string picture. Solicit numbers for all four regions of the picture. Frequently ask the students to explain how they know where a particular number belongs in the string picture. A possible dialogue is given here.

A student puts 20
in the string picture.

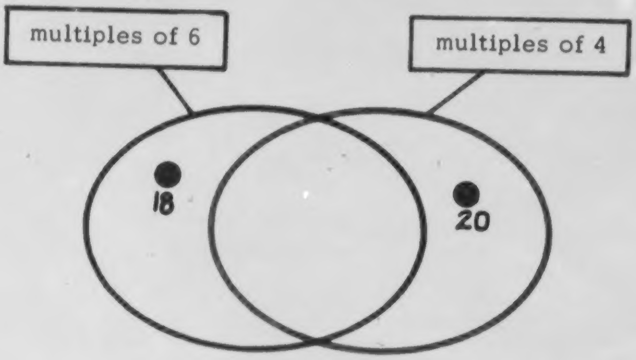

multiples of 6
multiples of 4
20

T: How do you know that 20 is a multiple of 4 but not a multiple of 6?

S: A blue arrow ends at 20, but none of the red arrows end at 20.

T: We know that 20 is a multiple of 4, because it is on the +4 arrow road which starts at 0. We also know that 20 is not a multiple of 6, because it is not on the +6 arrow road which starts at 0.

A student puts 18
in the string picture.


multiples of 6
multiples of 4
18
20

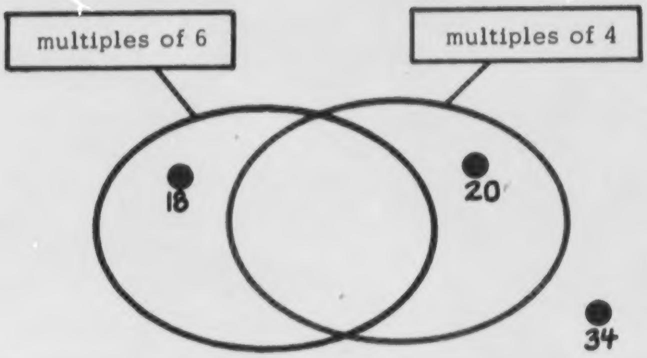
T: How do you know that 18 is a multiple of 6 but not a multiple of 4?

S: 18 is on the red arrow road but not on the blue arrow road.

T: Are there any numbers from 0 to 48 which go outside both the red string and the blue string?

N 36

A student puts 34 in the string picture.

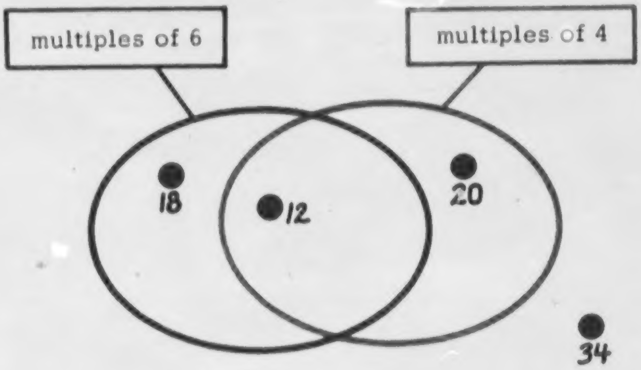

multiples of 6
multiples of 4
18
20
34

T: How can we be sure that 34 is neither a multiple of 6 nor a multiple of 4?

S: Both the red and the blue arrow roads skip over 34.

T: Are there any numbers which will be inside both the red string and the blue string, that is, in the middle?

A student puts 12 in the string picture.


multiples of 6
multiples of 4
18
12
20
34

T: Is 12 a multiple of 4 and also a multiple of 6? How do you know?

S: Yes, 12 is on both of the arrow roads.

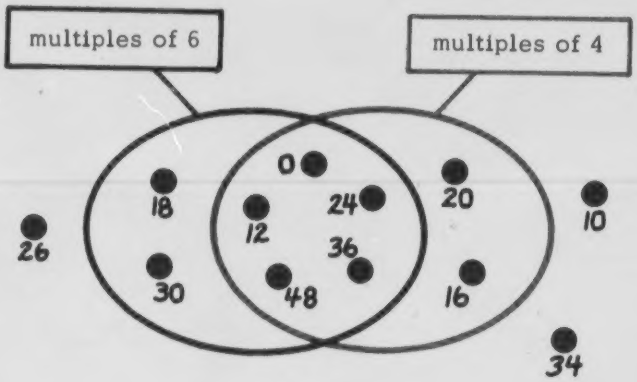
T: How many + 6 jumps would we make to go from 0 to 12? (Two)
2 × 6 = 12.

N 36

T: How many +4 jumps do we make to go from 0 to 12? (Three)
3 x 4 = 12.
Are there any other numbers from 0 to 48 which are both multiples of 6 and multiples of 4? (Yes) What are some of them?

Record in the string picture the common multiples of 4 and 6 which the students suggest. If no one suggests that 0 is a common multiple of 4 and 6, mention this yourself.

Perhaps your string picture will look like this one.



T: Look at the common multiples of 6 and 4. Do you see any patterns?

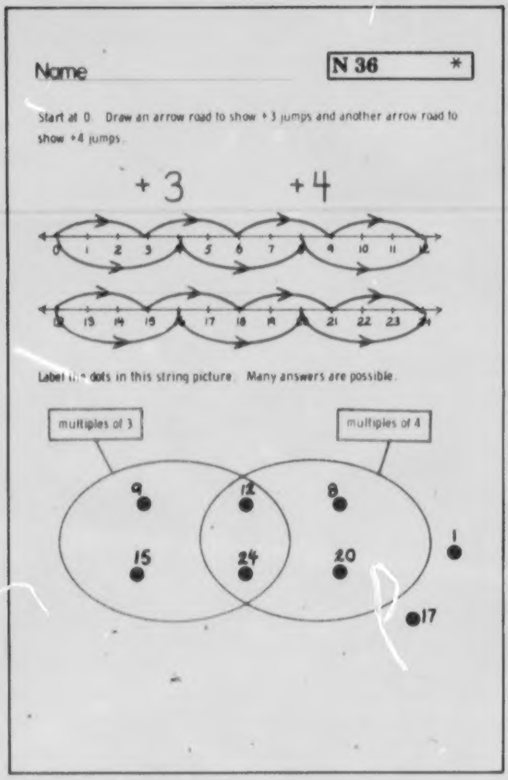
S: 0 + 12 = 12; 12 + 12 = 24; 24 + 12 = 36; and 36 + 12 = 48.

T: Are there any numbers which are not on this worksheet which are also common multiples of 4 and 6? (Yes)

Encourage the students to suggest several common multiples of 6 and 4 including some negative numbers.

Perhaps one of your students will observe that the numbers in the intersection of the two sets are the multiples of 12. Commend this observation but do not make it yourself.

Worksheets N36* and ** are available for individual work.



Name

N 36 *

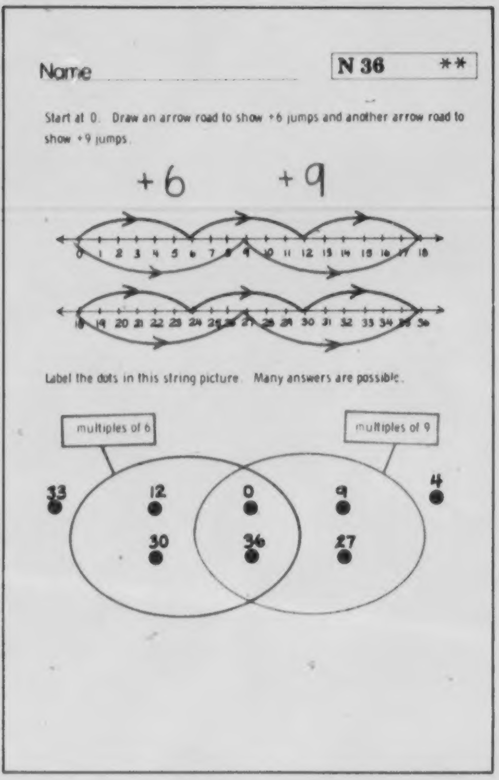
Start at 0. Draw an arrow road to show +3 jumps and another arrow road to show +4 jumps.

+3 +4

0
1
2
3
5
6
7
9
10
11
12
13
14
15
17
18
21
22
23
24

Label the dots in this string picture. Many answers are possible.

multiples of 3
multiples of 4
9
12
8
15
24
20
1
17



Name

N 36 **

Start at 0. Draw an arrow road to show +6 jumps and another arrow road to show +9 jumps.

+6 +9

0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
27
29
30
31
32
33
34
36

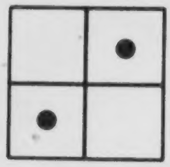
Label the dots in this string picture. Many answers are possible.

multiples of 6
multiples of 9
33
12
0
9
4
30
36
27

APPENDIX A: STANDARD CONFIGURATION AND STANDARD MINICOMPUTER MOVES

A number can be represented by putting checkers[*] on the squares of the Minicomputer. Each square has a numerical (positional) value, so that the number represented by a checker depends on which square the checker is placed. If several checkers are put on the Minicomputer, the number shown is the sum of the numbers represented by the checkers. A number can be put on the Minicomputer in a variety of ways, but the representation which uses at most one checker on each square and uses checkers to represent a digit 9 or less on each board is usually the easiest to read. In this case, we say that the number is in standard configuration. Standard configurations for the numbers 1 - 9 soon become as familiar to the students as the usual numerals so that they no longer need to do any mental calculation for such configurations.

For example,



. . . is seen as a name for 6 without doing the calculation "4 + 2 = 6".

Doing calculations on the Minicomputer involves putting one or more checkers on the various squares and making "plays" to get a specific configuration. In this context, a play is an exchange of checkers in a configuration which does not change the number represented. Some plays are natural to the design of the Minicomputer; we call these standard or usual plays (these are described in detail in this section). There are advantages to establishing such a set of plays:

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[*] In this appendix we refer only to whole numbers and, therefore, use only regular (positive) checkers.

- They become routine, and a large group of students with varying abilities can follow a calculation without needing to verify every play which is made.
- They lead to standard configurations of numbers.

These are the standard Minicomputer moves.

1 + 1 = 2
10 + 10 = 20
100 + 100 = 200
1,000 + 1,000 = 2,000

two checkers on the white square
of a board
=
one checker on the red square
of the same board

2 = 1 + 1
20 = 10 + 10
200 = 100 + 100
2,000 = 1,000 + 1,000

one checker on the red square
of a board
=
two checkers on the white square
of the same board

2 + 2 = 4
20 + 20 = 40
200 + 200 = 400
2,000 + 2,000 = 4,000

two checkers on the red square
of a board
=
one checker on the purple square
of the same board

4 = 2 + 2
40 = 20 + 20
400 = 200 + 200
4,000 = 2,000 + 2,000

one checker on the purple square
of a board
=
two checkers on the red square
of the same board

$4 + 4 = 8$
$40 + 40 = 80$
$400 + 400 = 800$
$4{,}000 + 4{,}000 = 8{,}000$

two checkers on the purple square
of a board
=
one checker on the brown square
of the same board

$8 = 4 + 4$
$80 = 40 + 40$
$800 = 400 + 400$
$8{,}000 = 4{,}000 + 4{,}000$

one checker on the brown square
of a board
=
two checkers on the purple square
of the same board

$8 + 2 = 10$
$80 + 20 = 100$
$800 + 200 = 1{,}000$
$8{,}000 + 2{,}000 = 10{,}000$

one checker on each of the brown
and red squares of a board
=
one checker on the white square
of the next board to the left

$10 = 8 + 2$
$100 = 80 + 20$
$1{,}000 = 800 + 200$
$10{,}000 = 8{,}000 + 2{,}000$

one checker on the white square
of a board
=
one checker on each of the brown
and red squares of the next board
to the right[*]

---

[*] This provides the link that motivates the introduction of the decimal numbers. In particular, we have that one checker on the white square of the ones' board equals one checker on each of the brown and red squares of a special board (the tenths' board - or as it is often referred to in the lessons, the dimes' board) located to the right of the ones' board and separated from it by a special barrier.

After many experiences with the Minicomputer, some students might suggest "non-standard" plays, e.g. "8 + 4 = 10 + 2". When students suggest such plays, ask them to convince the class that the plays are indeed correct. There will be a natural acceptance or rejection of non-standard plays by the students, sometimes depending on the situation.

Some "non-standard" plays will be introduced for specific purposes; for example, ten checkers on a square can be exchanged for one checker on the same color square of the next board to the left. Such plays serve to reinforce the support that the Minicomputer gives to decimal place value or to provide insight into a certain kind of problem. They are inviting in particular situations, but do not play the same "routine" role that the usual plays do.

APPENDIX B: MENTAL ARITHMETIC

Mental arithmetic activities are short, fast-moving question and answer sessions where students are asked to calculate mentally, preferably without the aid of paper and pencil. Often a mental arithmetic session consists of several sequences in which the answers to one or more questions lead to the answer of another, more difficult question; for example,

$2 \times 3 = ?$ (6)
$2 \times 10 = ?$ (20)
$2 \times 13 = ?$ (26)

There are specific lessons which call for mental arithmetic; however, we suggest you try to include such activities for five to ten minutes at least twice a week. Use mental arithmetic as a warm-up activity to a lesson, as a transition between two exercises in a lesson, as a conclusion to a lesson, or as a quick change of pace at any time during the day.

Why is Mental Arithmetic Important?

1) It develops mental computational skills.
2) It provides an opportunity to involve and help students at all ability levels.
3) It creates an awareness of patterns and mathematical relationships.
4) It helps students to be able eventually to recall arithmetic facts easily.
5) It provides an opportunity to keep alive and to reinforce concepts previously introduced.
6) It helps the students to see that there is often more than one way to solve a problem.

Hints for Mental Arithmetic Activities

1) Vary the level of difficulty of questions throughout a session.

2) Involve many students.

3) Follow a pattern for awhile and then start a new pattern.

4) Keep a brisk pace, although you should allow students time for thought.

5) Give extra attention to a student who shows signs of improvement.

6) Illustrate, whenever possible, the value of estimation in making calculations.

7) Occasionally ask a student to explain an answer.

8) Occasionally use the chalkboard to emphasize a pattern.

9) Occasionally ask students to whisper an answer to you so that many students will have an opportunity to answer.

SUGGESTED MENTAL ARITHMETIC ACTIVITIES

This section contains specific examples that illustrate patterns in arithmetic. You should elaborate and expand upon ideas depending upon the abilities and interests of your students. As you work with your class on the regularly scheduled mathematics lessons, you might make a note of computational skills you would like to improve; then plan a short mental arithmetic activity which develops those skills. Do not expect everyone of your students to be good at all of these suggested activities, even at the end of the year.

Counting Activities

1) Count by twos, threes, fives, or tens collectively, everyone counting together. Refer to the 0-109 chart as the class counts if you wish.

2) Count by twos, threes, fives, or tens as a group activity with each student saying one number. Use a natural seating order in your class (such as going up and down rows) and ask each student in turn to say the number which comes next. If you are counting by threes, the first student will say "3"; the second will say "6"; and so on. Repeat the counting but start with a different student.

3) Count by twos, threes, fives, or tens, starting at 1 or any number other than 0.

4) Count backward by twos, fives, or tens. Do this immediately following a similar forward count.

5) Play "BUZZ". The students count by ones starting at 0 but skip every fourth number by saying "buzz". This should be done rapidly. This can be done collectively or as a group activity with each student in turn saying the number that comes next or "buzz".

More challenging variations of this activity involve:

- skipping every sixth number rather than every fourth number;
- starting the counting at 2 and skipping every fourth number;
- counting by twos or fives and skipping every fourth number.

Addition and Subtraction

The first fourteen examples are lists of calculations to be done mentally. Each list suggests a pattern in arithmetic which students should begin to use in doing such calculations.

1) 6 + 4
6 + 5
6 + 6
6 + 7

8 + 2
8 + 3
8 + 4
8 + 5

20 + 20
20 + 21
20 + 22
20 + 23

10 - 4
10 - 5
10 - 6
10 - 7

40 - 1
40 - 2
40 - 3
40 - 4

20 - 10
20 - 11
20 - 12
20 - 13

2) 2 + 5
20 + 50
200 + 500
2,000 + 5,000
2,000,000 + 5,000,000

8 + 8
80 + 80
800 + 800
8,000 + 8,000
8,000,000 + 8,000,000

3) 10 + 5
5 + 10
10 + 7
7 + 10

10 + 6
20 + 6
30 + 6
40 + 6

30 + 6
31 + 6
40 + 5
43 + 5

4) 5 + 3
8 - 3
7 + 6
13 - 6

10 + 20
30 - 20
90 + 40
130 - 40

106 + 105
211 - 105
150 + 150
300 - 150

5) 4 + 10
14 + 10
24 + 10
44 + 10

7 + 10
17 + 10
117 + 10
1,017 + 10

6 + 10
18 + 10
33 + 10
71 + 10

5 + 20
15 + 20
25 + 20
35 + 20

3 + 20
13 + 20
113 + 20
1,013 + 20

19 + 20
100 + 20
600 + 20
619 + 20

6) 7 - 5
17 - 5
27 - 5
37 - 5

10 - 3
20 - 3
30 - 3
40 - 3

100 - 2
200 - 2
300 - 2
400 - 2

7) 7 + 4
7 + 14
7 + 24
7 + 34

10 + 18
20 + 18
30 + 18
40 + 18

10 + 13
12 + 13
10 + 14
13 + 14

8) 44 - 10
44 - 20
44 - 30
44 - 40

52 - 10
52 - 20
52 - 30
52 - 40

86 - 10
186 - 10
186 - 20
186 - 30

307 - 100
307 - 200
307 - 300

520 - 100
520 - 200
520 - 300

1,000 - 100
1,000 - 200
1,000 - 300

9) 23 - 10
230 - 100
2,300 - 1,000

51 - 10
510 - 100
5,100 - 1,000

98 - 20
980 - 200
9,800 - 2,000

10) 5 + 2
5 + 1
5 + 0
$5 + \widehat{1}$
$5 + \widehat{2}$

16 + 2
16 + 1
16 + 0
$16 + \widehat{1}$
$16 + \widehat{2}$

64 + 2
64 + 1
64 + 0
$64 + \widehat{1}$
$64 + \widehat{2}$

11) $2 + \widehat{3}$
$20 + \widehat{30}$
$200 + \widehat{300}$

$\widehat{5} + \widehat{1}$
$\widehat{50} + \widehat{10}$
$\widehat{500} + \widehat{100}$

$8 + \widehat{5}$
$80 + \widehat{50}$
$800 + \widehat{500}$

12) 6 + 10
6 + 9
8 + 10
8 + 9

27 + 10
27 + 9
44 + 10
44 + 9

100 - 10
100 - 9
1,000 - 10
1,000 - 9

13) $7 + \widehat{9}$
$\widehat{7} + 9$

$3 + \widehat{8}$
$\widehat{3} + 8$

$12 + \widehat{15}$
$\widehat{12} + 15$

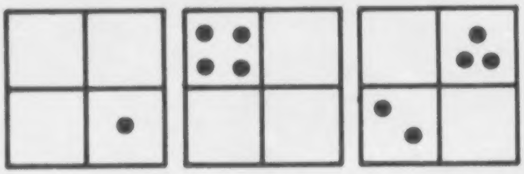
14) 50 + 40
8 + 2
58 + 42

30 + 20
5 + 7
35 + 27

40 + 60
9 + 4
49 + 64

15) Choose some numbers and ask the students to add 1 to each number. For example, if you say "108", a student responds "109". You can vary this activity by asking the students to add 2, 3, 5, or 10 to each of your numbers or to subtract 1, 2, 3, or 10 from each of your numbers.

16) Conduct an activity similar to an exercise entitled "Transforming a Number" in Lesson N1. Put six to ten checkers on the ones' and the tens' boards. Move one checker from some square (it makes no difference which square you choose) to another square. Ask the students how much larger or smaller the new number is. For example, suppose this configuration is on the Minicomputer.



Move a checker from the 4-square to the 10-square. When the students tell you that the new number is larger, ask them how much larger this number is than the previous number. (6 larger)

17) Ask the students to give various names for a particular number. Encourage them to suggest names involving subtraction and also names involving addition of three numbers. For example, names for 20 include: 16 + 4; 30 - 10; 4 + 4 + 12; $21 + \hat{1}$; 4 × 5; $\frac{1}{2} \times 40$.

Multiplication

The first ten examples are lists of calculations to be done mentally. Each list suggests a pattern in arithmetic which students should begin to use in doing such calculations.

1) 3 + 3
2 × 3
5 + 5
2 × 5

7 + 7
2 × 7
20 + 20
2 × 20

100 + 100
2 × 100
200 + 200
2 × 200

2) 1 × 5
2 × 5
3 × 5
4 × 5

1 × 10
2 × 10
3 × 10
4 × 10

1 × 20
2 × 20
3 × 20
4 × 20

3) 2 × 3
2 × 30
2 × 300
2 × 3,000
2 × 3,000,000

2 × 5
2 × 50
2 × 500
2 × 5,000
2 × 5,000,000

3 × 4
3 × 40
3 × 400
3 × 4,000
3 × 4,000,000

4) 2 × 3
2 × 33
2 × 333
2 × 3,333

3 × 3
3 × 33
3 × 333
3 × 3,333

2 × 5
2 × 55
2 × 555
2 × 5,555

2 × 10
2 × 3
2 × 13

2 × 40
2 × 2
2 × 42

2 × 20
2 × 6
2 × 26

2 × 60
2 × 4
2 × 64

2 × 100
2 × 7
2 × 107

2 × 200
2 × 30
2 × 1
2 × 231

5)	6 + 6	10 + 10	50 + 50
	2 × 6	2 × 10	2 × 50
	$\frac{1}{2} \times 12$	$\frac{1}{2} \times 20$	$\frac{1}{2} \times 100$
6)	10 × 2	10 × 5	10 × 8
	10 × 20	10 × 50	10 × 80
	10 × 200	10 × 500	10 × 800
7)	$\frac{1}{2} \times 6$	$\frac{1}{2} \times 10$	$\frac{1}{2} \times 24$
	$\frac{1}{2} \times 60$	$\frac{1}{2} \times 100$	$\frac{1}{2} \times 240$
	$\frac{1}{2} \times 600$	$\frac{1}{2} \times 1,000$	$\frac{1}{2} \times 2,400$
	$\frac{1}{2} \times 6,000$	$\frac{1}{2} \times 10,000$	$\frac{1}{2} \times 24,000$
	$\frac{1}{2} \times 6,000,000$	$\frac{1}{2} \times 10,000,000$	$\frac{1}{2} \times 24,000,000$
8)	$\frac{1}{2} \times 20$	$\frac{1}{2} \times 6$	$\frac{1}{2} \times 10$
	$\frac{1}{2} \times 30$	$\frac{1}{2} \times 16$	$\frac{1}{2} \times 100$
	$\frac{1}{2} \times 40$	$\frac{1}{2} \times 26$	$\frac{1}{2} \times 108$
	$\frac{1}{2} \times 50$	$\frac{1}{2} \times 36$	$\frac{1}{2} \times 148$
9)	2 × 13	2 × 35	2 × 104
	$\frac{1}{2} \times 26$	$\frac{1}{2} \times 70$	$\frac{1}{2} \times 208$
	3 × 10	3 × 5	3 × 105
	$\frac{1}{3} \times 30$	$\frac{1}{3} \times 15$	$\frac{1}{3} \times 315$

10) $\frac{1}{2} \times 14$

$\frac{1}{2} \times 15$

$\frac{1}{2} \times 20$

$\frac{1}{2} \times 21$

$\frac{1}{2} \times 7$

$\frac{1}{2} \times 9$

$\frac{1}{2} \times 11$

$\frac{1}{2} \times 13$

$\frac{1}{2} \times 100$

$\frac{1}{2} \times 101$

$\frac{1}{2} \times 102$

$\frac{1}{2} \times 103$

11) Ask for a multiple of 3 (or 4) between two other numbers; for example, ask for a multiple of 3 between 20 and 25.

Estimation

Ask the students to estimate a calculation and to explain their answers. For example,

T: Is 218 + 157 more than 300? How do you know?

S: Yes, because 200 + 100 = 300.

T: Is it more than 500? How do you know?

S: No, because 300 + 200 = 500. 218 is less than 300 and 157 is less than 200, so 218 + 157 must be less than 500.

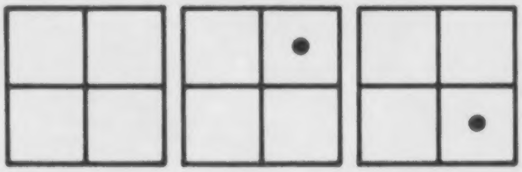
T: Is it more than 400?

S: No, because 18 + 57 is less than 100.

During the lessons, students are often asked to estimate a number displayed on the Minicomputer. See Lesson N1 for an example.

Other Mental Arithmetic Activities

1) Use the Minicomputer to give visual clues for addition, subtraction, and multiplication problems. For example, suppose this configuration is on the Minicomputer.



T: What number is on the Minicomputer? (41)

Hold a checker over the 8-square.

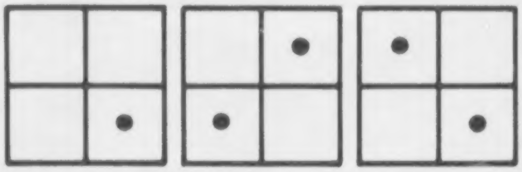
T: What number is 41 + 8? (49)

Hold a checker over the 10-square.

T: What number is 41 + 10? (51)

After a few such clues, hold a checker over the square only if the class has difficulty.

The next example involves subtraction.



T: What number is on the Minicomputer? (169)

Lift the checker temporarily from the 8-square.

T: What number is 169 - 8? (161)

Lift the checker temporarily from the 20-square.

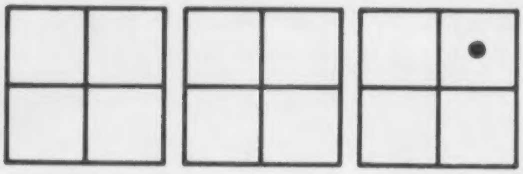
T: What number is 169 - 20? (149)

Lift the checkers temporarily from the 20-square and the 1-square.

T: What number is 169 - 21? (148)

After a few such clues, lift the checkers only if the class has difficulty.

The next example involves multiplication.



T: What number is on the Minicomputer? (4)

Lift the checker.

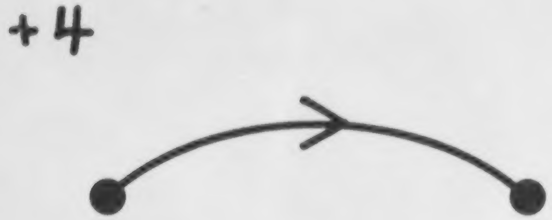
T: What number is 10 × 4?

S: 40.

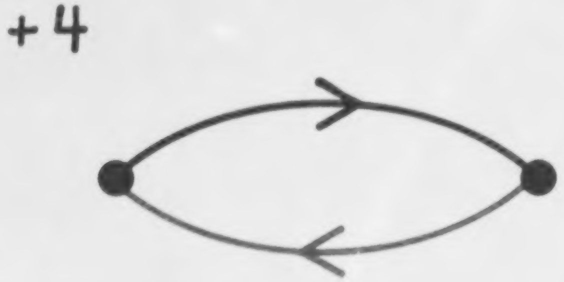
Put the checker on the 40-square.

Repeat this activity starting with the checker on another square of the ones' board.

2) Use an arrow picture to pose mental arithmetic questions. For example, draw this arrow picture on the board.



Ask what number is at the right if the number at the left is, for example, 7. Repeat this activity several times and then draw the -4 return arrow.



T: What could this blue return arrow be for?

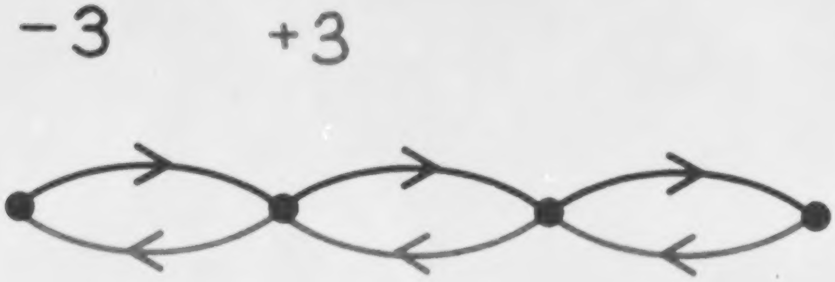
S: -4.

Ask the students what number is at the left if the number at the right is, for example, 10. Repeat this activity several times.

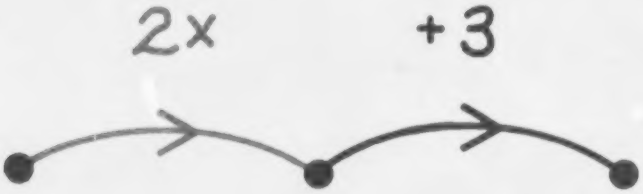
There are many variations of this activity; for example,

- use larger starting and ending numbers;
- use functions other than +4 and -4;

- use two or more arrows of the same function;



- use a composition of two functions.



NOTE: In this last variation, choose the ending number carefully when asking the class for the starting number. For example, if the ending number is 16, then 16 - 3 = 13 and "$\frac{1}{2}$ x 13" might be too difficult a calculation for most students, especially early in the year.

3) Use the suggestion of an arrow picture to pose place value questions.

- Imagine an arrow road starting at 0 and having four +10 arrows and seven +1 arrows. What is the ending number?
- Imagine an arrow road starting at 0 and having two +100 arrows, five +10 arrows, and two +1 arrows. What is the ending number?

4) Play "Guess My Rule". Choose a secret rule such as +10. A student suggests a number to you and you answer with the number which is 10 more. This continues until the students discover your rule; for example,

S: 34.

T: 44.

S: 1.

T: 11.

S: 100.

T: 110.

S: 50.

T: 60. Does any one know my secret rule?

S: +10.

T: How do you know?

S: Each time you answered a number which was 10 more than the number we gave you.

Choose other secret rules such as +3, +5, -2, -1, 2×, 3×, and 10×.

Hand Calculator Activities

Use the hand calculator (i) to develop and reinforce counting skills; (ii) to emphasize counting patterns; and (iii) to improve estimation skills.

1) Use one or more hand-calculators in the class to support counting activities in which patterns are generated. For example, such an activity might be similar to the first clue in "Detective Story #2", from Lesson W6. Start with any whole number on the display of your calculator and press $\boxed{+}$ $\boxed{5}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ .... Record on the board the number that appears on the display of the hand calculator each time you press $\boxed{=}$. As soon as most of your students are able to predict the sequence quickly, you can abandon the calculator. Ask students to explain the pattern.

There are many variations of this activity, such as:

- press $\boxed{+}$ $\boxed{2}$ (or $\boxed{3}$ or $\boxed{4}$ or $\boxed{1}$ $\boxed{0}$) $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ ...;
- start with a large number and press $\boxed{-}$ $\boxed{5}$ (or $\boxed{2}$ or $\boxed{1}$ $\boxed{0}$ or $\boxed{1}$ $\boxed{0}$ $\boxed{0}$) $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ ....

2) Play "Let's Concentrate" using one or more hand-calculators in the class. Ask the students who have the calculators to put a number of your choice on the display and then to hide the display with one hand. Give a sequence of two operations for the students to do by pressing the appropriate keys and without looking at the display. Then ask what number is on the display and let the students check to see if they are correct. Continue with the new number and another sequence of two operations. For example,

Start with 4 on the display.

Press $\boxed{+}$ $\boxed{3}$ $\boxed{+}$ $\boxed{2}$ $\boxed{=}$. The resulting number is 9.
(Students check that 9 is on the display.)

Press $\boxed{+}$ $\boxed{1}$ $\boxed{\times}$ $\boxed{2}$ $\boxed{=}$. The resulting number is 20.
(Students check that 20 is on the display.)

Press $\boxed{-}$ $\boxed{3}$ $\boxed{+}$ $\boxed{1}$ $\boxed{=}$. The resulting number is 18.
(Students check that 18 is on the display.)

3) Play "Hand-Calculator Golf" with a small group of students, each one having a calculator. Ask the students to put a small whole number of your

choice on the display of the calculators. Then let the students take turns suggesting some numbers to add or to subtract until a particular goal, such as 200, is reached.

A possible game where 200 is the goal and four students are playing is described below.

		Display
Start with 12 on the display.		12
First player:	+ 9	21
Second player:	+72	93
Third player:	+17	110
Fourth player:	+ 7	117
First player:	+93	210
Second player:	-10	200

Some variations of this game include:

- change the starting number;
- make the goal a larger multiple of 100;
- allow the use of $\boxed{x}$ $\boxed{\text{some number between 0 and 10}}$

Comprehensive School Mathematics Program
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Final Experimental Version
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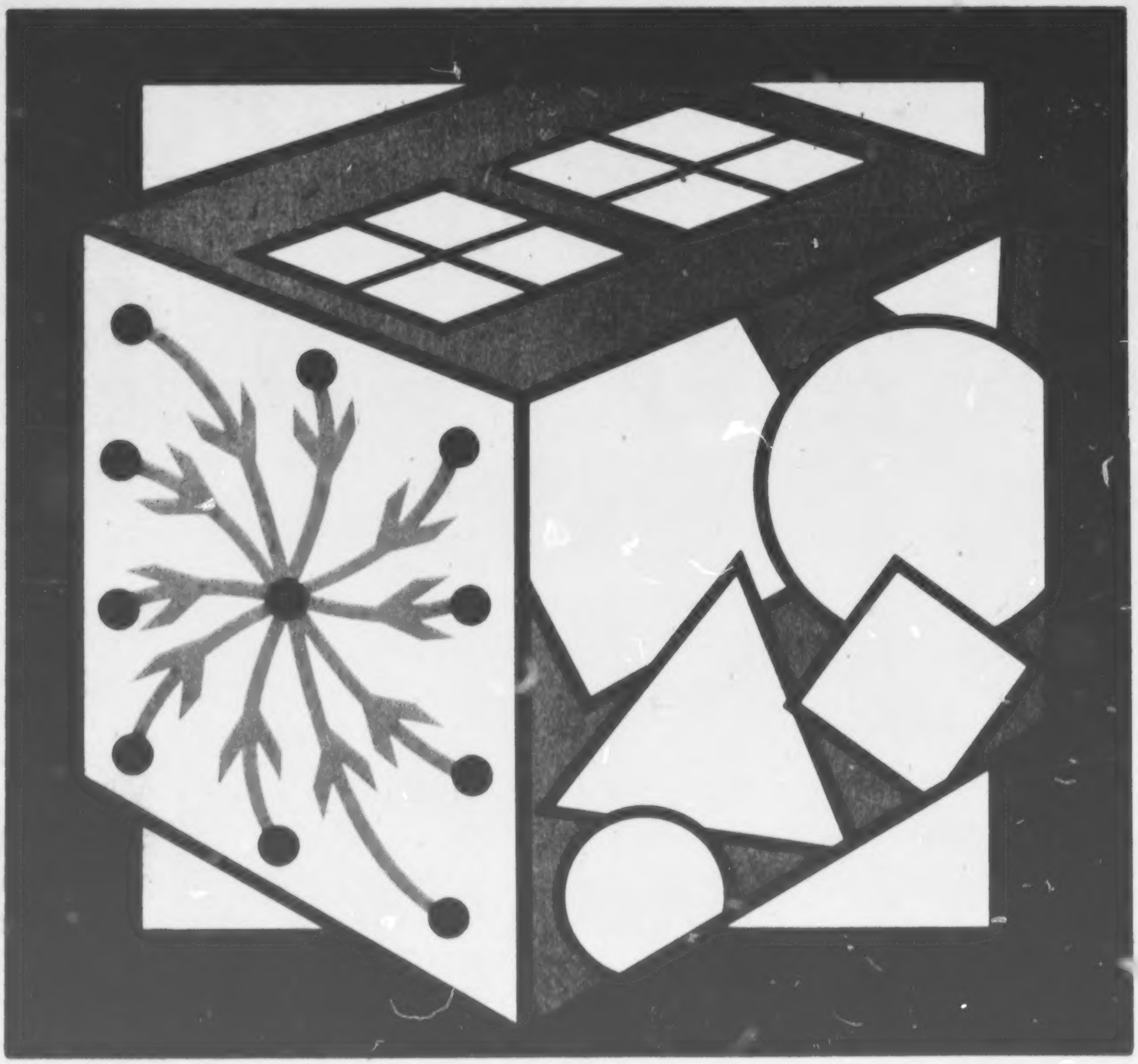
CSMP Mathematics for the Upper Primary Grades
Part II

Name

Worksheets



WHAT'S IN THIS BOOK?

This book contains all the worksheets you will need for Part II of the CSMP Upper Primary Grades Curriculum. Worksheets are labeled with the same letter and number as the lessons with which they are used. In this book, they are in the following order:

1) All the "N" worksheets: N1* through N36**
2) All the "L" worksheets: L1 through L14
3) All the "G" worksheets: G1* through G13**
4) All the "W" worksheets: W4 through W6****

HOW DO YOU REMOVE PAGES?

Step 1: Find the worksheet you want.
The number of each worksheet is in the top right corner of the page.

Step 2: Fold and crease the page along the perforation line.

Step 3: With your left hand index finger and thumb, press book firmly down left of the perforation line and along center fold, starting at the top of the book.

Step 4: With your right hand, hold top right corner of page and tear along the perforation line. Follow the tear with your left hand holding down center fold of book.

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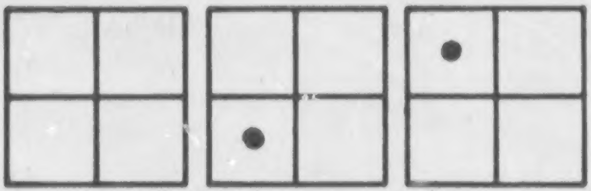
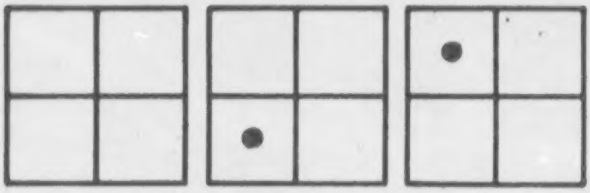
Name ______________________________ **N 1** *

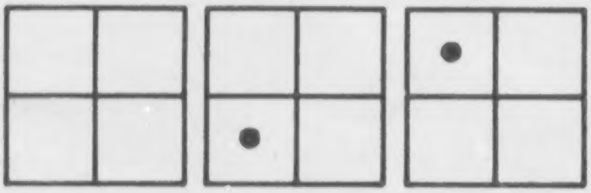
Complete.



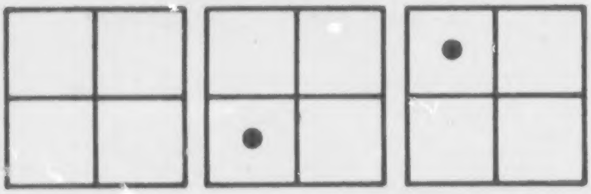
28 − 8 = ____



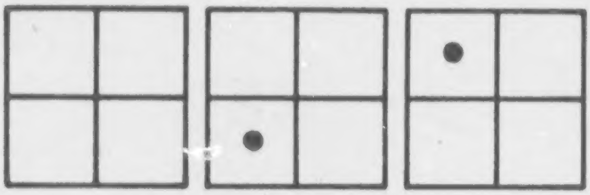
28 − 20 = ____



28 − 4 = ____



28 − 10 = ____

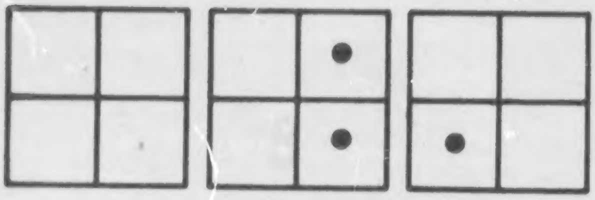


28 − 24 = ____

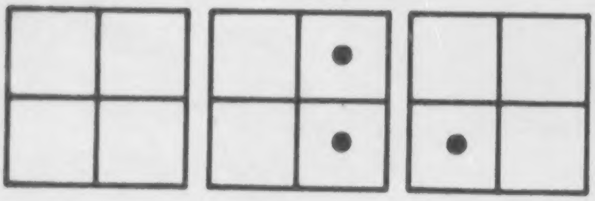
Name ______________________

N 1 **

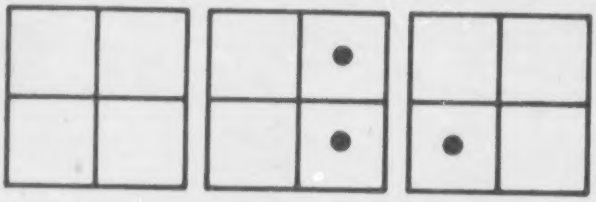
Complete.



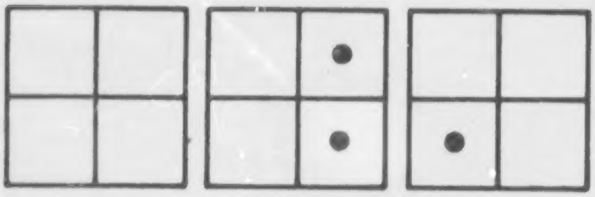
52 − 20 = ____



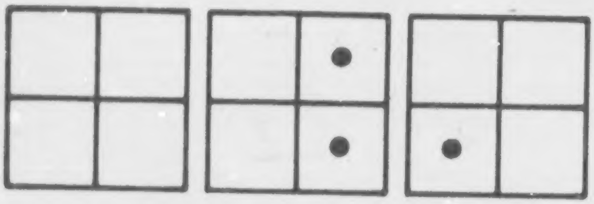
52 − 11 = ____



52 − 30 = ____



52 − 21 = ____

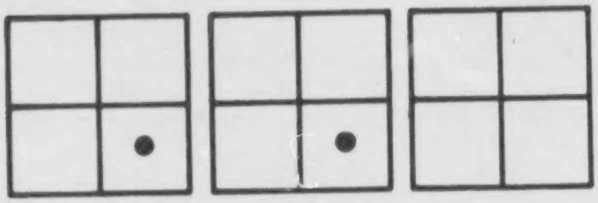


52 − 32 = ____

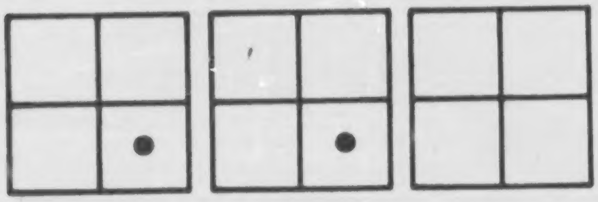
Name ______________________

N 1 ***

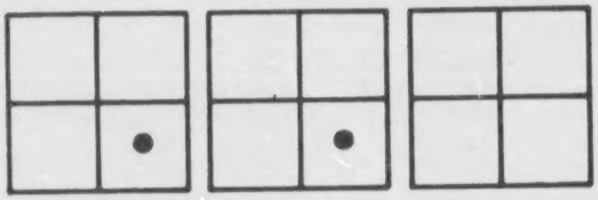
Complete.



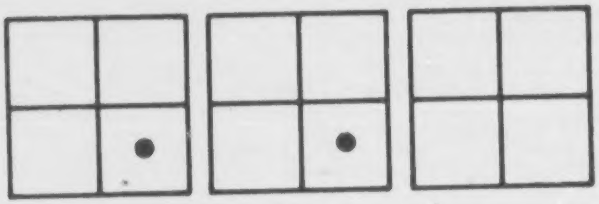
110 - 80 = ____



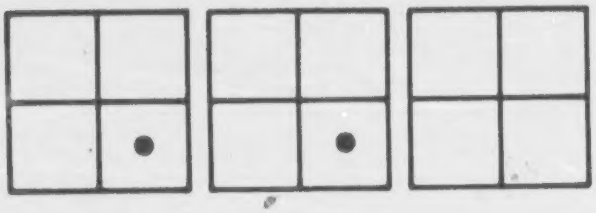
110 - 8 = ____



110 - 20 = ____



110 - 88 = ____

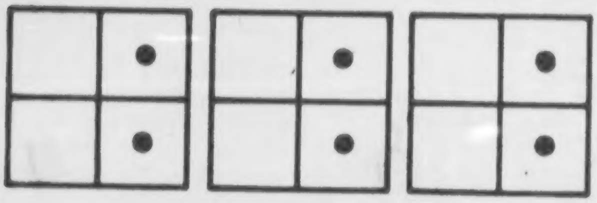


110 - 22 = ____

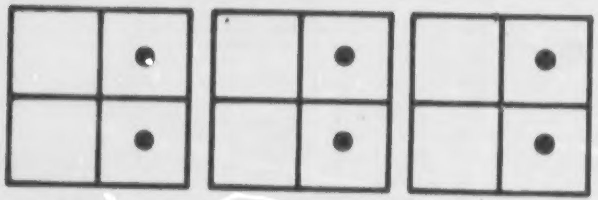
Name ____________________

N 1 ****

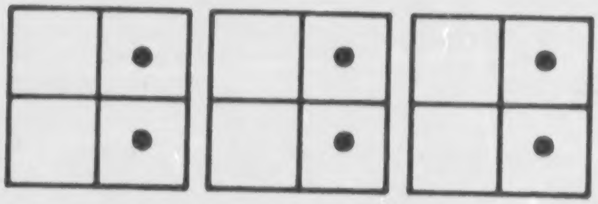
Complete.



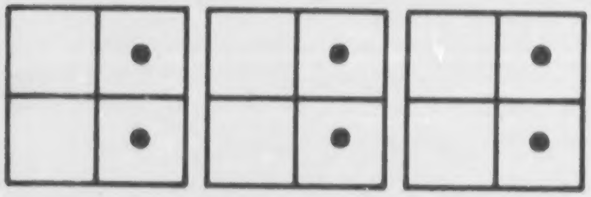
555 − 80 = ____



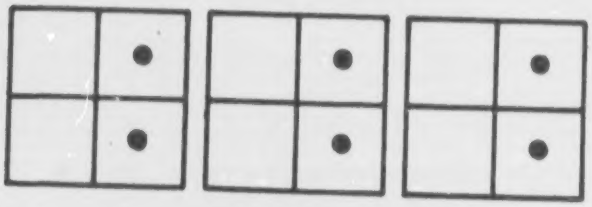
555 − 84 = ____



555 − 88 = ____



555 − 94 = ____

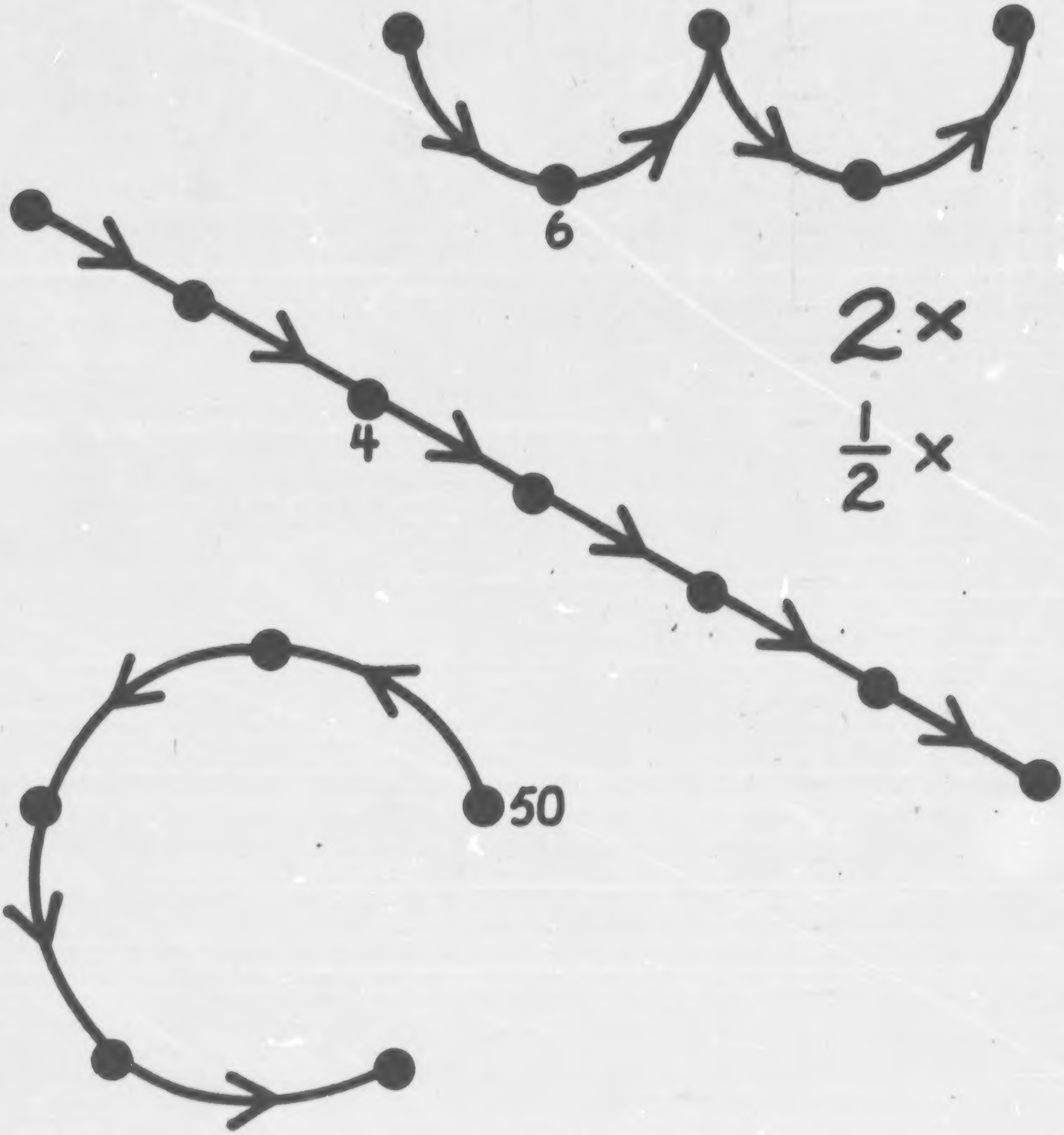


555 − 75 = ____

Name ____________________ **N 2** *

Label the dots.

Draw all the missing blue arrows.

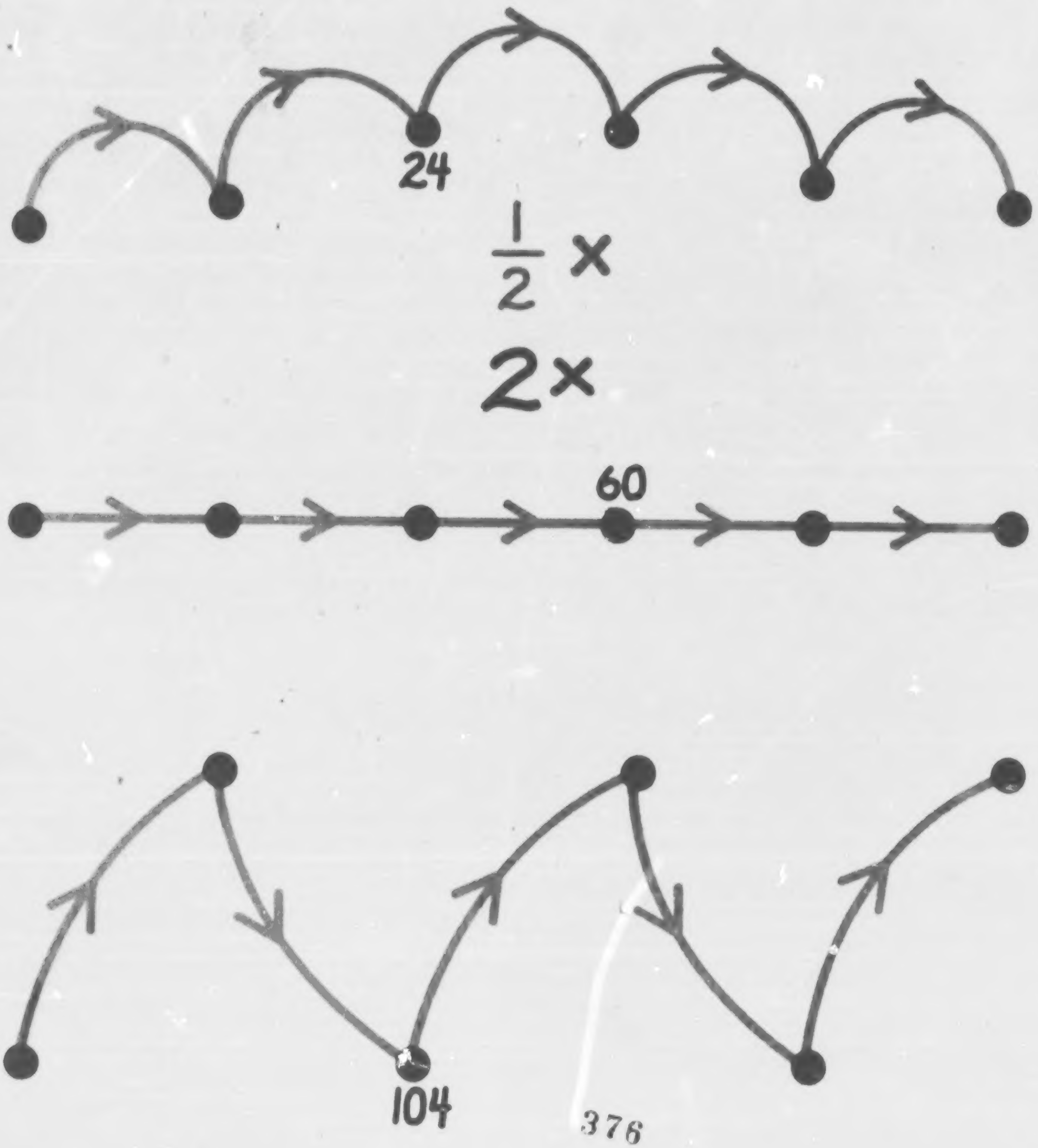

6
4
2x
$\frac{1}{2}$x
50

Name ______________________

N 2 **

Label the dots.

Draw all the missing red arrows.


24
$\frac{1}{2}\times$
$2\times$
60
104

Name ______________________________ **N 6 ***

Build a red-blue road from 0 to 42.

+1

+10

● 42

● 0

Name ______________________ **N 6** **

Build a red-blue road from 5 to 62.

5
●

+1

+10

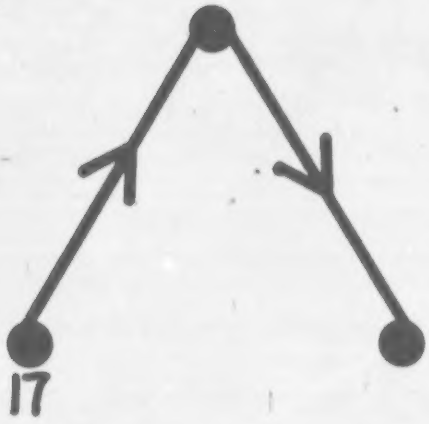
●62

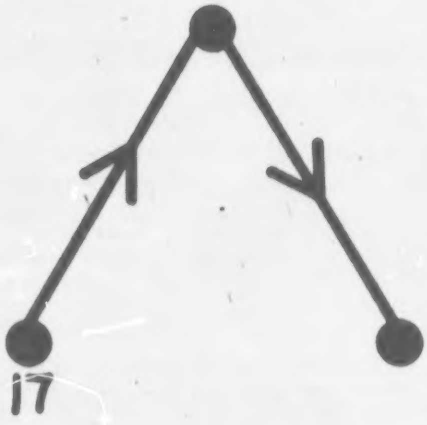
Name ________________________________

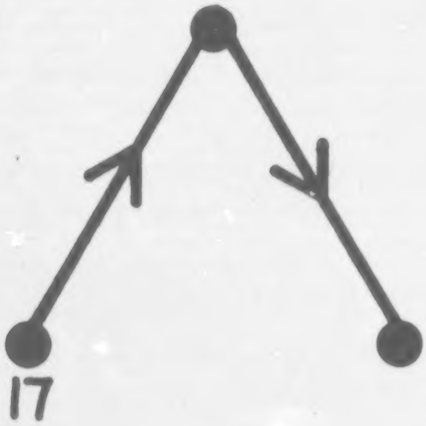
N 8 *

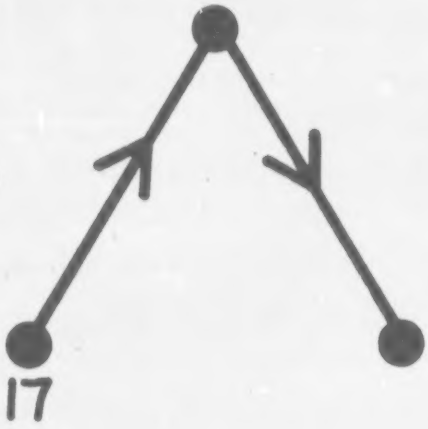
Label the dots.

+10 −1









The largest ending number is ____________.

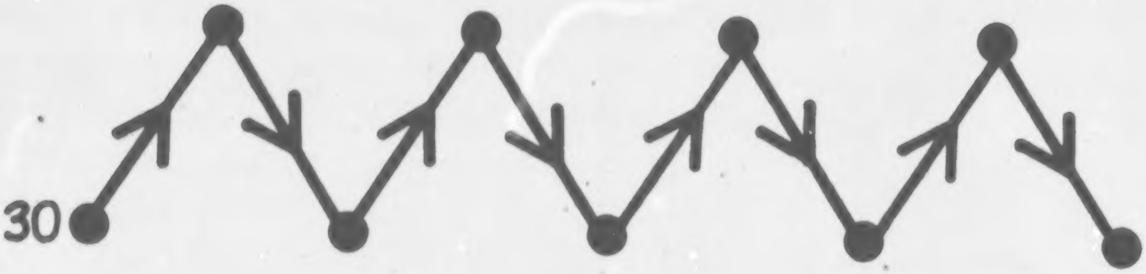
The smallest ending number is ____________.

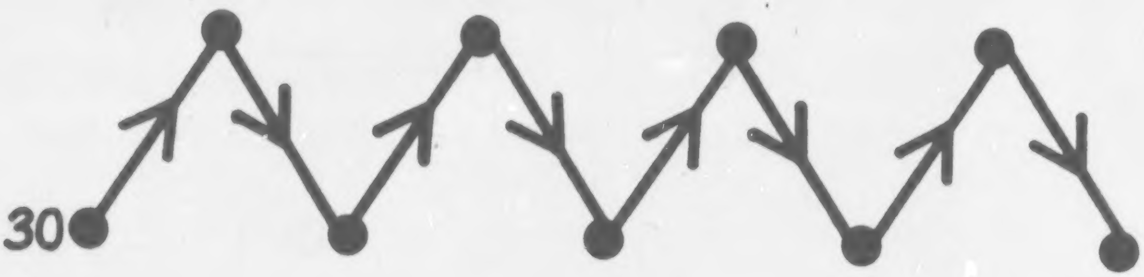
Name ______________________

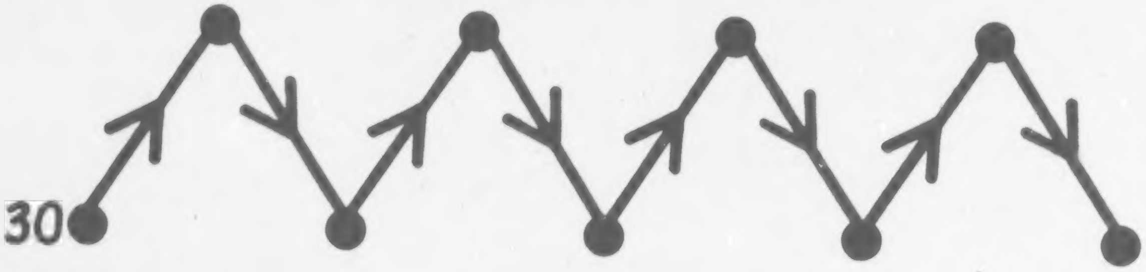
N 8 **

Label the dots.

+10 −1





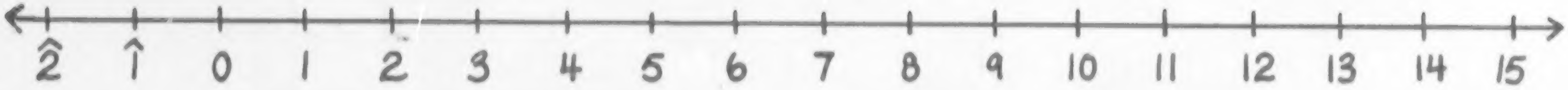


The largest ending number is ____________.

The smallest ending number is ____________.

N 10

Name ______________________________


2̂
1̂
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

Name ______________________________ **N 12** *

Build a red-blue road from 1 to 9.

1 ●

+1

2x

● 9

Name ______________________________

N 12 **

Build a red-blue road from 0 to 15.

+1

2x

●15

0●

Name ______________________

N 13 *

$\frac{1}{2} \times 4 =$ ______

$\frac{1}{2} \times 6 =$ ______

$\frac{1}{2} \times 8 =$ ______

$\frac{1}{2} \times 2 =$ ______

$\frac{1}{2} \times 20 =$ ______

$\frac{1}{2} \times 200 =$ ______

$\frac{1}{2} \times 12 =$ ______

$\frac{1}{2} \times 14 =$ ______

$\frac{1}{2} \times 16 =$ ______

$\frac{1}{2} \times 10 =$ ______

$\frac{1}{2} \times 40 =$ ______

$\frac{1}{2} \times 100 =$ ______

Name ______________________

N 13 **

$\frac{1}{2} \times 24 =$ ______

$\frac{1}{2} \times 28 =$ ______

$\frac{1}{2} \times 26 =$ ______

$\frac{1}{2} \times 42 =$ ______

$\frac{1}{2} \times 48 =$ ______

$\frac{1}{2} \times 82 =$ ______

$\frac{1}{2} \times 64 =$ ______

$\frac{1}{2} \times 46 =$ ______

$\frac{1}{2} \times 60 =$ ______

$\frac{1}{2} \times 66 =$ ______

$\frac{1}{2} \times 68 =$ ______

$\frac{1}{2} \times 86 =$ ______

$\frac{1}{2} \times 240 =$ ______

$\frac{1}{2} \times 608 =$ ______

$\frac{1}{2} \times 206 =$ ______

$\frac{1}{2} \times 480 =$ ______

Name ____________________

N 13 ***

$\frac{1}{2} \times 286 =$ ______

$\frac{1}{2} \times 828 =$ ______

$\frac{1}{2} \times 262 =$ ______

$\frac{1}{2} \times 684 =$ ______

$\frac{1}{2} \times 200 =$ ______

$\frac{1}{2} \times 208 =$ ______

$\frac{1}{2} \times 210 =$ ______

$\frac{1}{2} \times 212 =$ ______

$\frac{1}{2} \times 108 =$ ______

$\frac{1}{2} \times 110 =$ ______

$\frac{1}{2} \times 112 =$ ______

$\frac{1}{2} \times 118 =$ ______

$\frac{1}{2} \times 30 =$ ______

$\frac{1}{2} \times 32 =$ ______

$\frac{1}{2} \times 34 =$ ______

$\frac{1}{2} \times 38 =$ ______

Name ____________________

N 13 ****

$\frac{1}{2} \times 50 =$ ______

$\frac{1}{2} \times 52 =$ ______

$\frac{1}{2} \times 56 =$ ______

$\frac{1}{2} \times 58 =$ ______

$\frac{1}{2} \times 70 =$ ______

$\frac{1}{2} \times 74 =$ ______

$\frac{1}{2} \times 90 =$ ______

$\frac{1}{2} \times 96 =$ ______

$\frac{1}{2} \times 120 =$ ______

$\frac{1}{2} \times 140 =$ ______

$\frac{1}{2} \times 160 =$ ______

$\frac{1}{2} \times 180 =$ ______

$\frac{1}{2} \times 142 =$ ______

$\frac{1}{2} \times 186 =$ ______

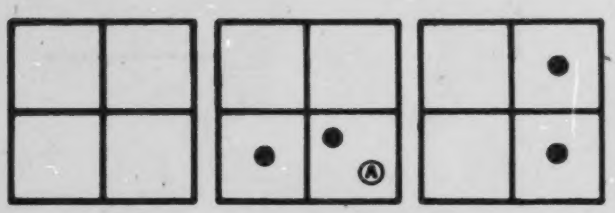
$\frac{1}{2} \times 320 =$ ______

$\frac{1}{2} \times 512 =$ ______

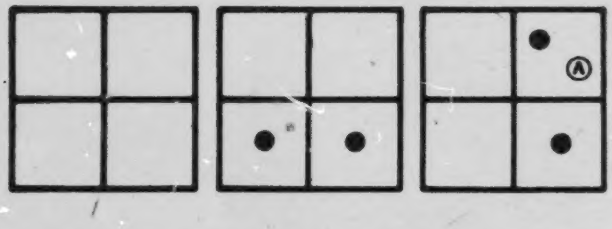
Name ______________________

N 17 *

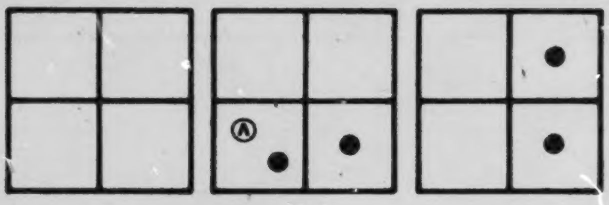
Complete.



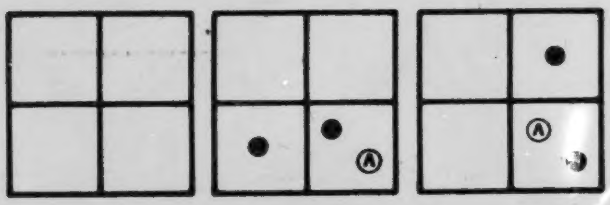
35 − 10 = ____



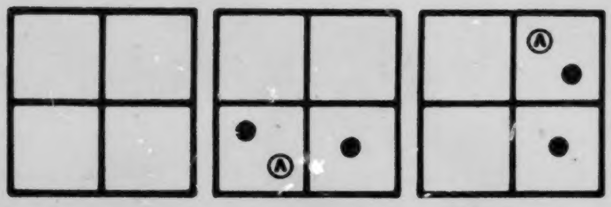
35 − 4 = ____



35 − 20 = ____



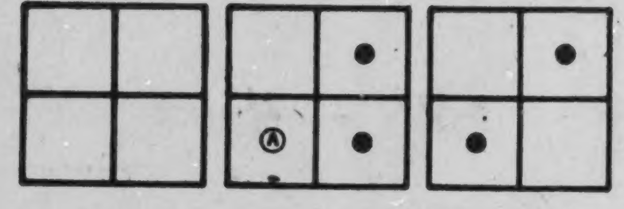
35 − 11 = ____



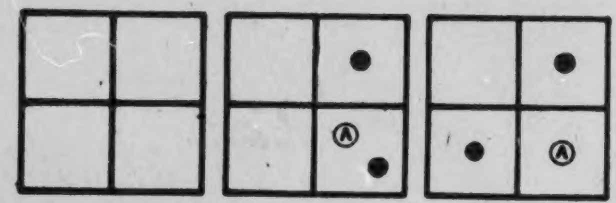
35 − 24 = ____

Name ______________________ **N 17 ****

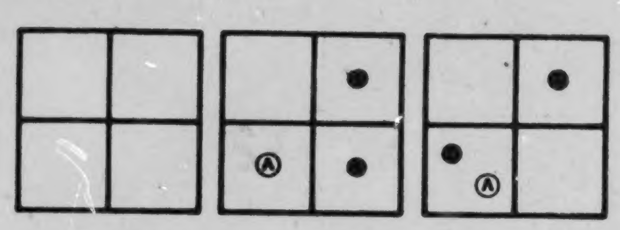
Complete.



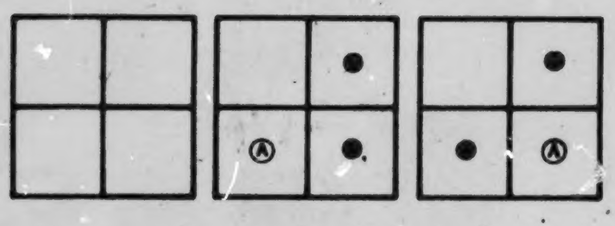
56 − 20 = ____



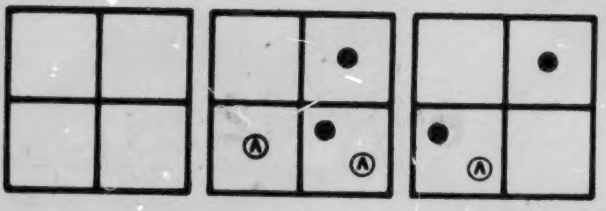
56 − 11 = ____



56 − 22 = ____



56 − 21 = ____

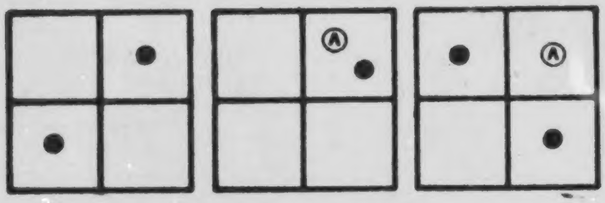


56 − 32 = ____

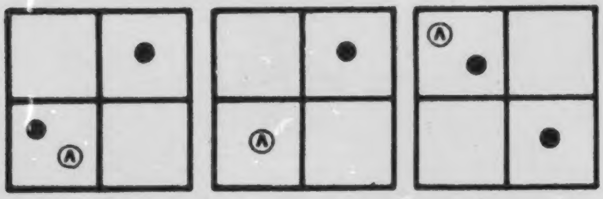
Name ____________________

N 17 ***

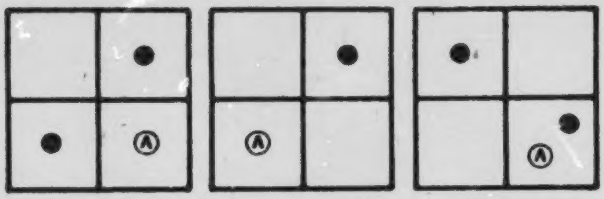
Complete.



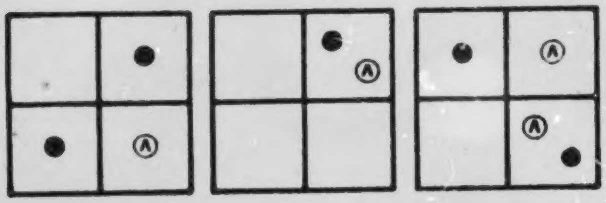
649 - 44 = ____



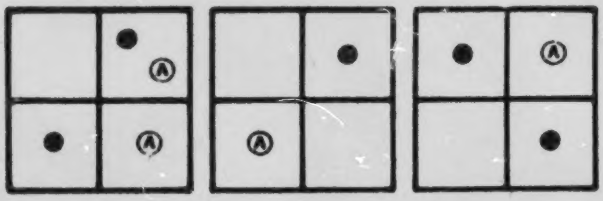
649 - 228 = ____



649 - 121 = ____



649 - 145 = ____

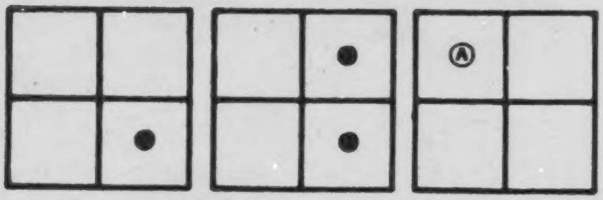


649 - 524 = ____

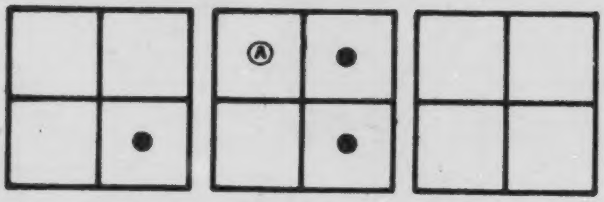
Name ______________________

N 17 **

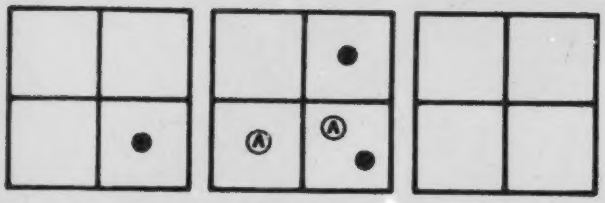
Complete .



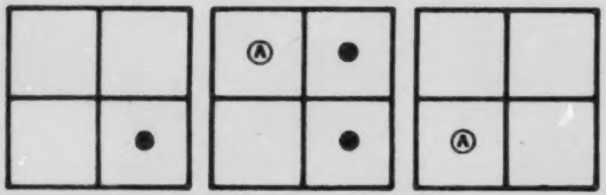
150 − 8 = ____



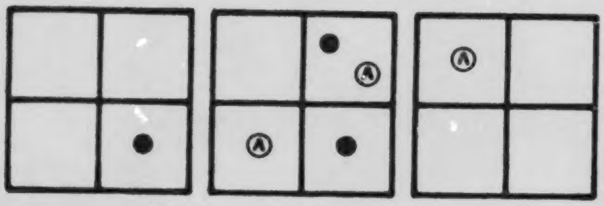
150 − 80 = ____



150 − 30 = ____



150 − 82 = ____

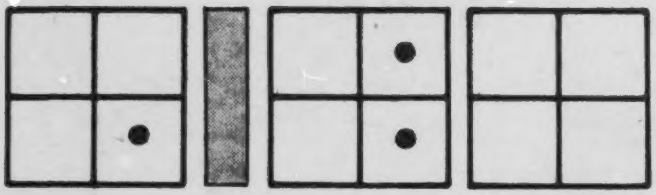


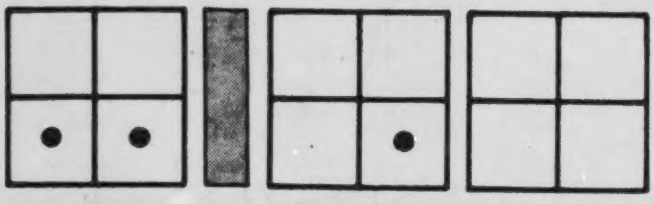
150 − 68 = ____

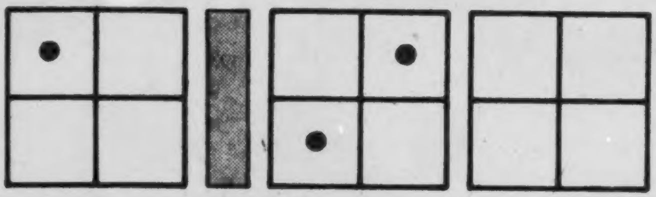
Name ______________________

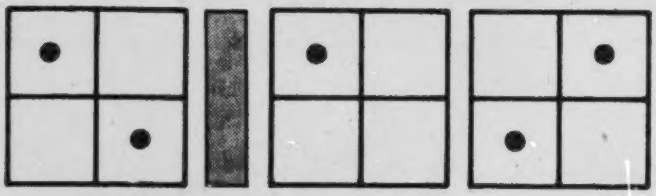
N 21 *

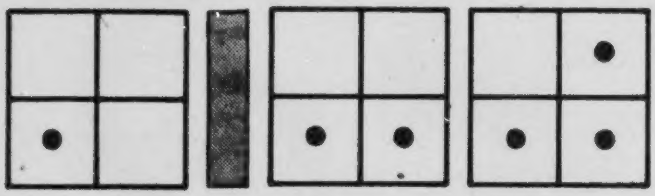
What number is on the Minicomputer ?

 = ______

 = ______

 = ______

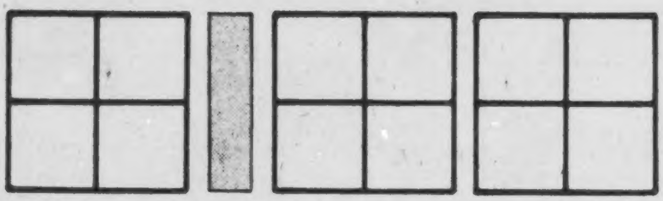
 = ______

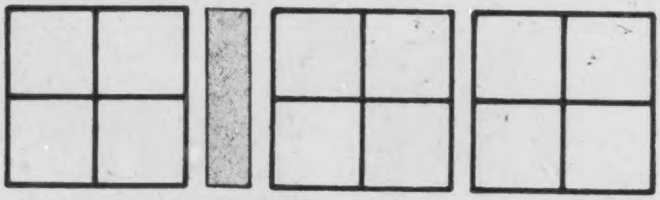
 = ______

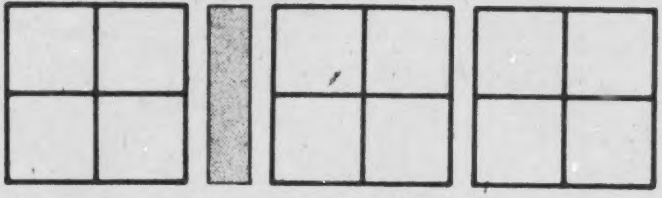
Name ______________________

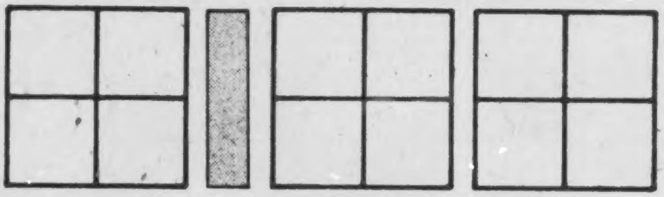
N 21 **

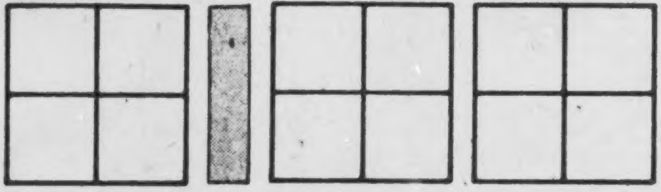
Put these numbers on the Minicomputer.

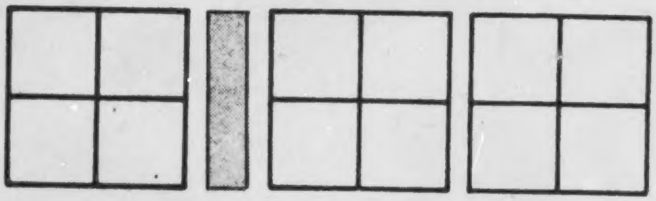
4.51 =


9.78 =


0.45 =


3.09 =


7.0 =


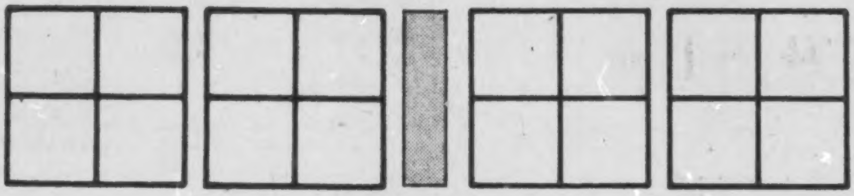
6.3 =


Name ________________________

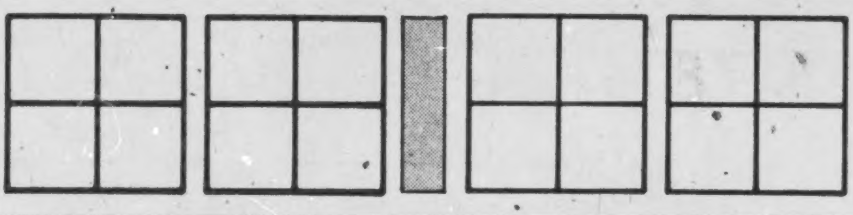
N 21 ***

Put these numbers on the Minicomputer.

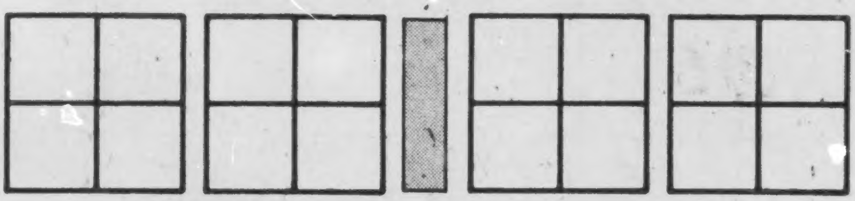
21.48 =



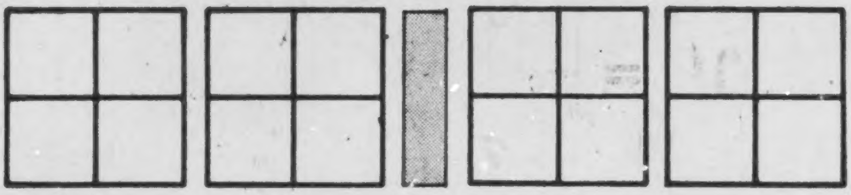
7.62 =



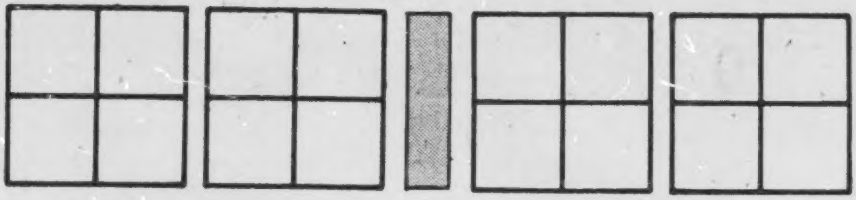
19.3 =



0.4 =



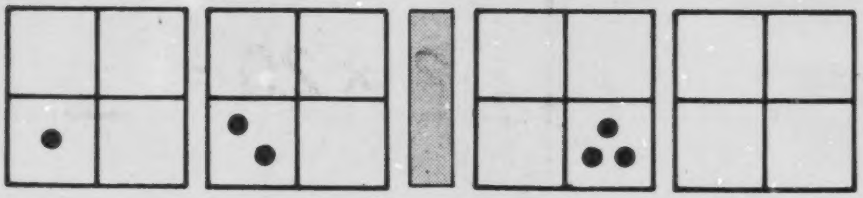
50 =



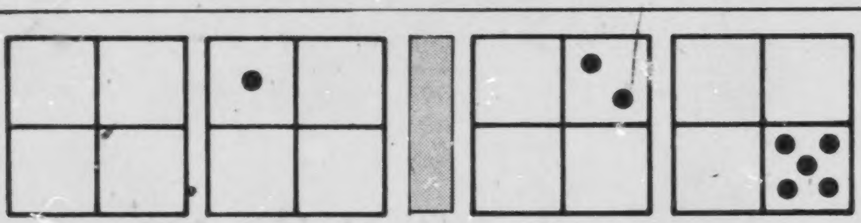
Name ______________________

N 21 **

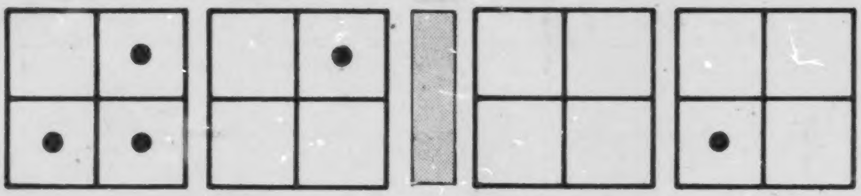
What number is on the Minicomputer ?



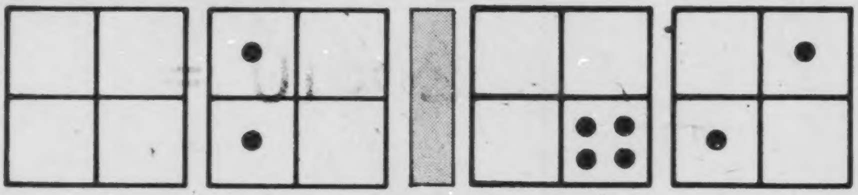
= ______



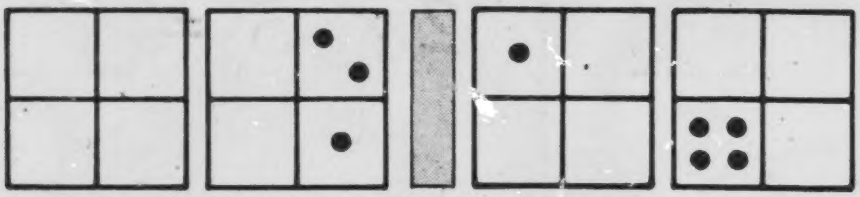
= ______



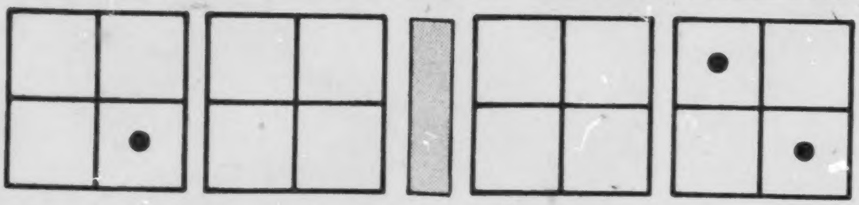
= ______



= ______



= ______



= ______

Name ____________________

N 23 *

Complete these number stories.

2 × 10 = ____

2 × 2 = ____

2 × 12 = ____

2 × 20 = ____

2 × 1 = ____

2 × 21 = ____

2 × 20 = ____

2 × 4 = ____

2 × 24 = ____

2 × 10 = ____

2 × 3 = ____

2 × 13 = ____

Name ______________________

N 23 **

Complete these number stories.

2 × 20 = ____

2 × 5 = ____

2 × 25 = ____

2 × 10 = ____

2 × 7 = ____

2 × 17 = ____

2 × 30 = ____

2 × 4 = ____

2 × 34 = ____

2 × 100 = ____

2 × 13 = ____

2 × 113 = ____

Name ____________________

N 23 ***

Complete these number stories.

2 × 4 = ____

2 × 14 = ____

2 × 24 = ____

2 × 34 = ____

2 × 44 = ____

2 × 5 = ____

2 × 15 = ____

2 × 25 = ____

2 × 35 = ____

2 × 45 = ____

2 × 6 = ____

2 × 16 = ____

2 × 26 = ____

2 × 36 = ____

2 × 46 = ____

2 × 23 = ____

2 × 123 = ____

2 × 223 = ____

2 × 423 = ____

2 × 523 = ____

Name ______________________

N 23 ****

Complete these number stories.

2× 18 = ____

2× 48 = ____

2× 148 = ____

2× 248 = ____

2× 548 = ____

2× 1,048 = ____

2× 35 = ____

2× 435 = ____

2× 1,035 = ____

2× 1,235 = ____

2× 4,035 = ____

2× 1,435 = ____

2× 70 = ____

2× 75 = ____

2× 175 = ____

2× 475 = ____

2× 675 = ____

2× 1,075 = ____

3× 54 = ____

3× 254 = ____

3× 354 = ____

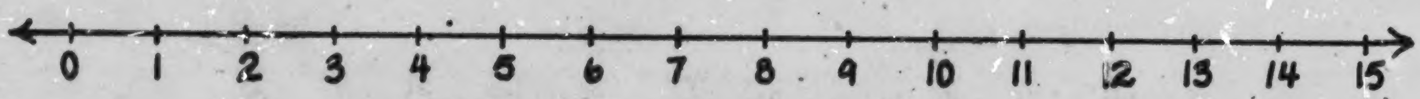
3× 454 = ____

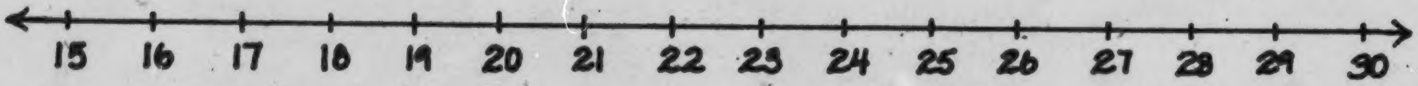
3× 1,054 = ____

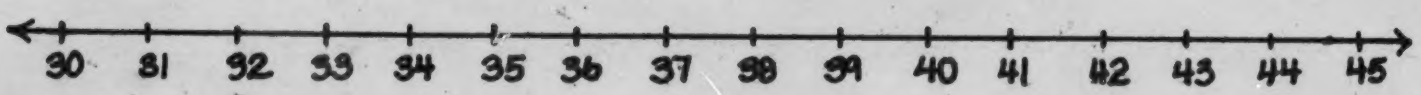
3× 1,354 = ____

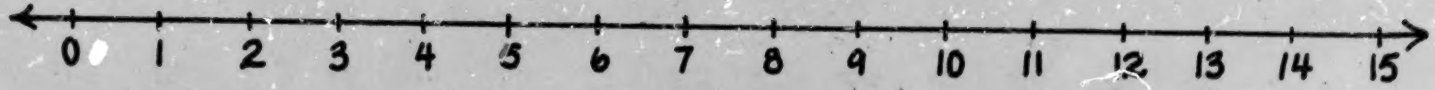
N 30

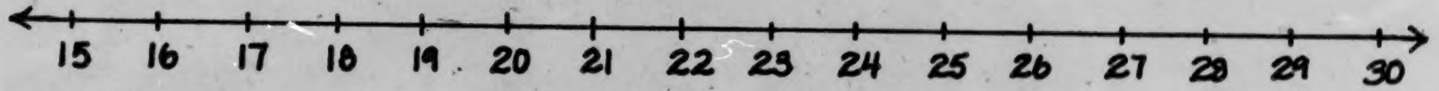
Name ______________________

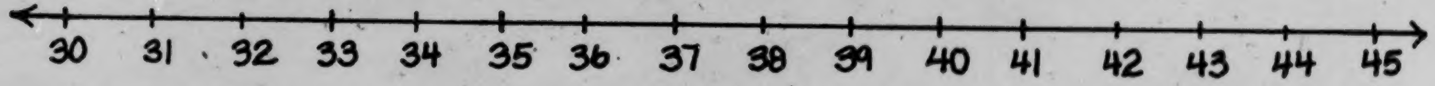










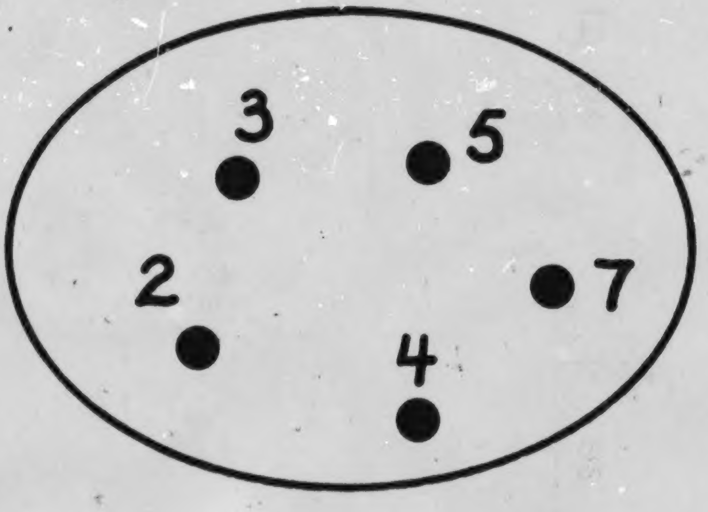


Name ______________________

N 31 *

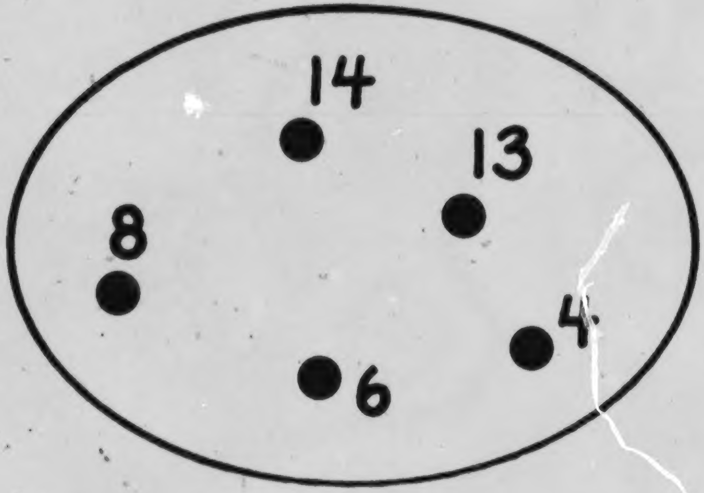
Janet buys two different candies and spends exactly 10¢.

Draw a red string around the prices of these two candies.



Mike buys two different balls and spends exactly 20¢.

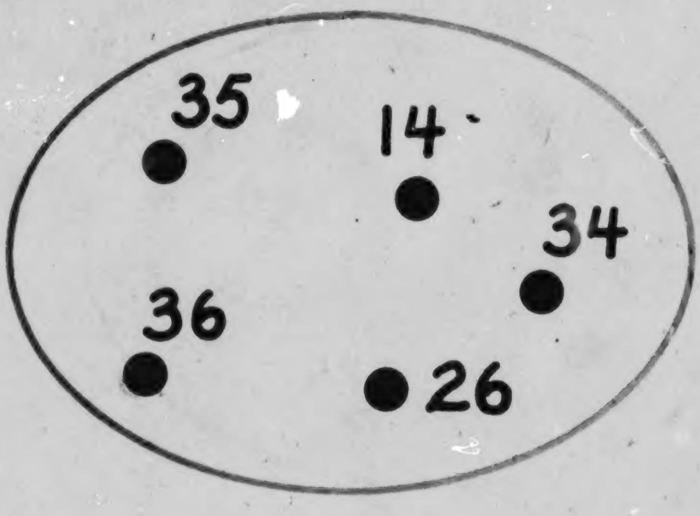
Draw a red string around the prices of these two balls.



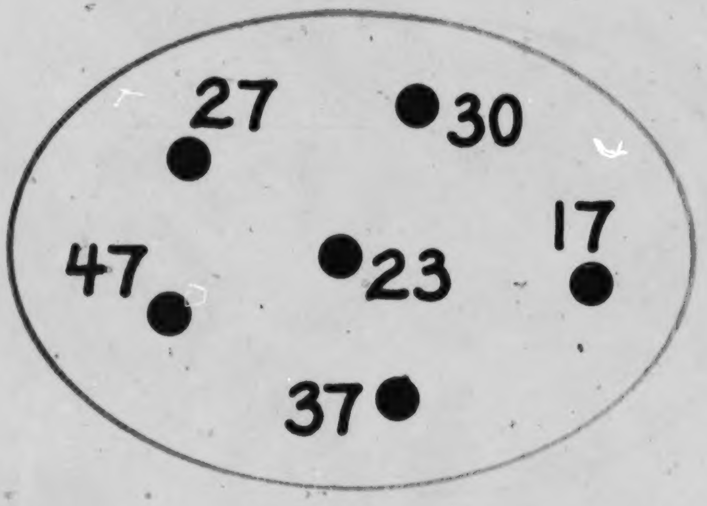
Name ______________________

N 31 **

Kenny buys two different cards and spends exactly 50¢.
Draw a red string around the prices of these two cards.


35
14
34
36
26

Lisa buys two different toys and spends exactly 60¢.
Draw a red string around the prices of these two toys.

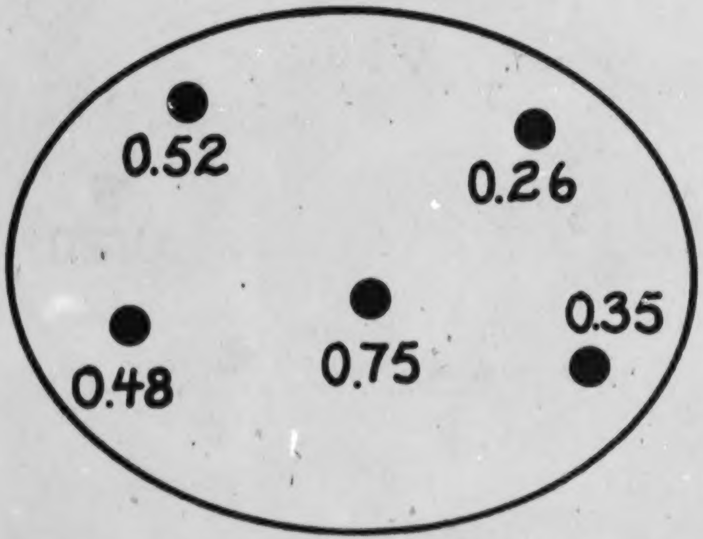

27
30
17
47
23
37

Name ______________________

N 31 ***

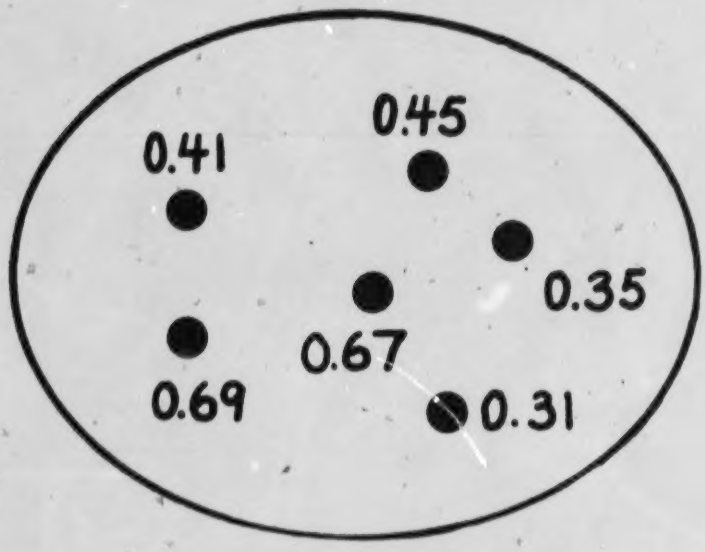
Robin buys two different books and spends exactly $1.

Draw a red string around the prices of these two books.


0.52
0.26
0.35
0.48
0.75

Roberto buys two different rings and spends exactly $1.

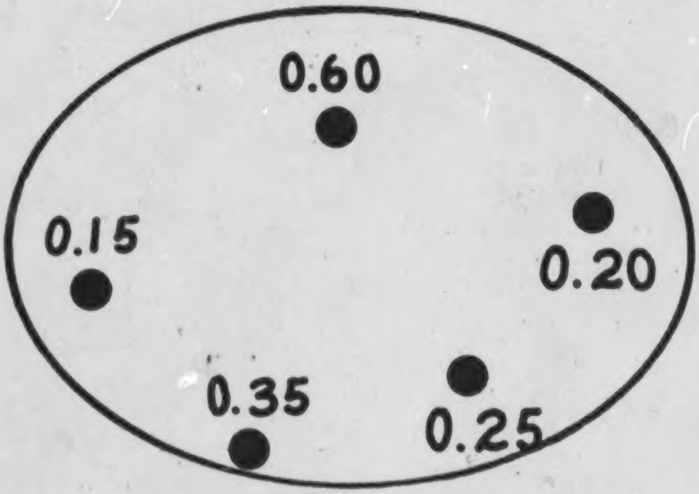
Draw a red string around the prices of these two rings.


0.45
0.41
0.35
0.67
0.69
0.31

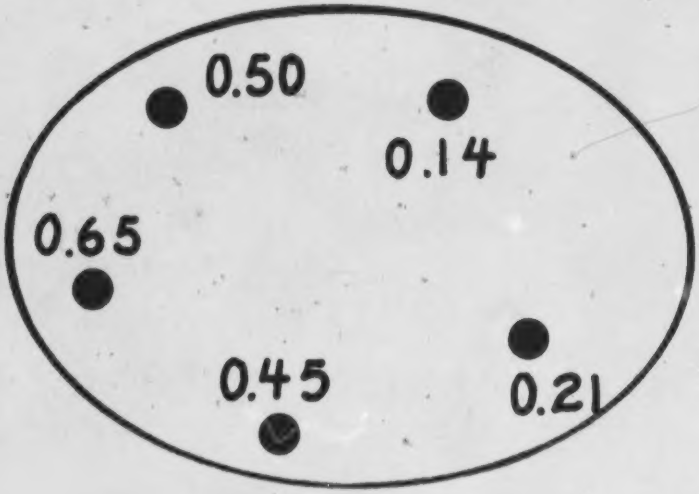
Name ______________________

N 31 ****

Daniel buys three different pieces of fruit and spends exactly $1.
Draw a red string around the prices of these three pieces of fruit.


0.60
0.15
0.20
0.35
0.25

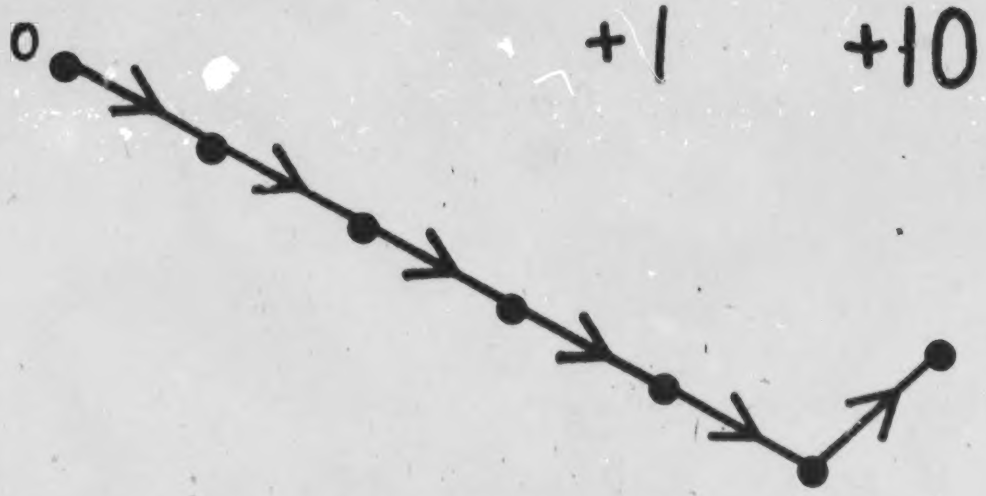
Sandra buys three different flowers and spends exactly $1.
Draw a red string around the prices of these three flowers.


0.50
0.14
0.65
0.45
0.21

Name ______________________

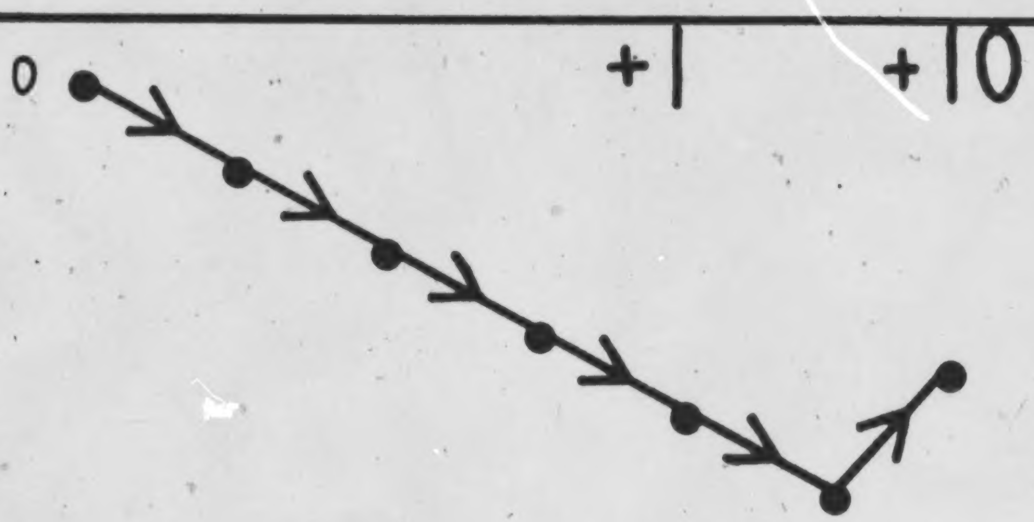
N 32 *

Label the dots in both arrow roads.



How many blue arrows are in this road? ______

How many red arrows are in this road? ______



How many blue arrows are in this road? ______

How many red arrows are in this road? ______

Name ______________________

N 32 **

Build a red-blue road from 0 to 62.

+1

+10

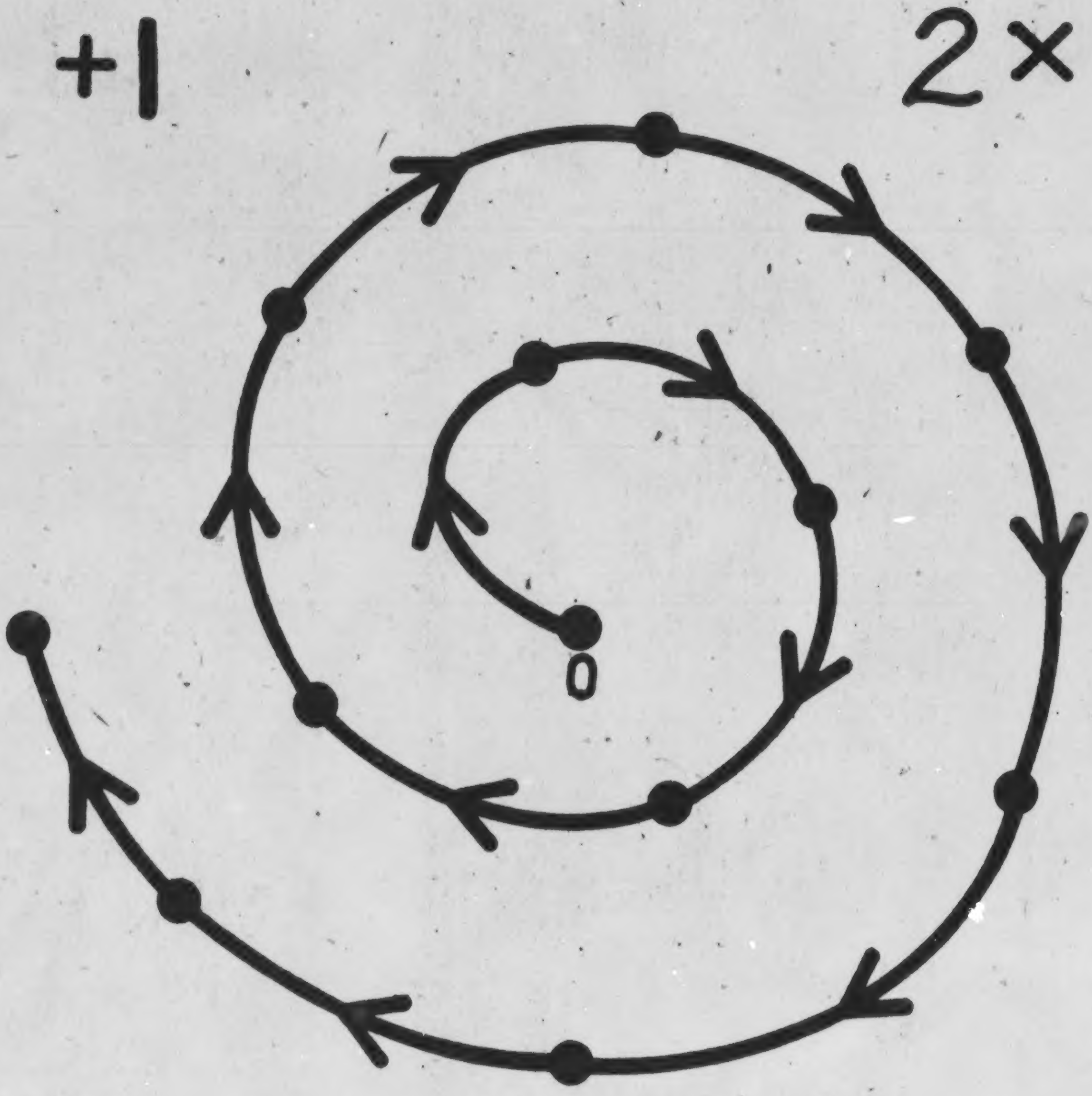
● 62

●
0

Name ________________________

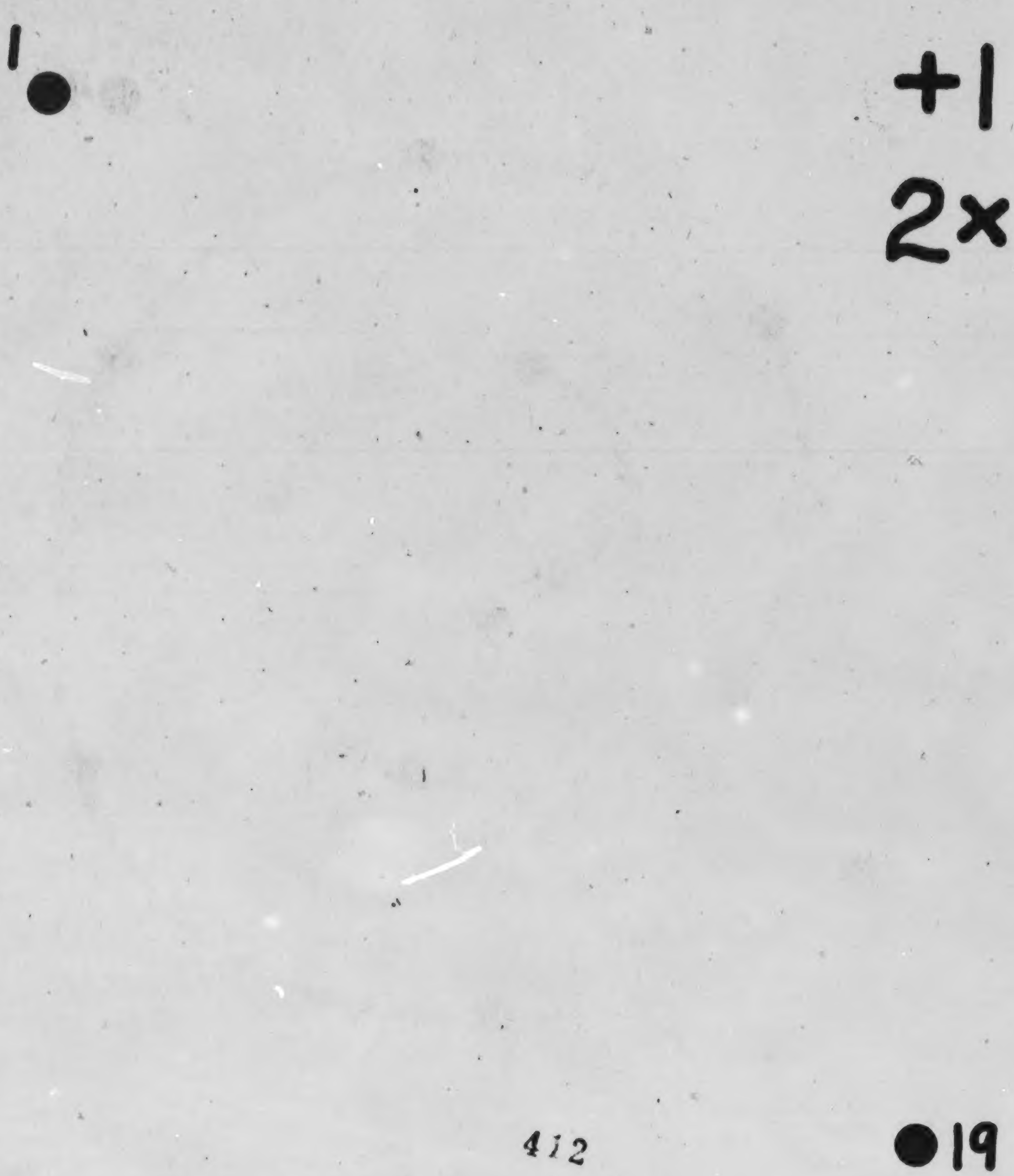
N 34 *

Label the dots.



Name ______________________ **N 34** **

Build a red-blue road from 1 to 19.



Name ____________________

N 34 ***

Build a red-blue road from 0 to 40.

+1

2x

●40

0●

Name ____________________ **N 34 ******

Build a red-blue road from 0 to 100.

+2

2×

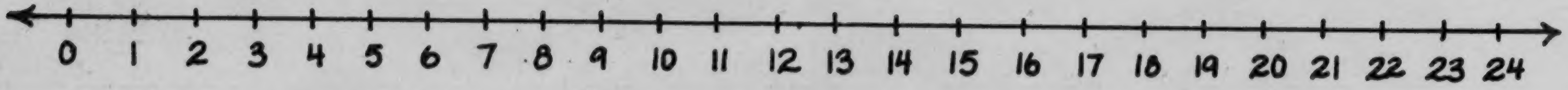
● 0

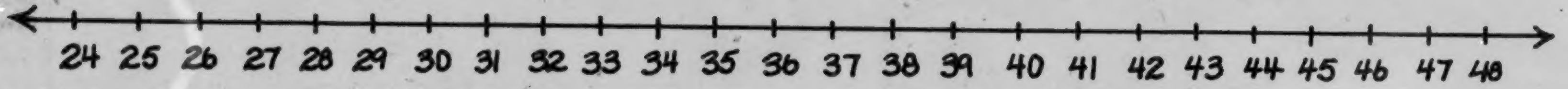
100

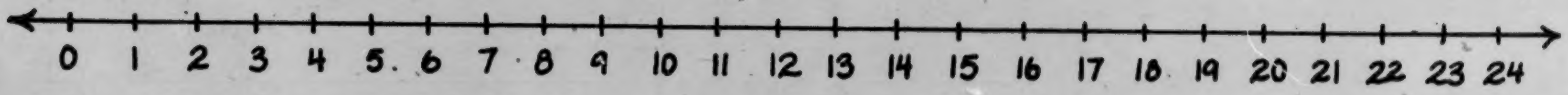
●

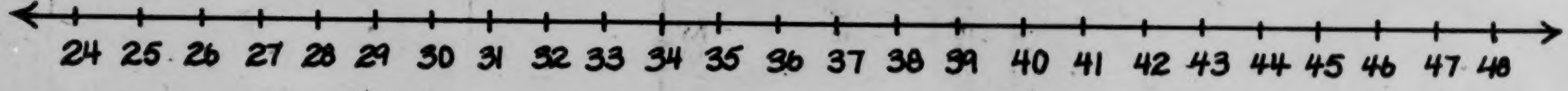
N 36

Name ____________________







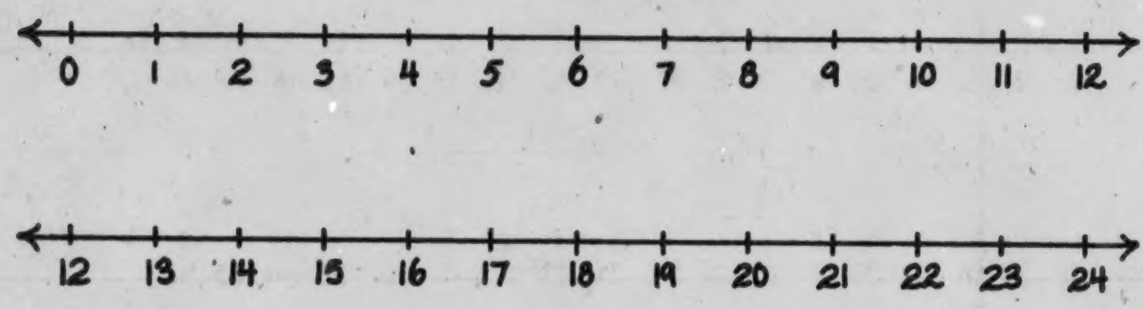


Name ______________________________

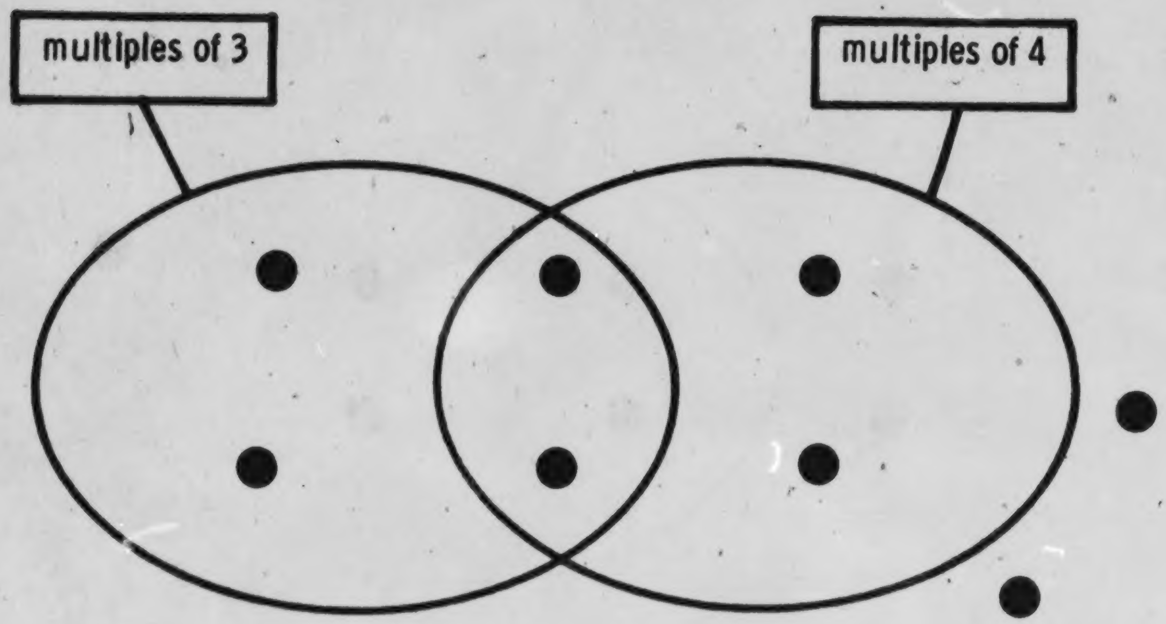
N 36 *

Start at 0. Draw an arrow road to show +3 jumps and another arrow road to show +4 jumps.

+3 +4


0
1
2
3
4
5
6
7
8
9
10
11
12
12
13
14
15
16
17
18
19
20
21
22
23
24

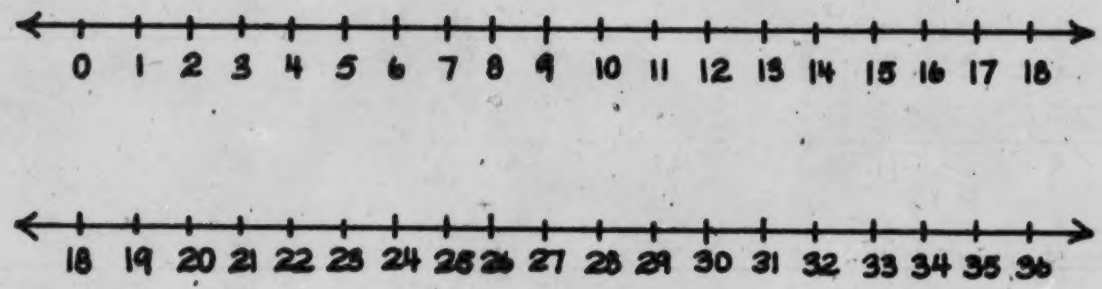
Label the dots in this string picture. Many answers are possible.


multiples of 3
multiples of 4

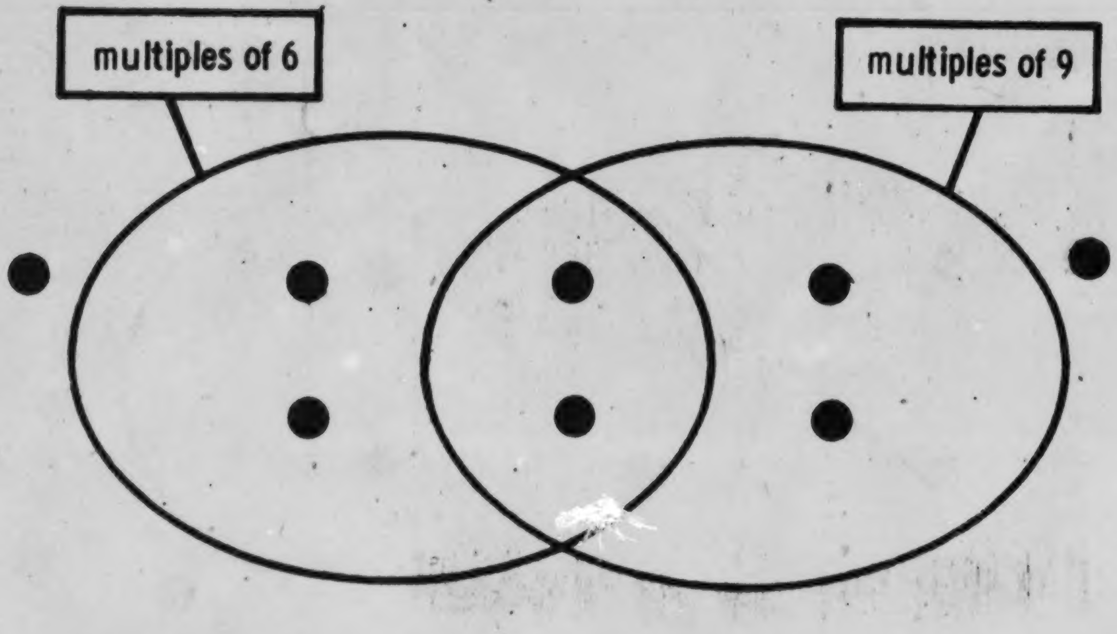
Name ____________________ **N 36** **

Start at 0. Draw an arrow road to show +6 jumps and another arrow road to show +9 jumps.

+6 +9


0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36

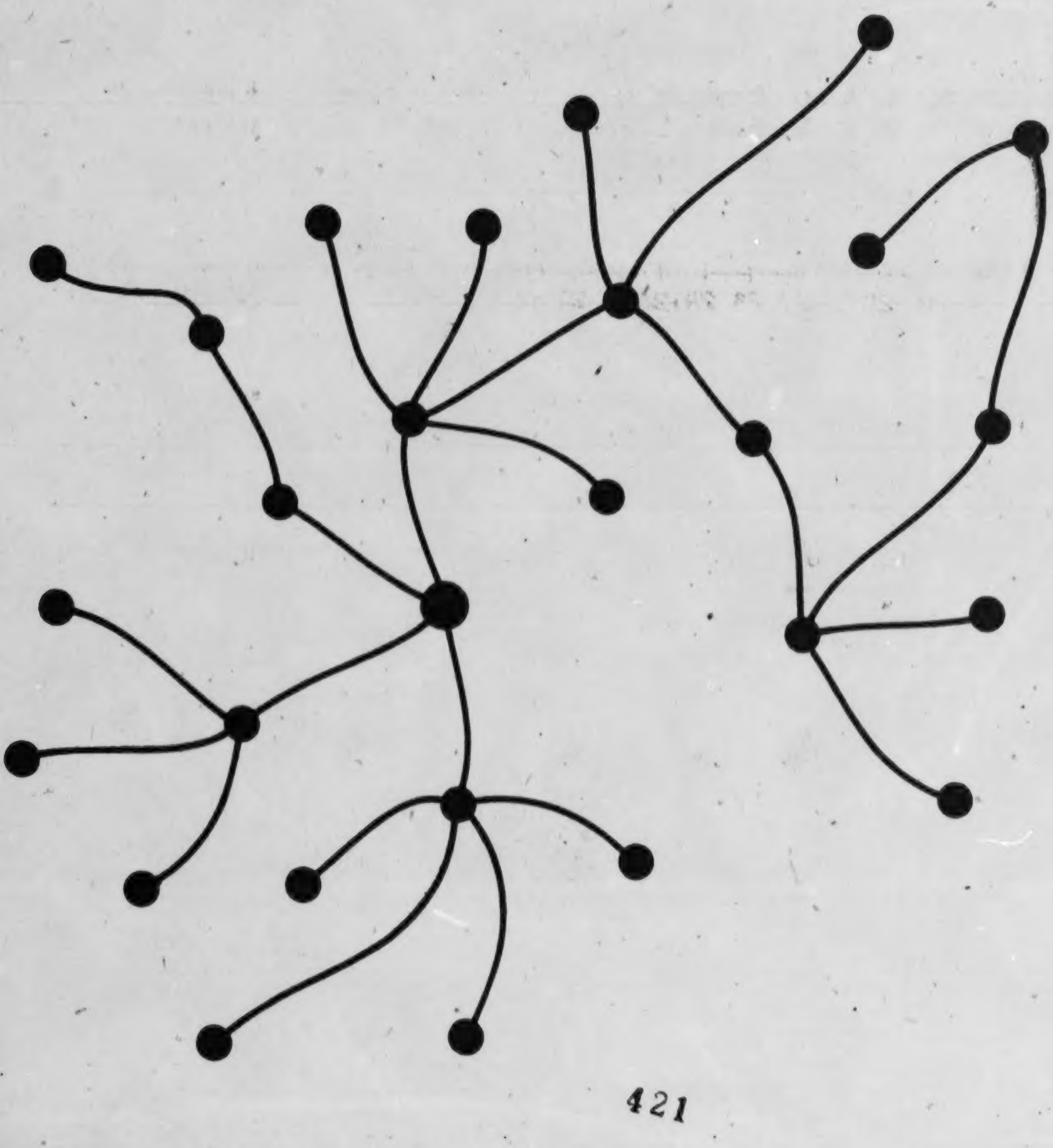
Label the dots in this string picture. Many answers are possible.


multiples of 6
multiples of 9

Name ______________________________ **L 1**

How many edges in this tree? __________ How many dots? __________

A root of the tree is circled red. Draw red arrowheads on the edge to show going away from the root.



Name ________________________________

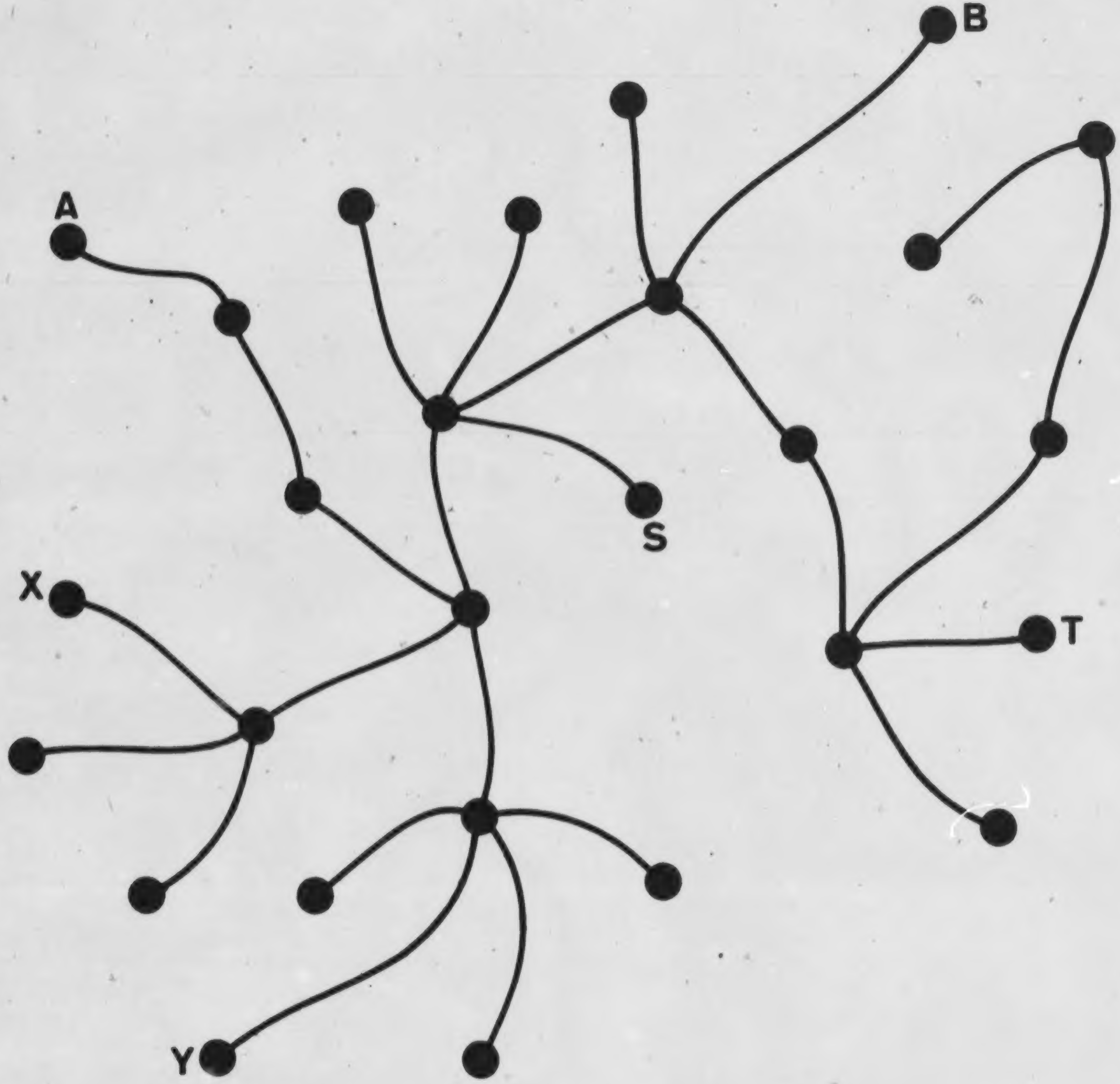
L 5 *

Use any colors you wish to draw these paths.

Draw a path between A and B. What is the distance between A and B? ________

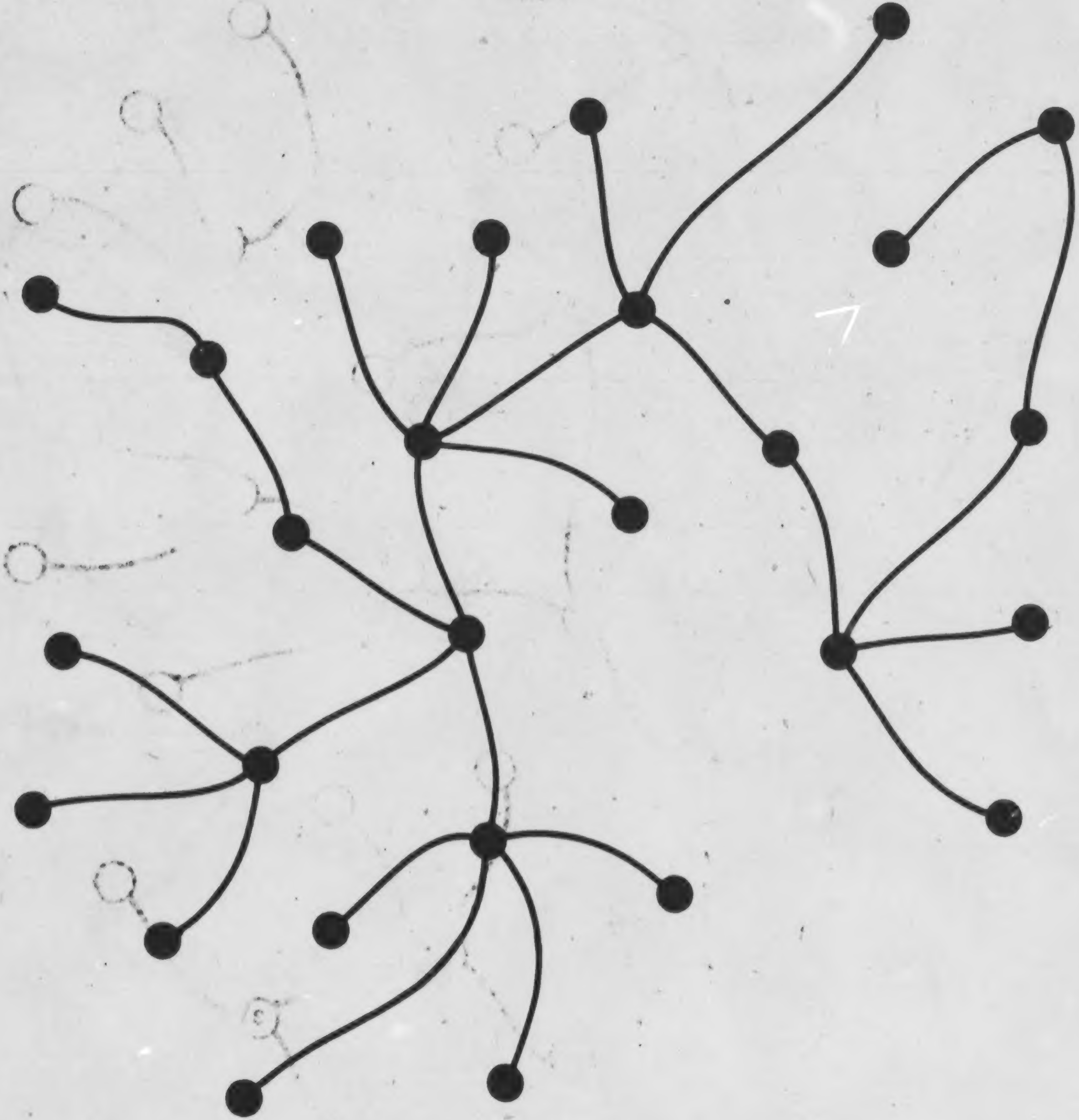
Draw a path between X and Y. What is the distance between X and Y? ________

Draw a path between S and T. What is the distance between S and T? ________



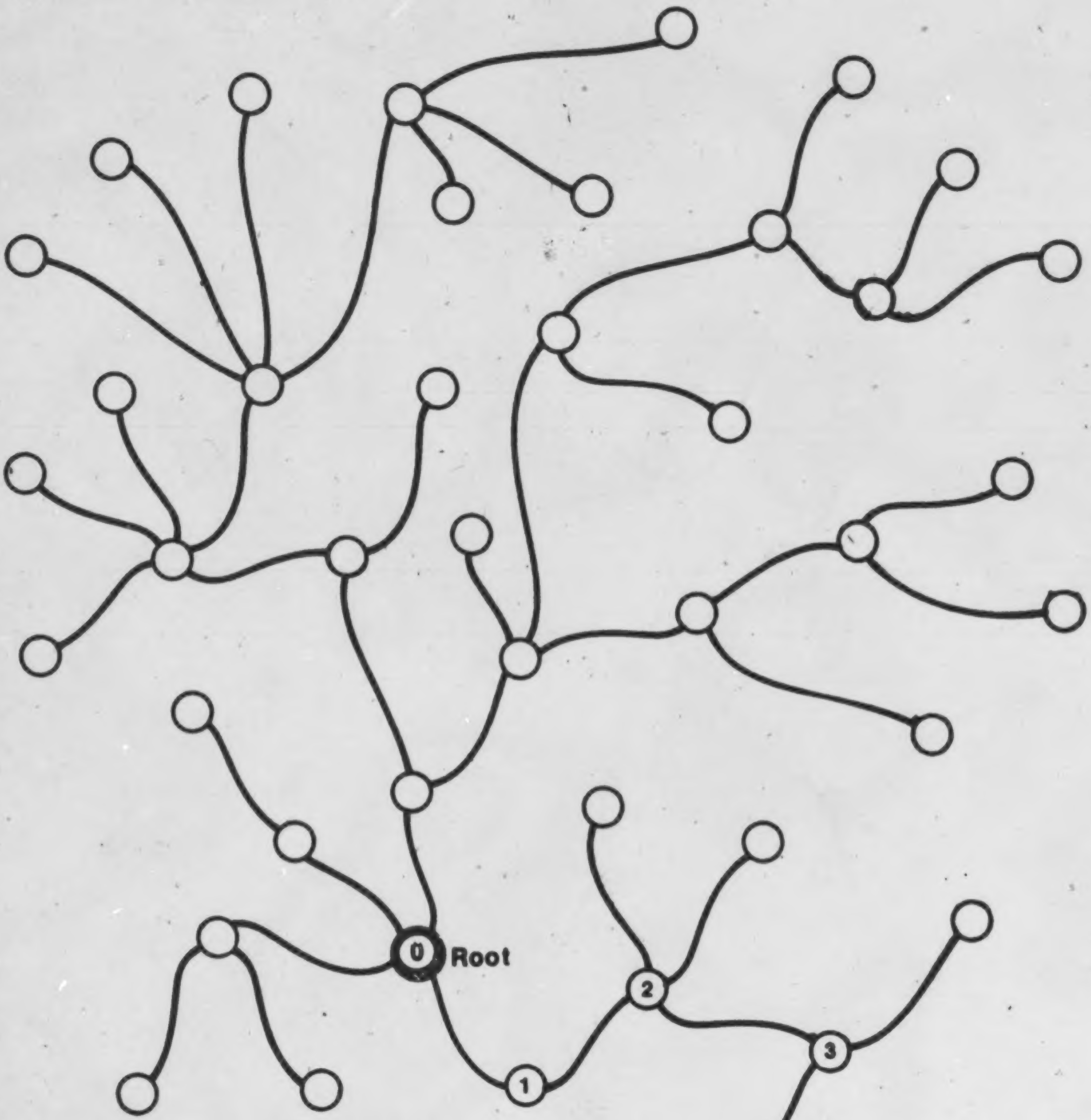
Name ______________________ **L 5 ****

Find the longest possible distance between two dots in this tree. Draw a path to show the longest possible distance. What is this distance? __________



Name ______________________________ **L 10**

Find the distance from the root (circled red) to each of the other dots in this tree. Write your answers in the circles. Some are done for you.



Name ____________________ **L 14**

2	3	4	5	6	7	8	9	10	11	12

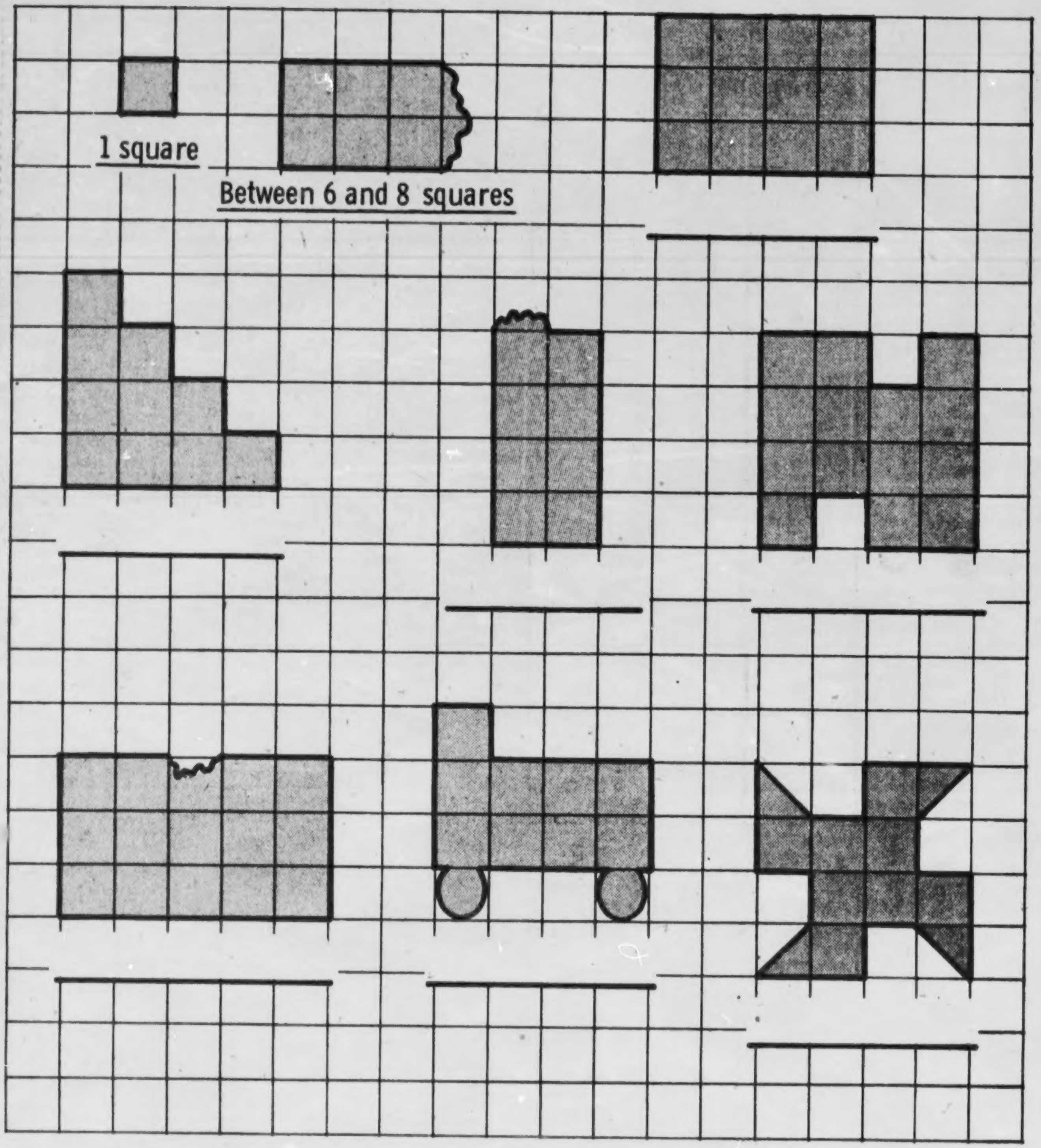
WHITE						
6						
5						
4						
3						
2						
1						
	1	2	3	4	5	6

RED

Name

G 1 *

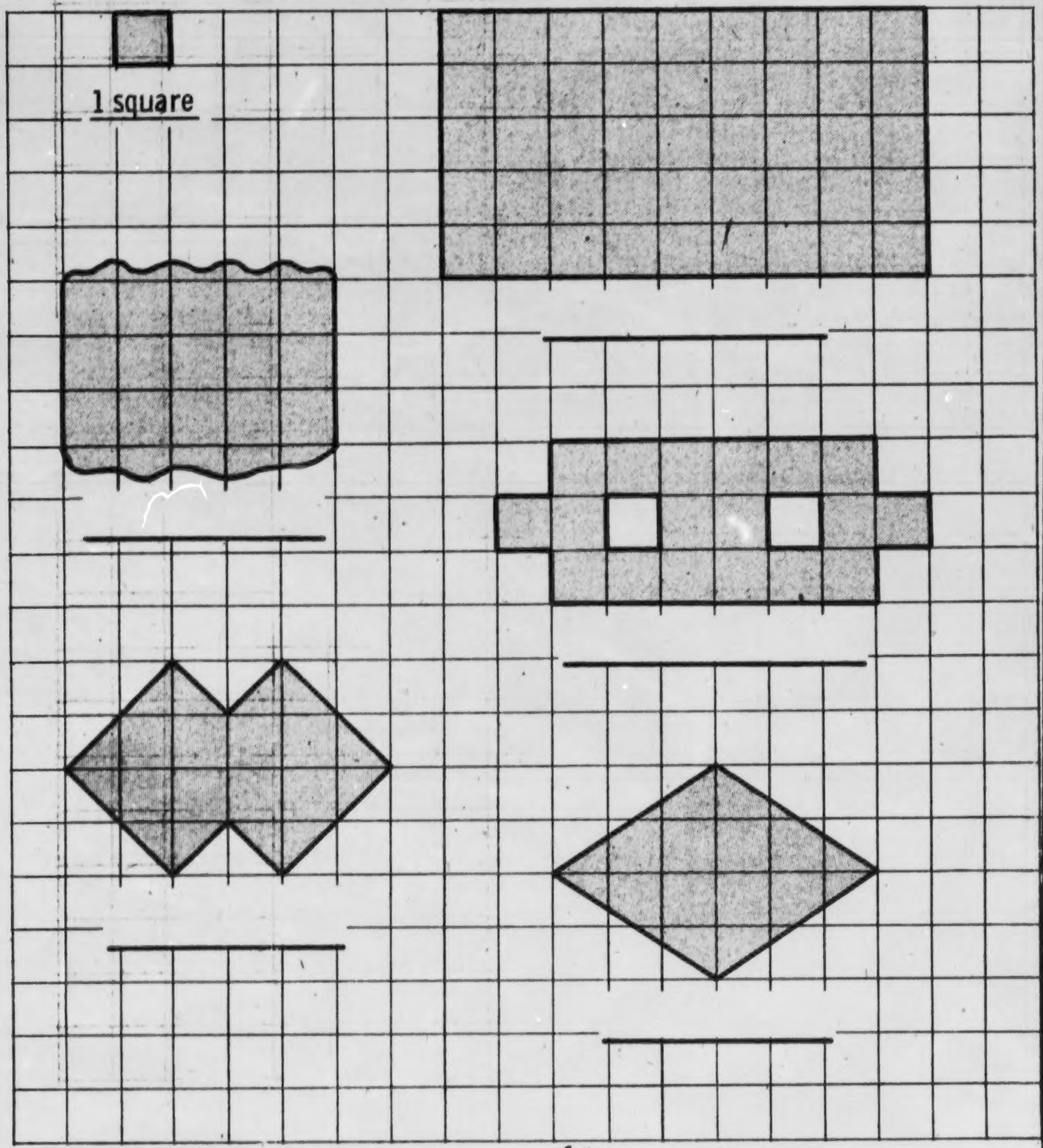
Find or estimate the area of each shape. One is done for you.


1 square
Between 6 and 8 squares

Name

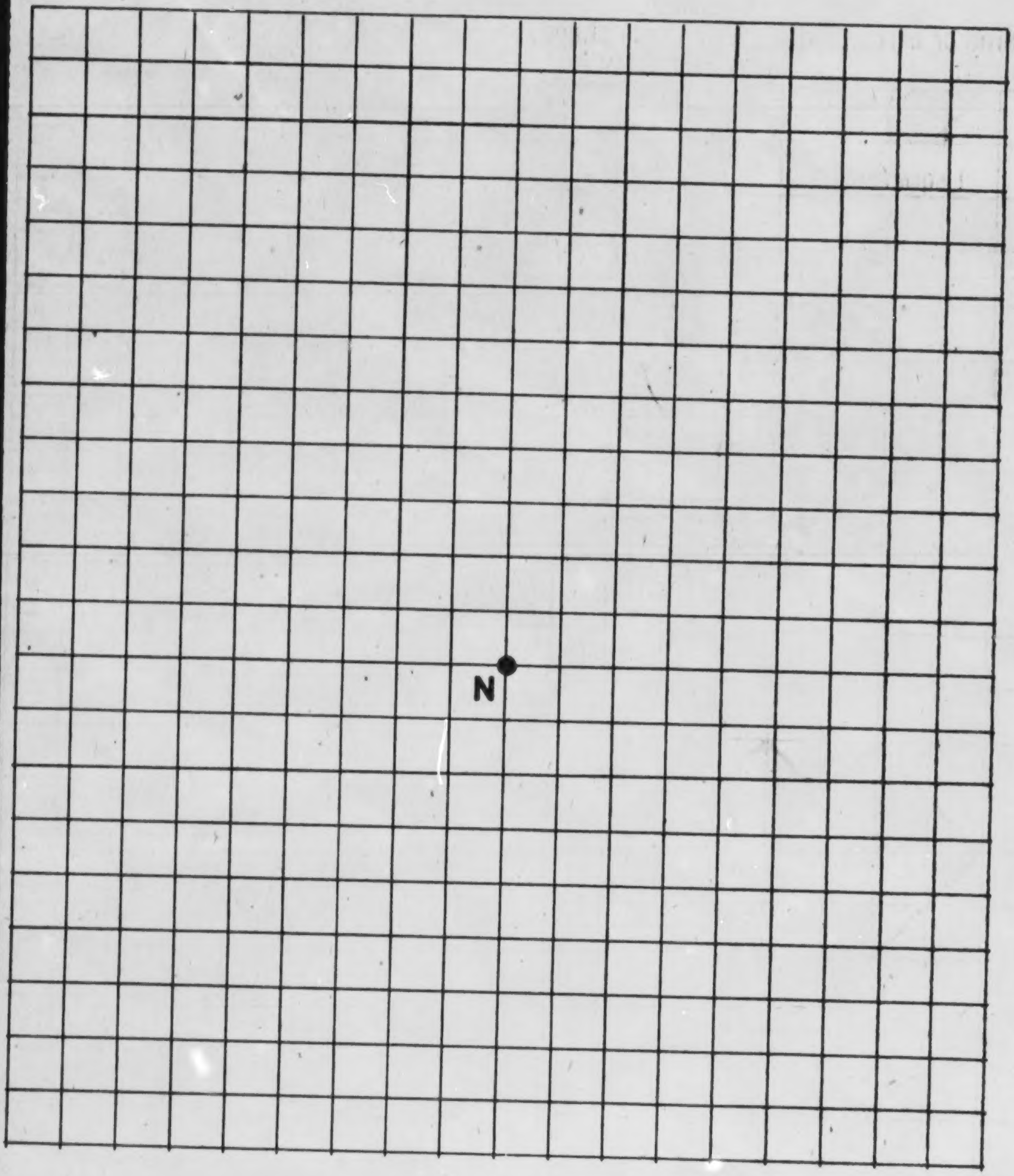
G 1 **

Find or estimate the area of each shape.



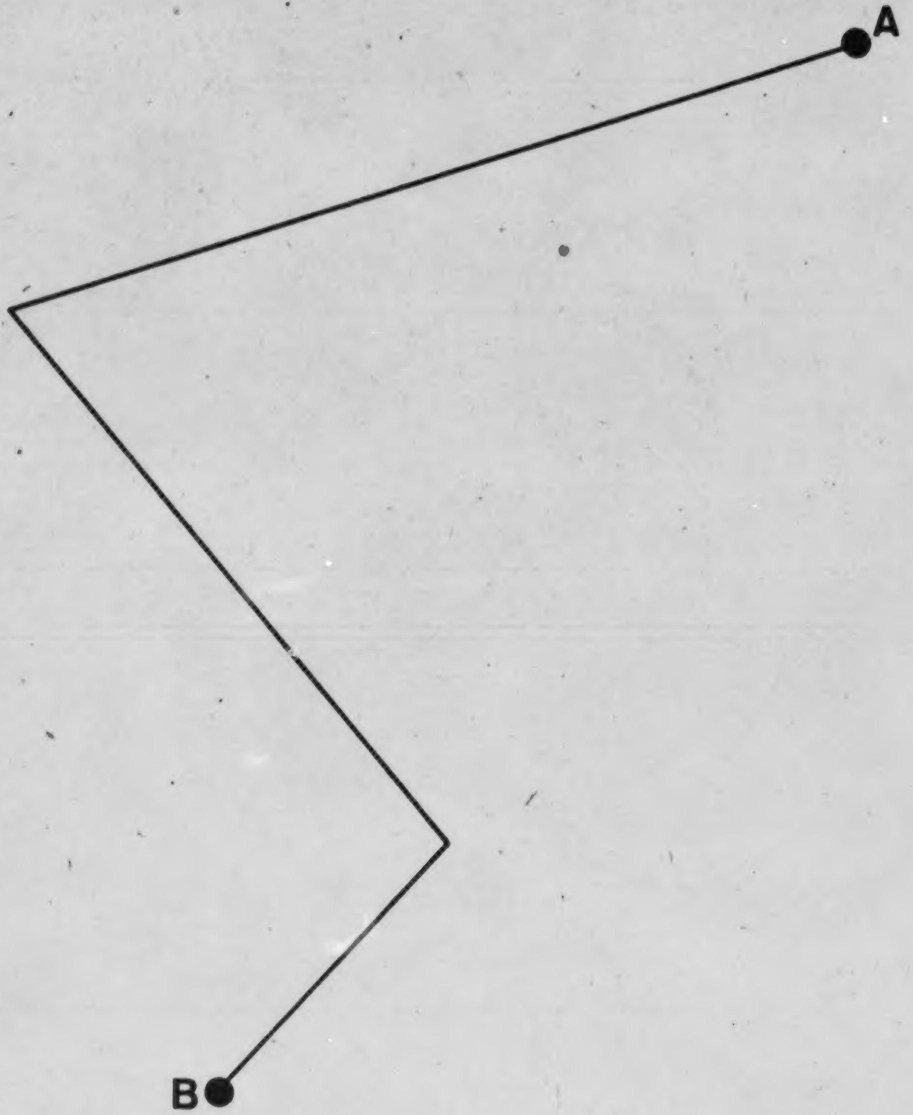
Name ____________________

G 3



Name ______________________

G 5 *



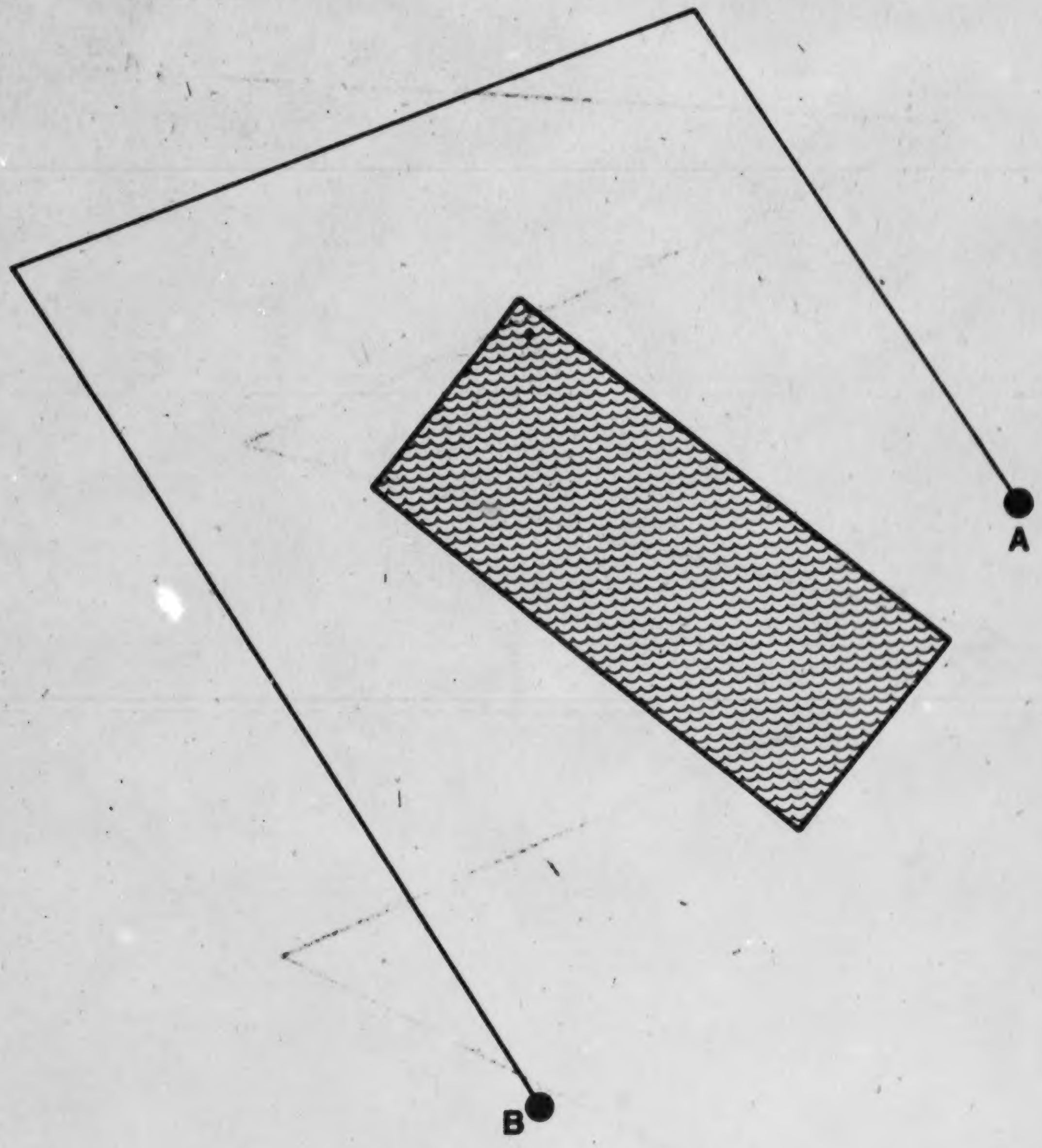
How long is this zigzag path from A to B? ____________ cm

Try to find a shorter path from A to B. Draw it.

How long is your path? ________ cm

Name ______________________

G 5 **



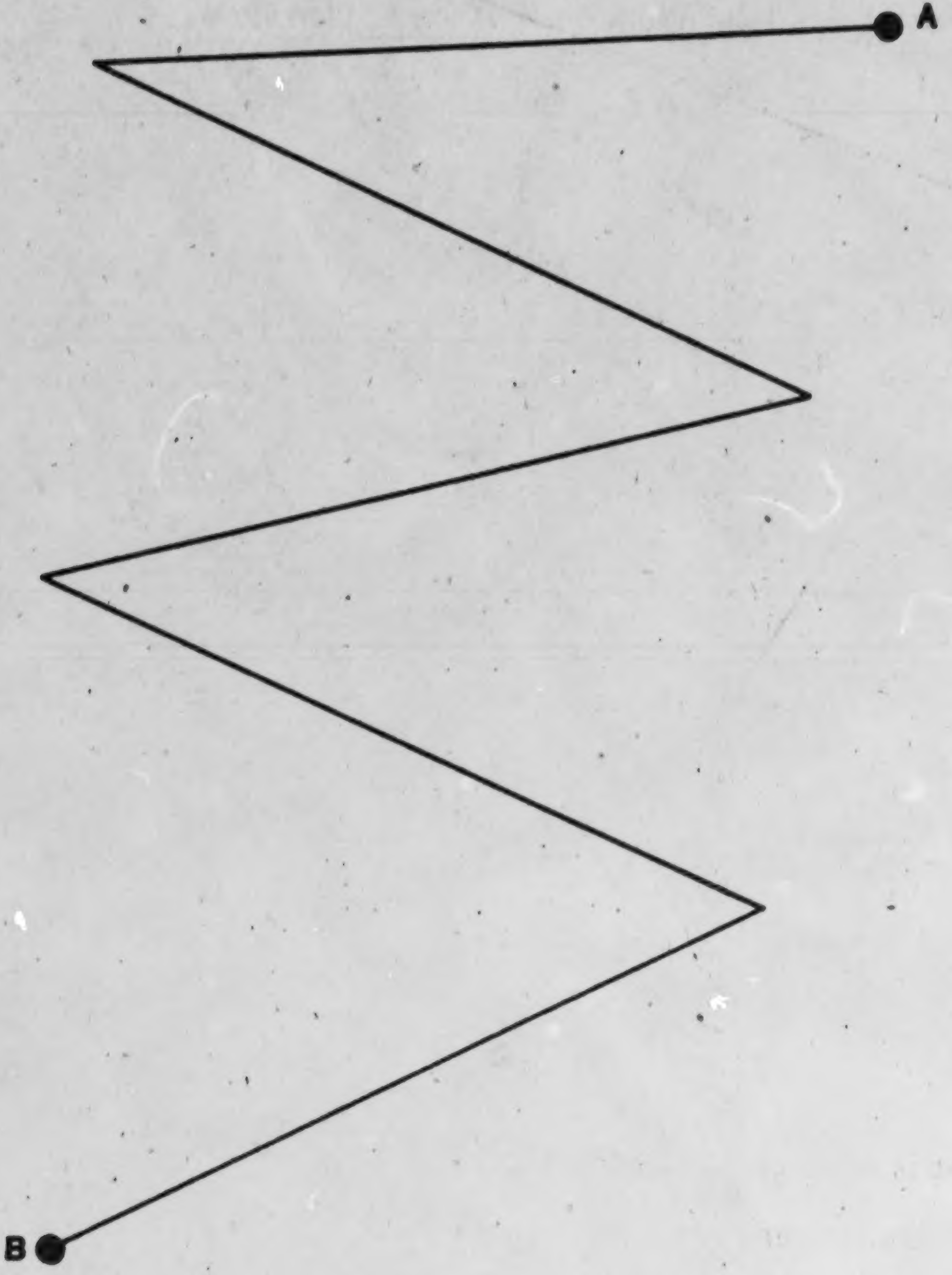
How long is this zigzag path from A to B? ________ cm

Try to find a shorter zigzag path. Draw it.

How long is your path? ________ cm

Name ____________________

G 5 ***

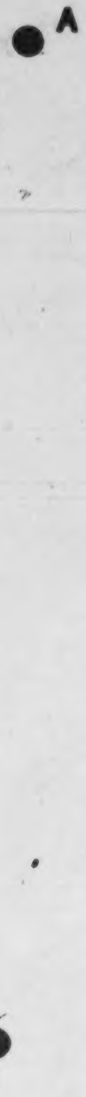


How long is this zigzag path from A to B? __________ cm

Name ____________________

G 5 ********

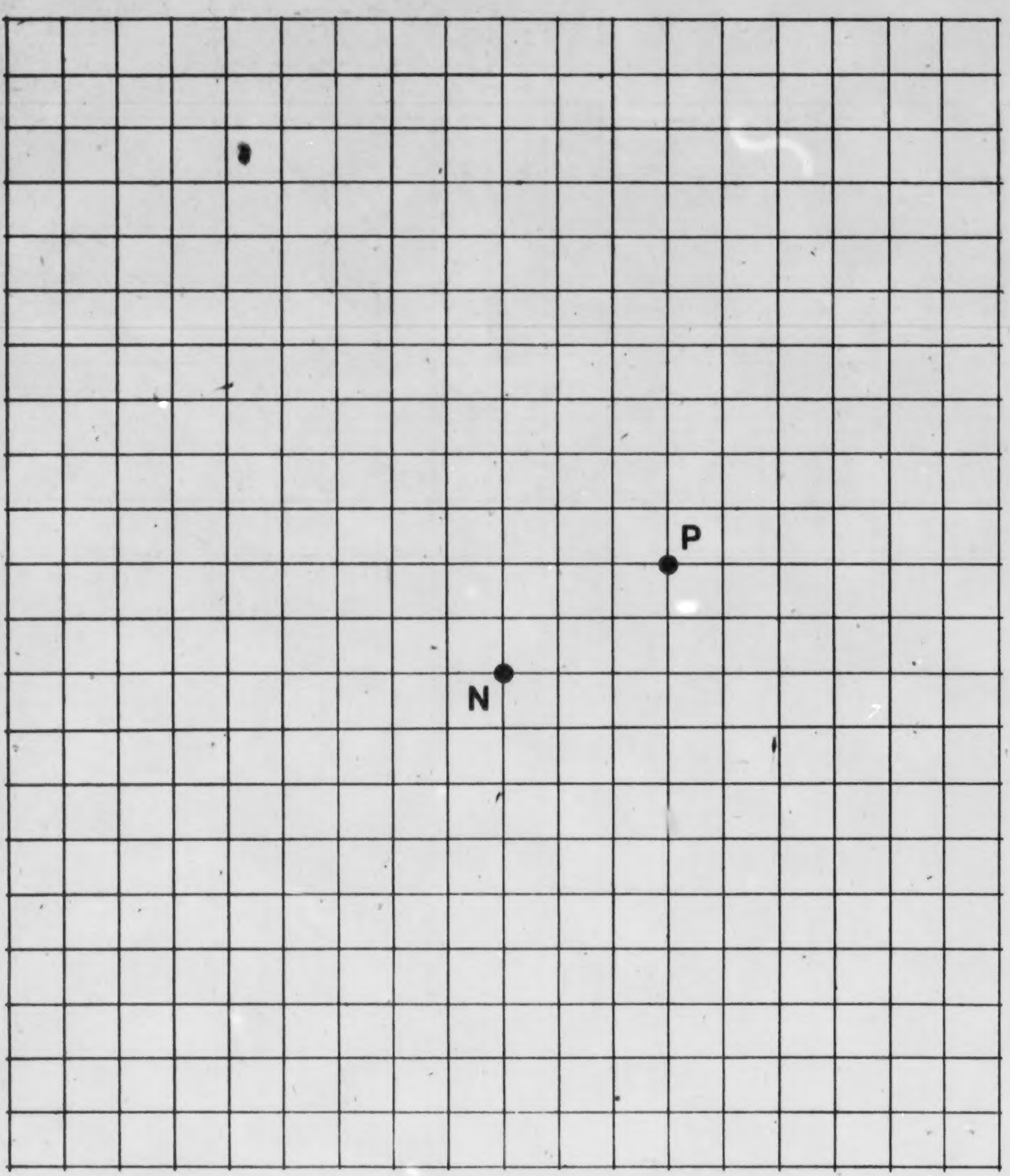
Draw a zigzag path from A to B which is longer than 50 cm.

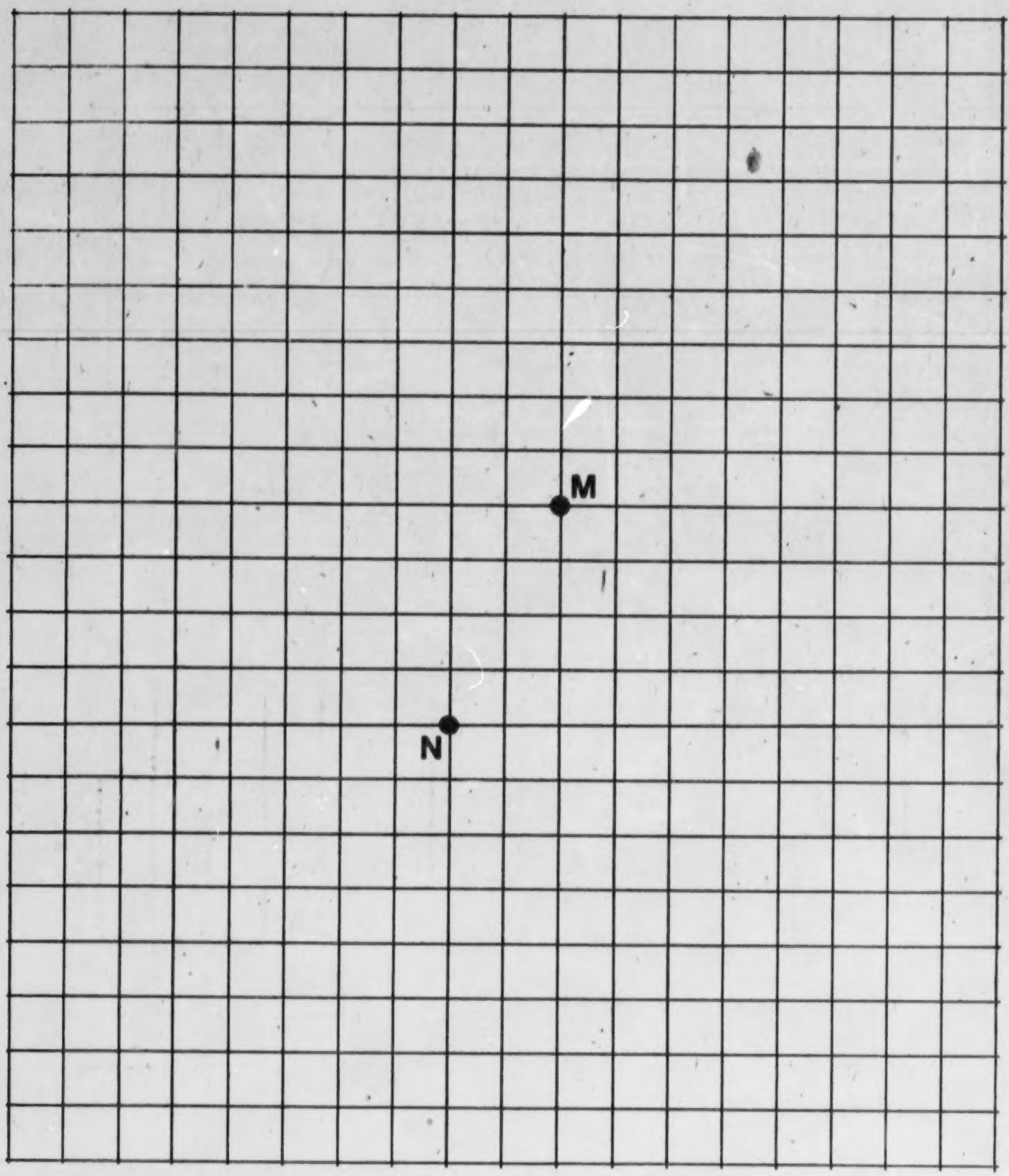


How long is your path? __________ cm

Name ______________________

G 6

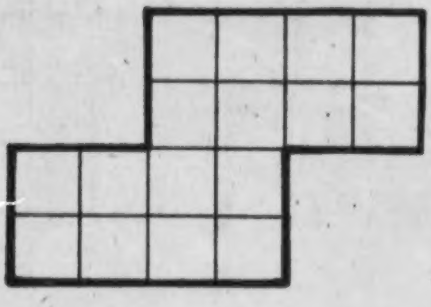



M
N

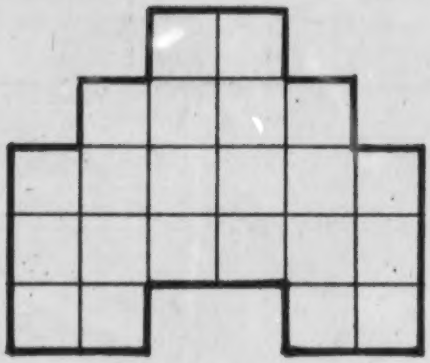
Name ______________________

G 7 *

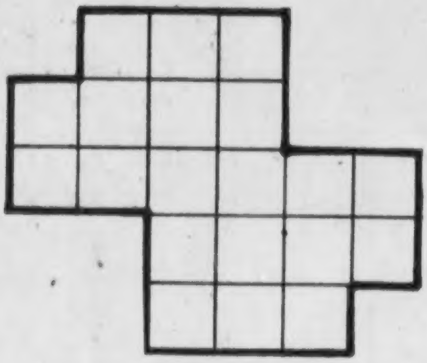
Color one - half of each shape and complete the number story.



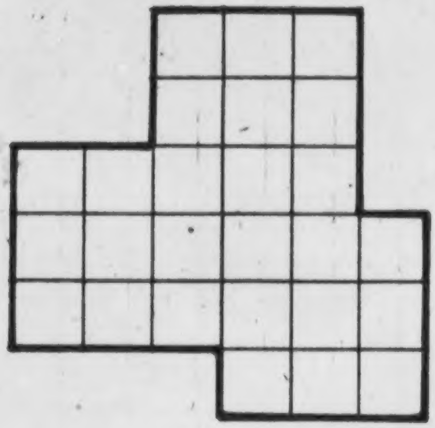
$\frac{1}{2} \times 16 =$ ____



$\frac{1}{2} \times 22 =$ ____



$\frac{1}{2} \times 20 =$ ____

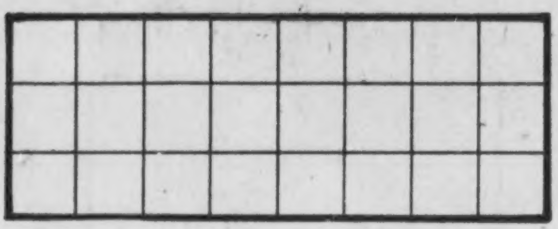


$\frac{1}{2} \times 26 =$ ____

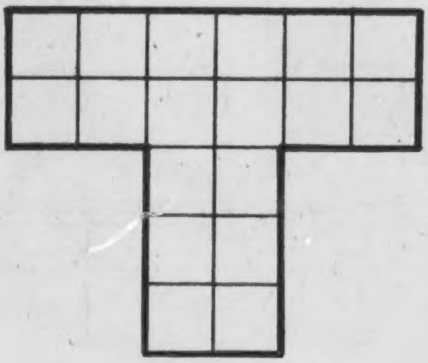
Name ____________________

G 7 **

Color one-half of each shape and complete the number story.

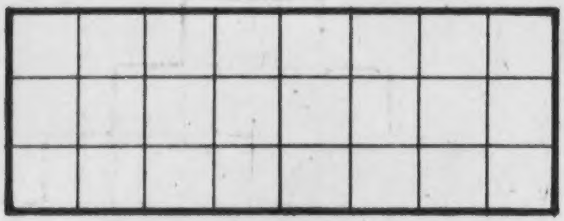


$\frac{1}{2} \times 24 =$ ____

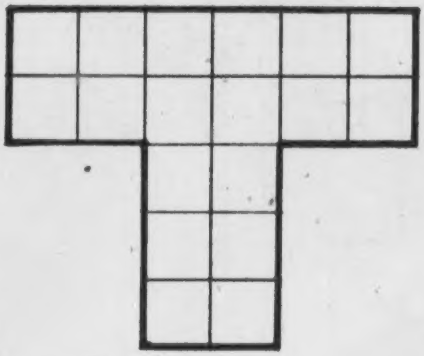


$\frac{1}{2} \times 18 =$ ____

Color one-third of each shape and complete the number story.



$\frac{1}{3} \times 24 =$ ____



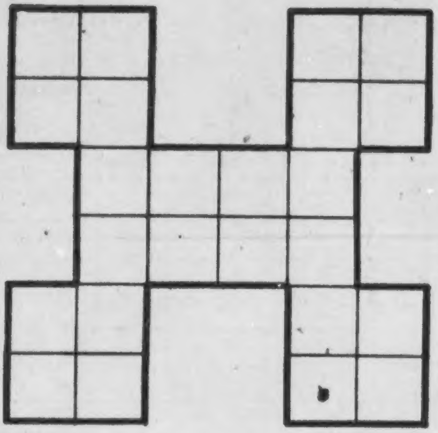
$\frac{1}{3} \times 18 =$ ____

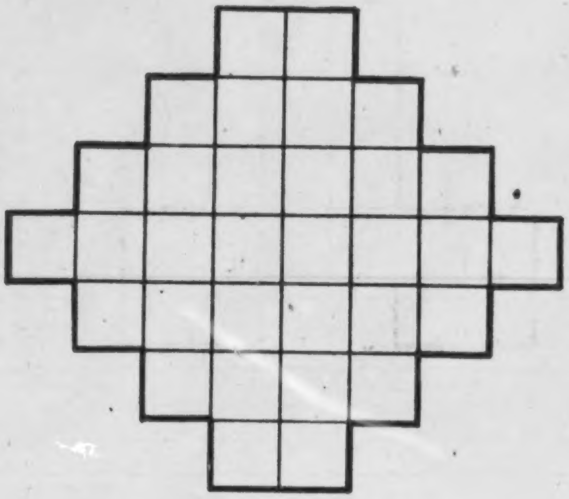
Name ______________________________

G 7 ***

Color one-half of each shape red.

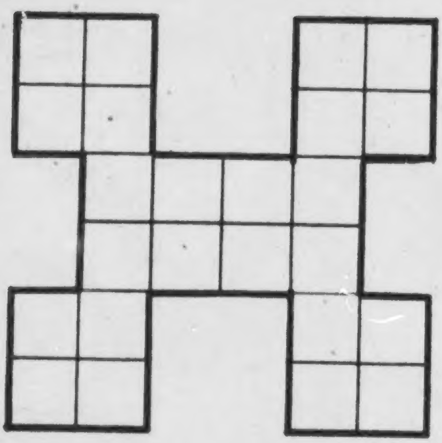
$\frac{1}{2}$

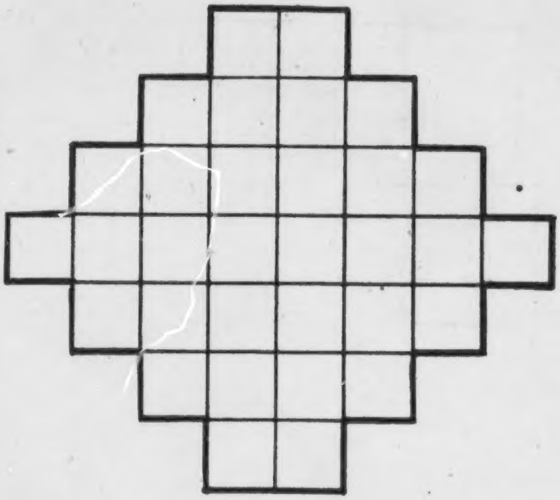




Color one-fourth of each shape blue.

$\frac{1}{4}$

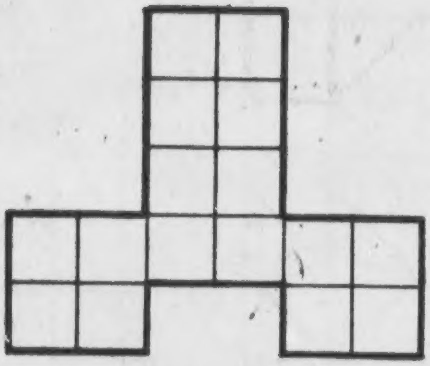




Name ______________________

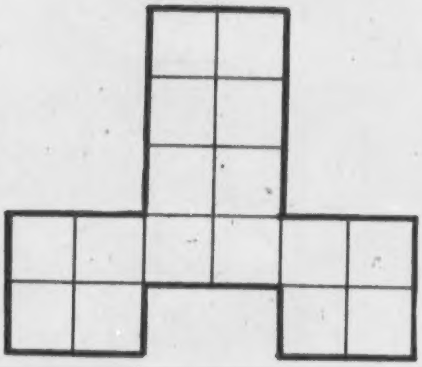
G 7 ****

Color one-half of this shape.



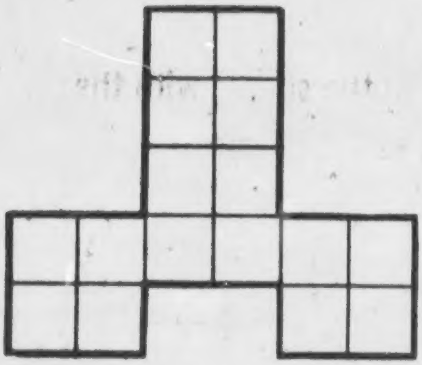
$\frac{1}{2}$

Color one-fourth of this shape.



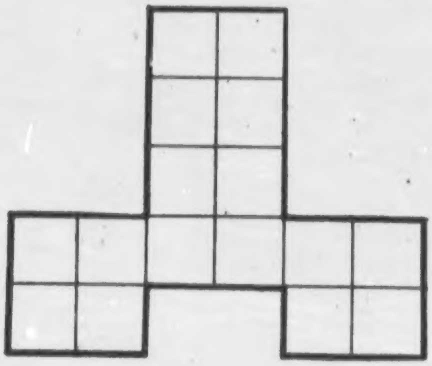
$\frac{1}{4}$

Color one-eighth of this shape.



$\frac{1}{8}$

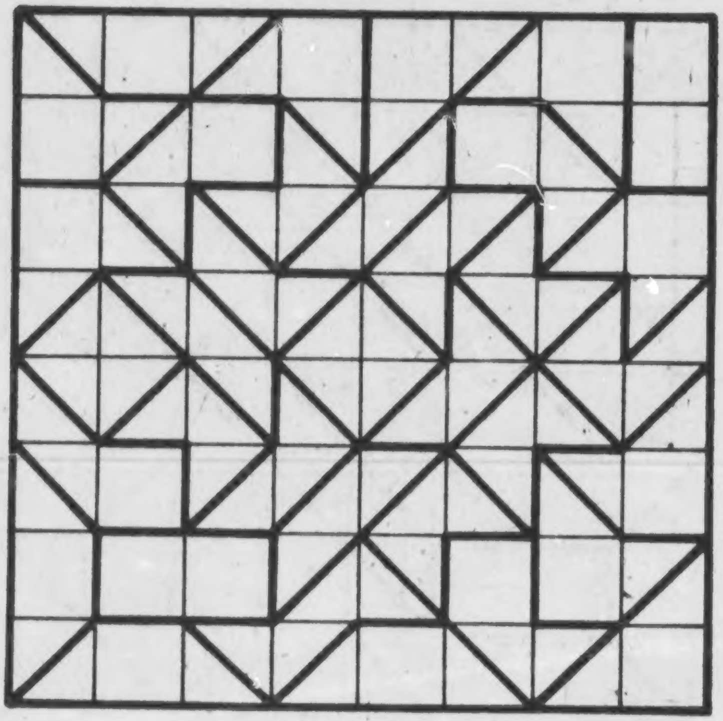
Color one-sixteenth of this shape.



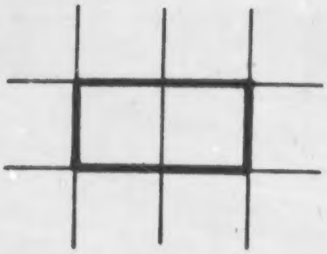
$\frac{1}{16}$

Name ____________________

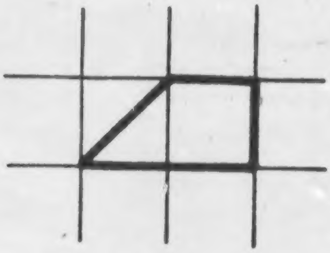
G 8

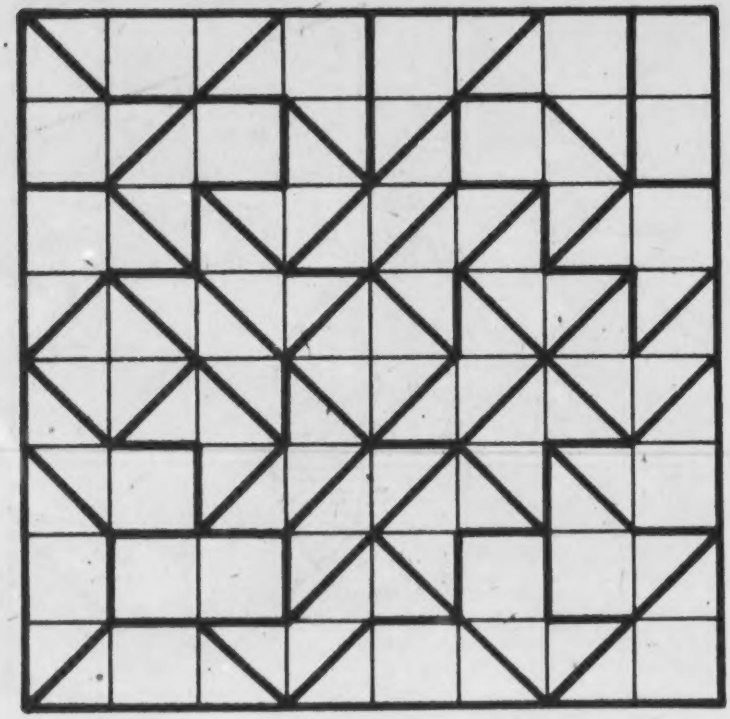


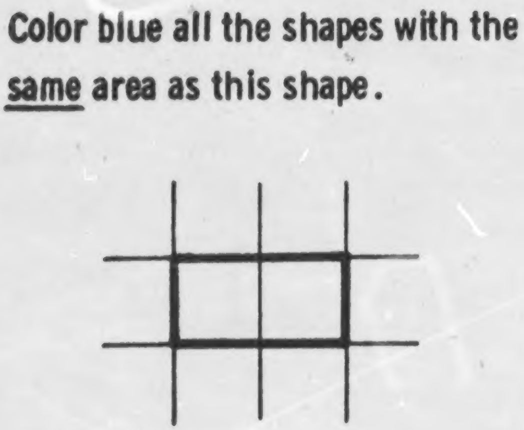
Color blue all the shapes with the same area as this shape.



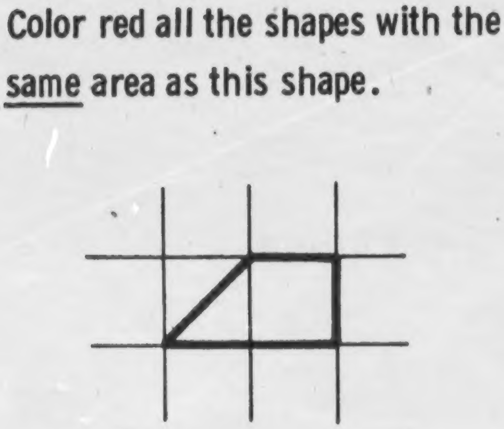
Color red all the shapes with the same area as this shape.







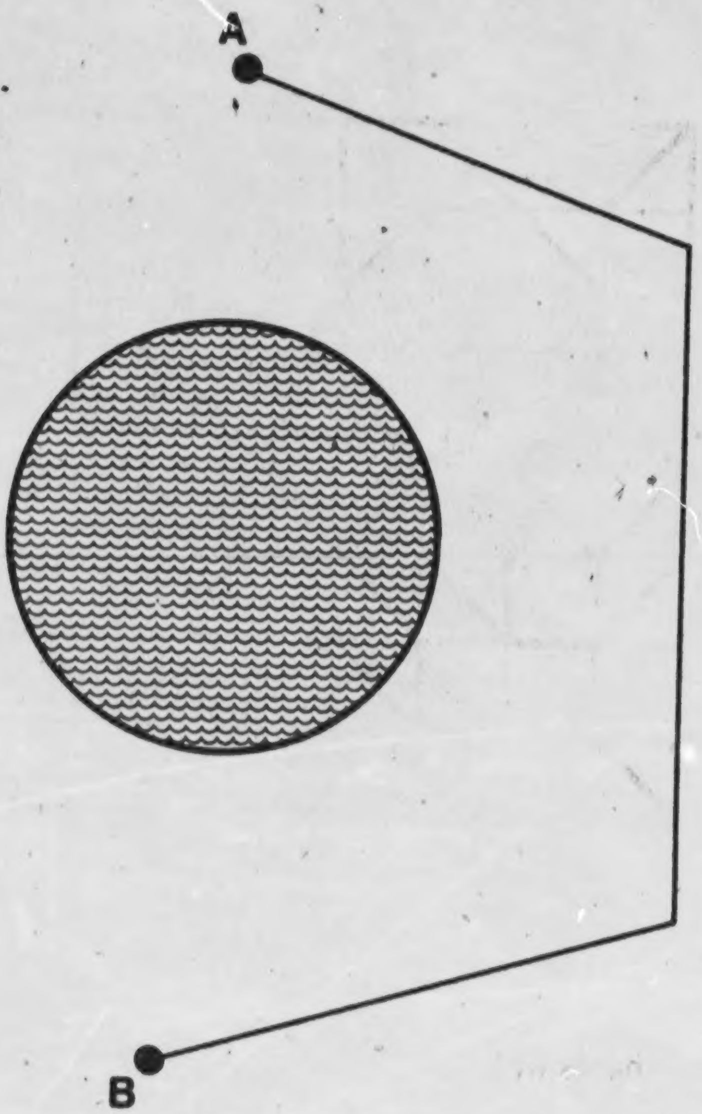
Color blue all the shapes with the same area as this shape.



Color red all the shapes with the same area as this shape.

Name ______________________

G 9 *



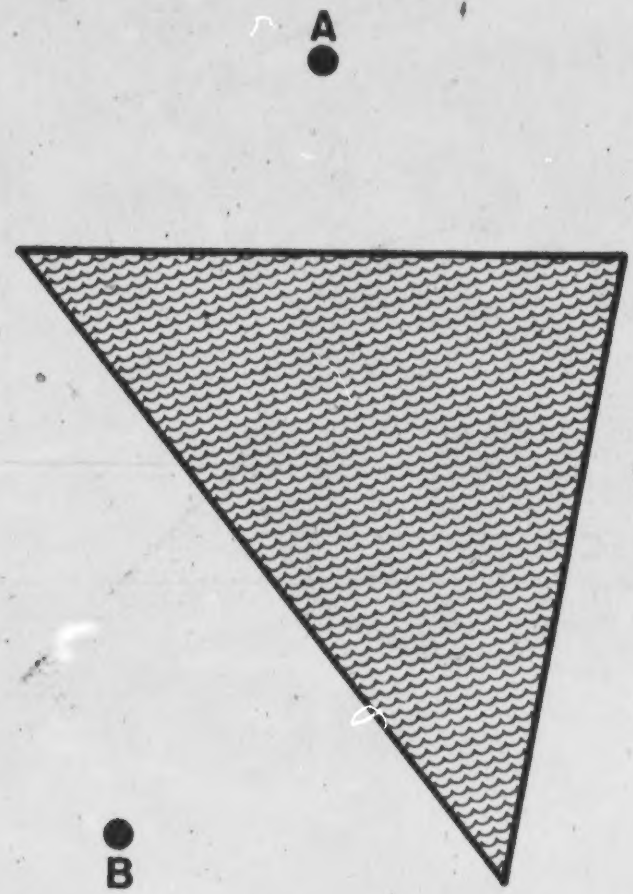
How long is this zigzag path from A to B? ____________ cm

Try to find a shorter zigzag path. Draw it.

How long is your zigzag path? ____________ cm

Name ____________________

G 9 **

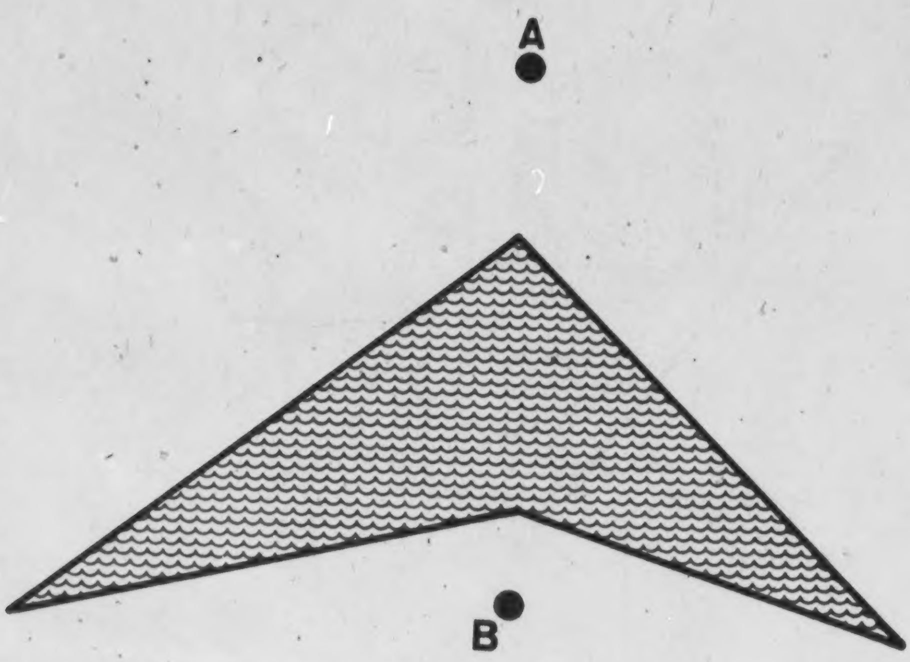


Draw a zigzag path from A to B. Try to make your zigzag path as short as possible.

How long is your zigzag path? __________ cm

Name ______________________________

G 9 ***



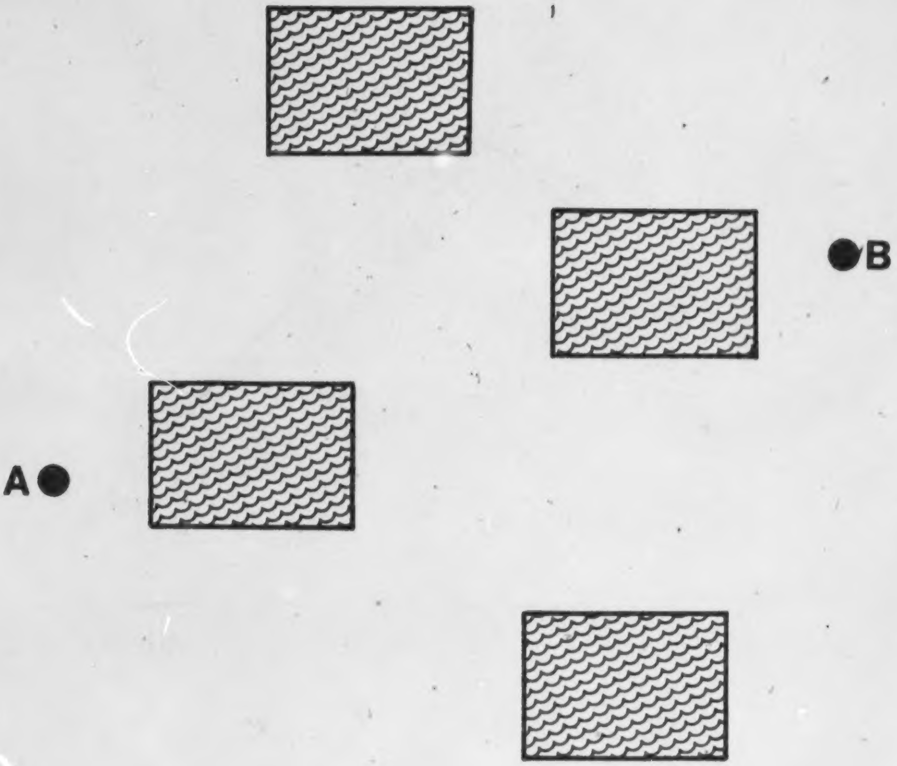
Draw a zigzag path from A to B. Try to make your zigzag path shorter than 18cm.

How long is your zigzag path? __________ cm

Name ____________________

G 9 ****

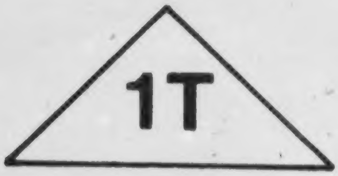
Draw a zigzag path from A to B. Make your zigzag path as short as possible.



How long is your zigzag path? __________ cm

Name ______________________

G 10 *

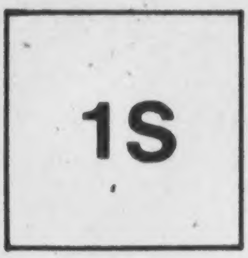


Use Tangram pieces to build:

a square with area 4T.	a triangle with area 4T.
a square with area 8T.	a triangle with area 8T.

Name ______________________________

G 10 **



Use Tangram pieces to build:

a rectangle with area 3 S.

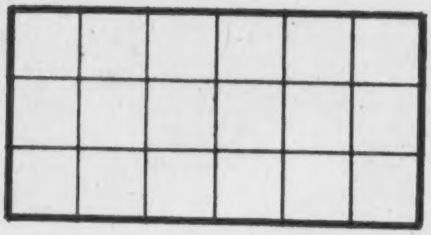
---

a rectangle with area 6 S.

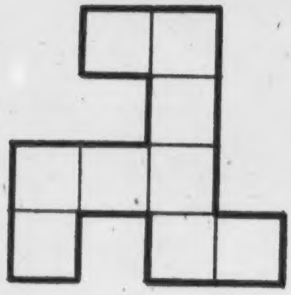
Name ______________________

G 11 *

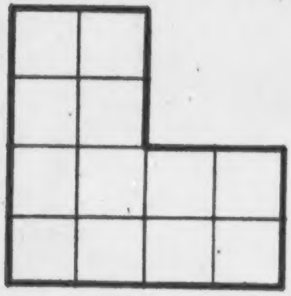
Color one-third of each shape and complete the number story.



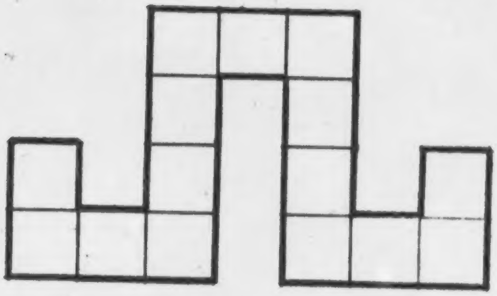
$\frac{1}{3} \times 18 =$ ______



$\frac{1}{3} \times 9 =$ ______



$\frac{1}{3} \times 12 =$ ______

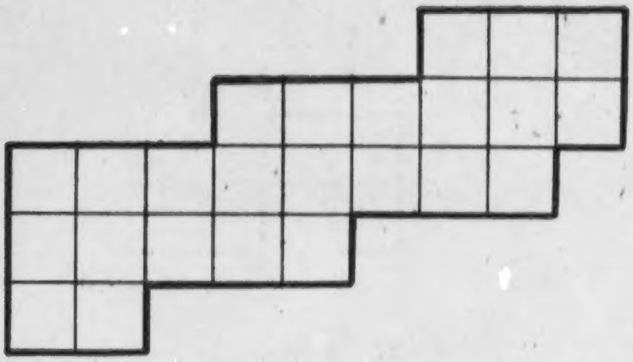


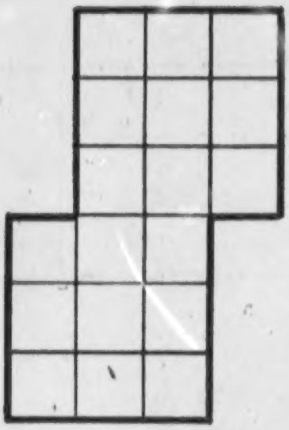
$\frac{1}{3} \times 15 =$ ______

Name ______________________________

G 11 **

Color one-half of each shape and complete the number story.

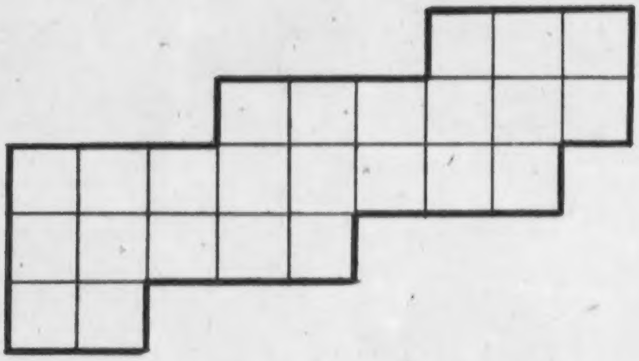


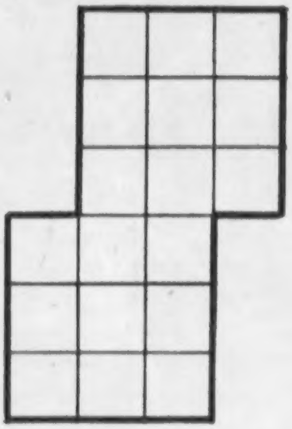


$\frac{1}{2} \times 24 =$ ______

$\frac{1}{2} \times 18 =$ ______

Color one-third of each shape and complete the number story.





$\frac{1}{3} \times 24 =$ ______

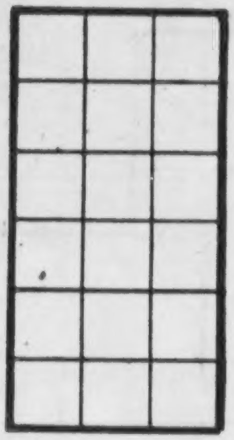
$\frac{1}{3} \times 18 =$ ______

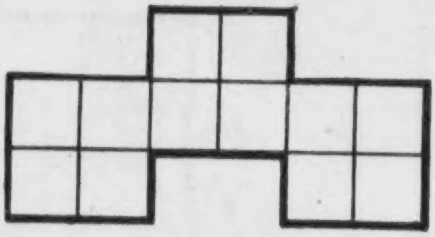
Name ______________________________

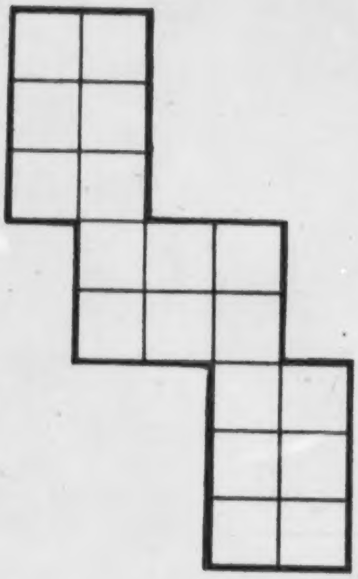
G 11 ＊＊＊

Color one-third of each shape.

$$\frac{1}{3}$$

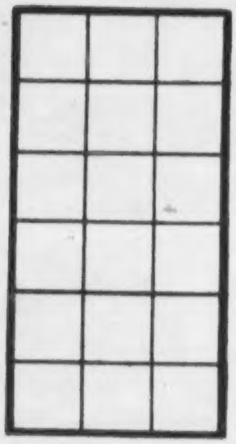


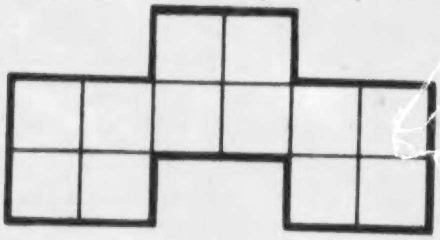


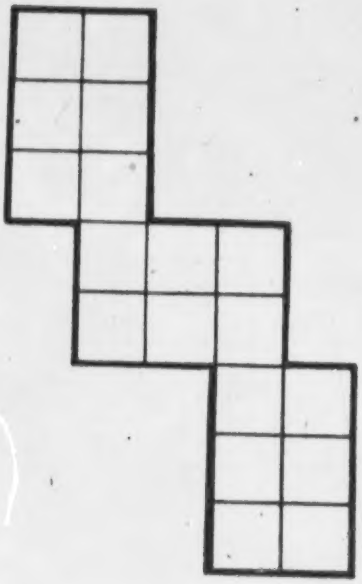


Color one-sixth of each shape.

$$\frac{1}{6}$$



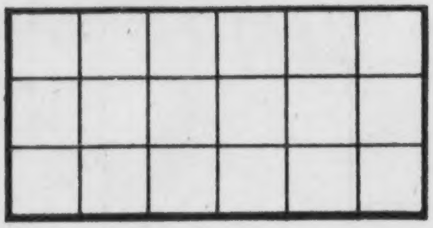


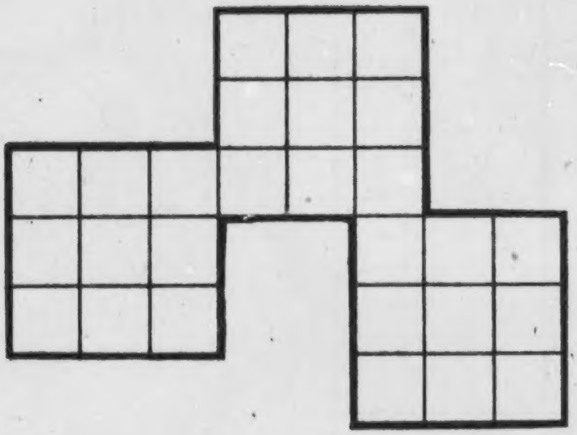


Name ______________________

G 11 ****

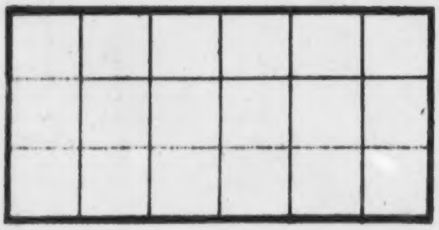
Color one-third of each shape.

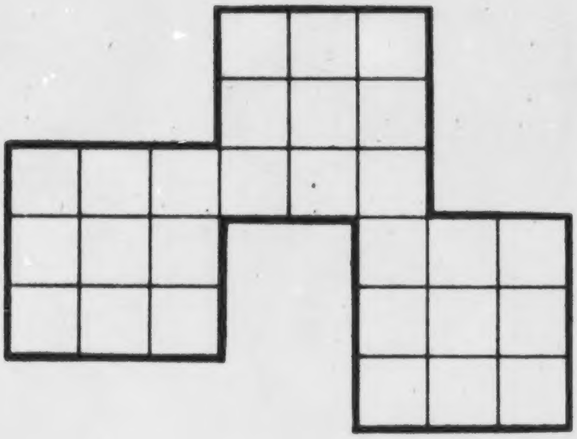




$\frac{1}{3}$

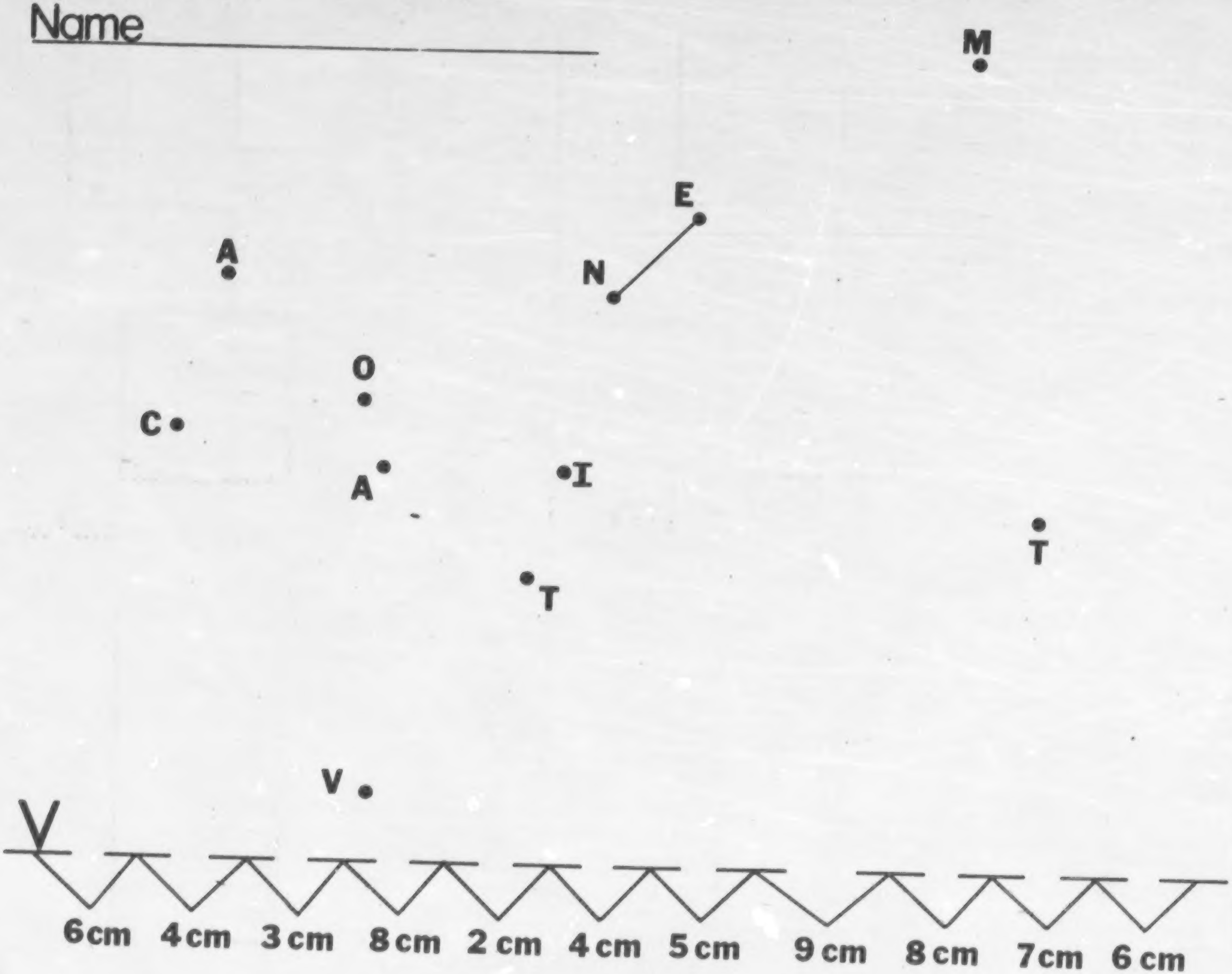
Color one-ninth of each shape.





$\frac{1}{9}$

Name ______________________

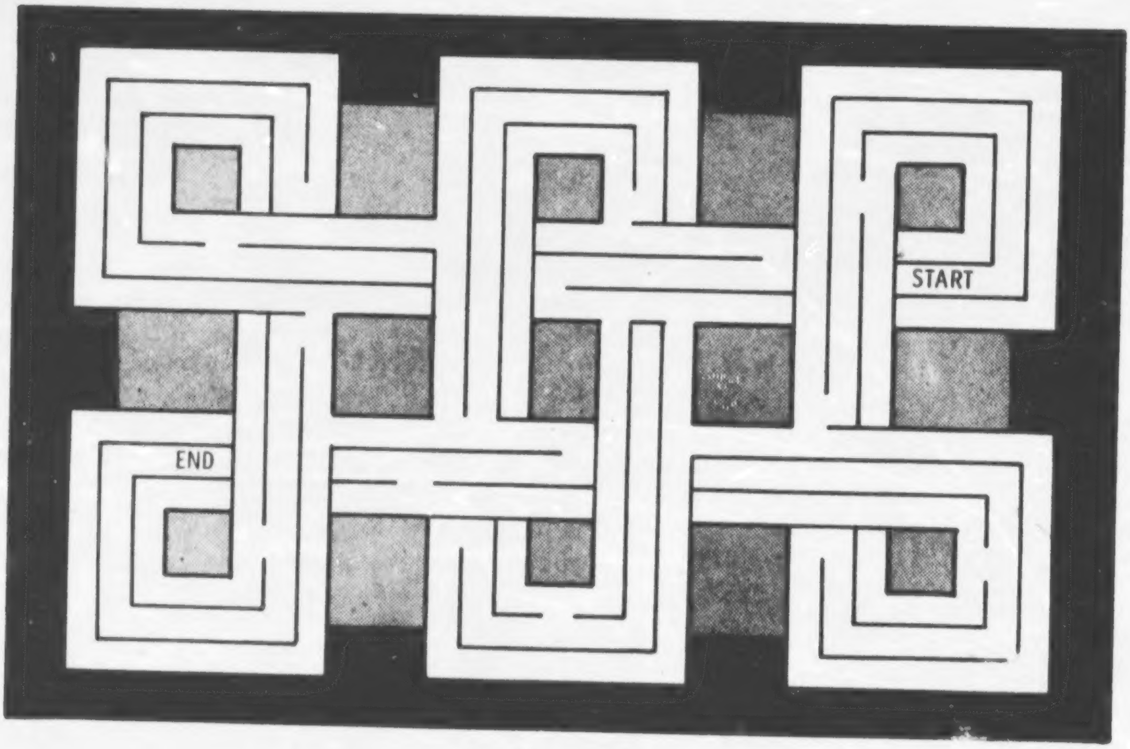

M
E
A
N
O
C
A
I
T
T
V
V
6cm
4cm
3cm
8cm
2cm
4cm
5cm
9cm
8cm
7cm
6cm

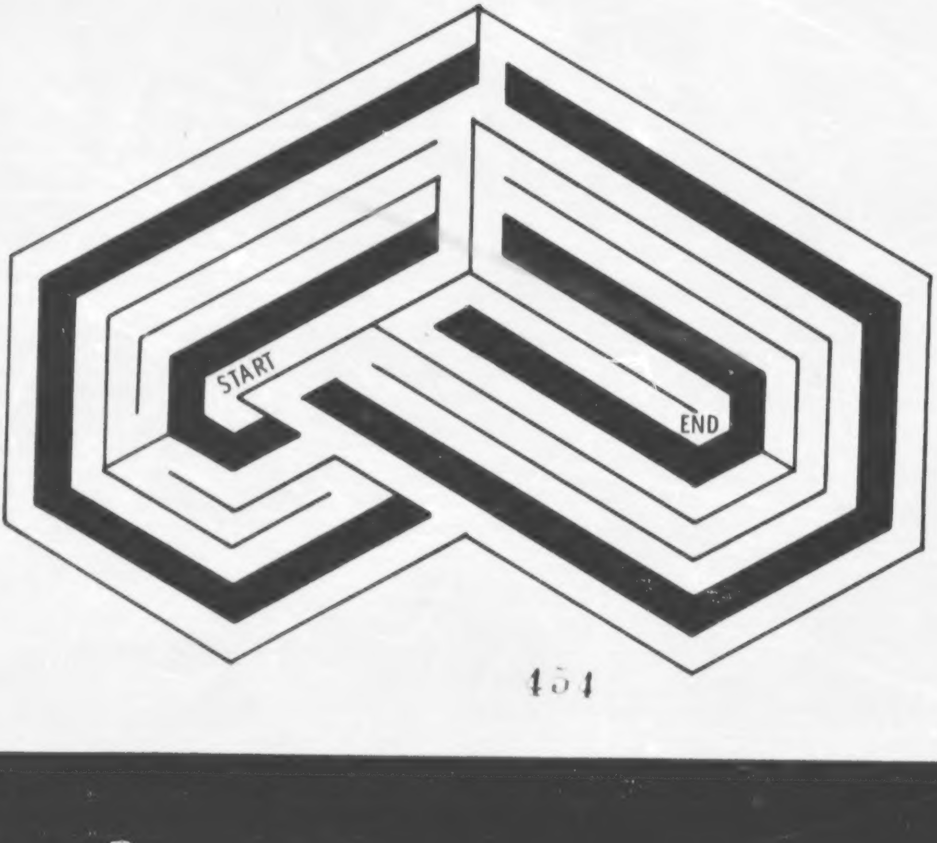
G12

Name ____________________

G 13 *

In each maze, find a path from start to end.

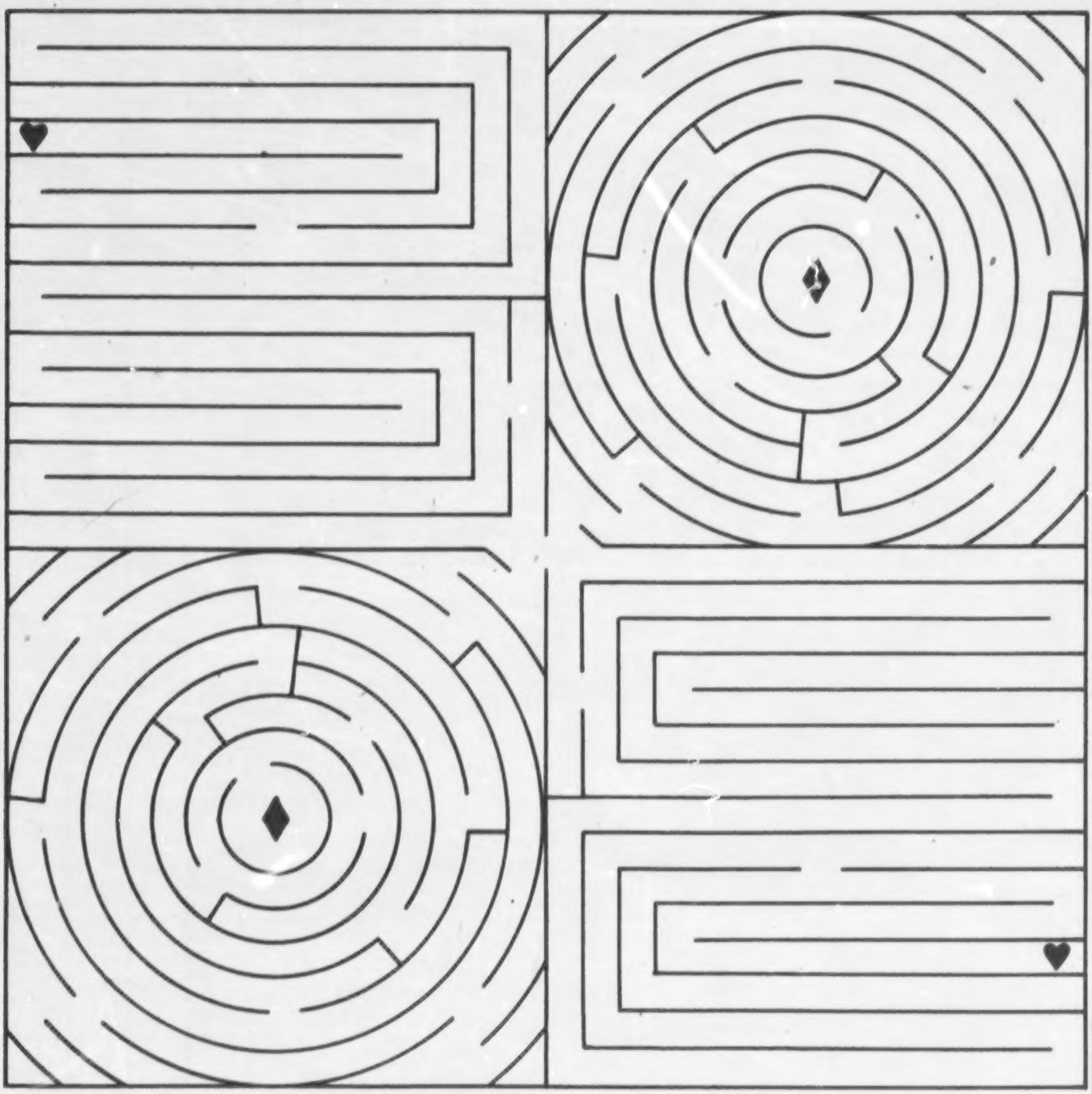




Name ______________________________

G 13 **

In this maze, find a path connecting the hearts ♥ and another path connecting the diamonds ♦ .



Name ______________________________

W 4

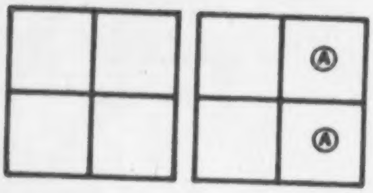
Name ______________________

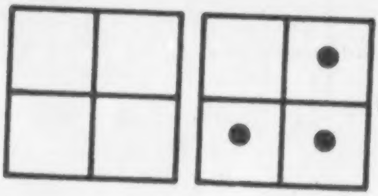
W 5

Name ______________________

W 6 *

What number is on the Minicomputer?


= ____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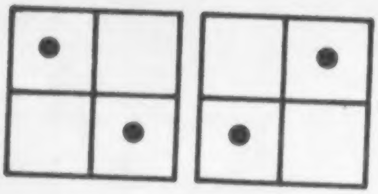
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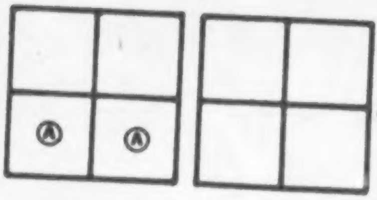
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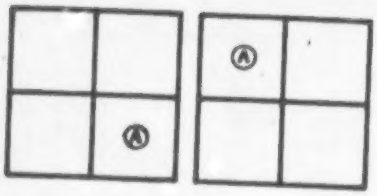
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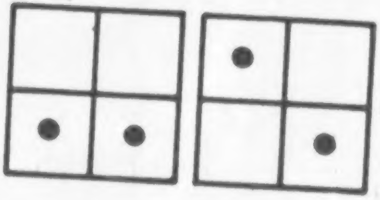
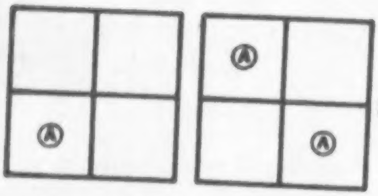
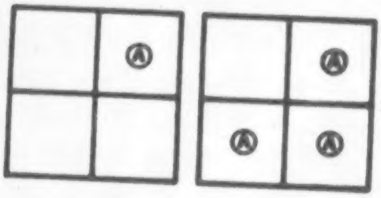
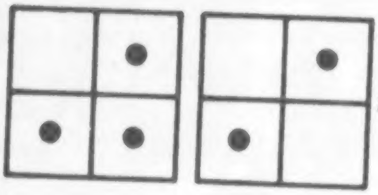
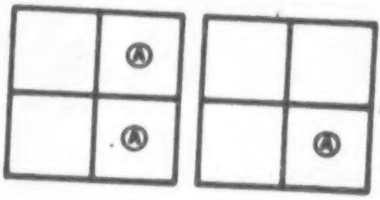

= ____


= ____


= ____

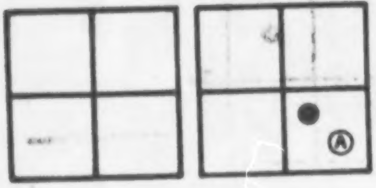

= ____


= ____

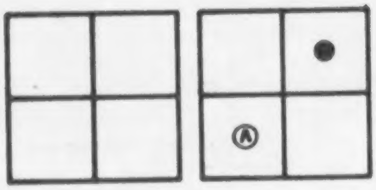
Name ________________________________

**W 6 **

What number is on the Minicomputer?



= ________



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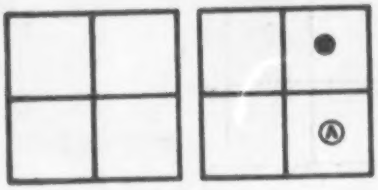

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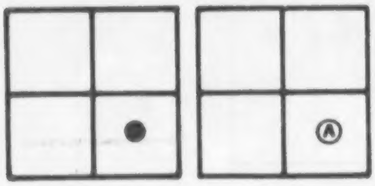
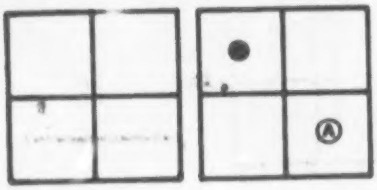
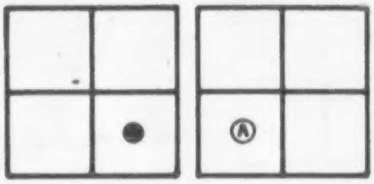


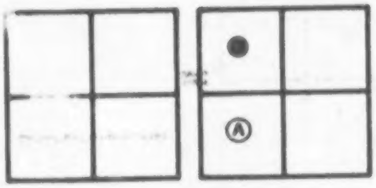
= ________



= ________



= ________



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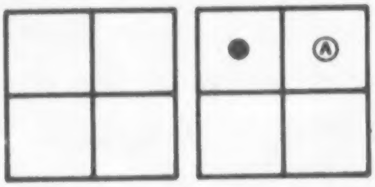

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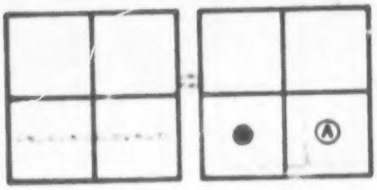

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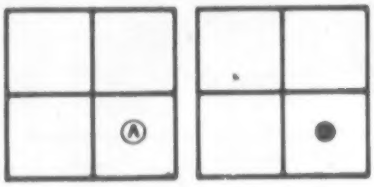
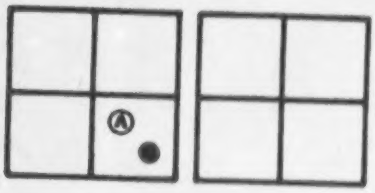


= ________

Name ________________________

W 6 ***

What number is on the Minicomputer?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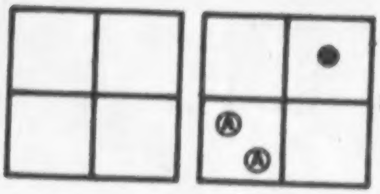
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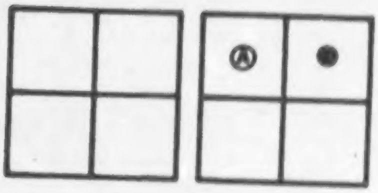
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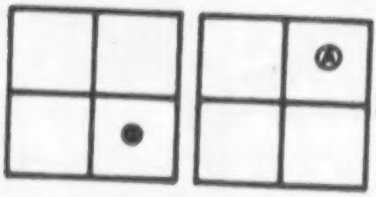
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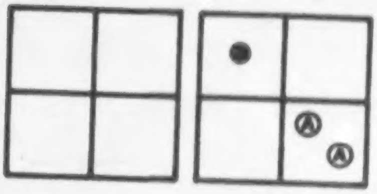
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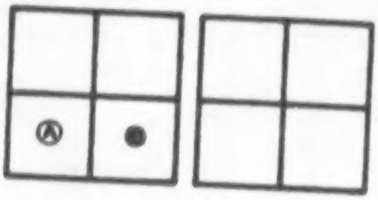
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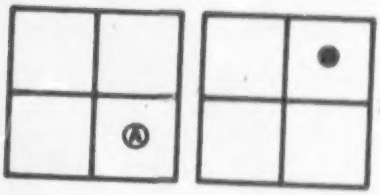
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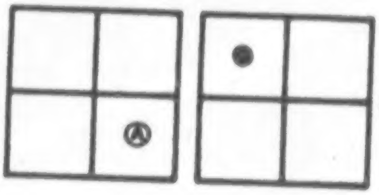
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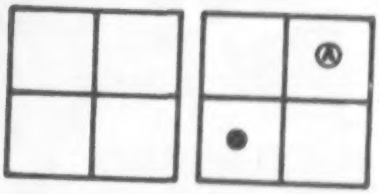
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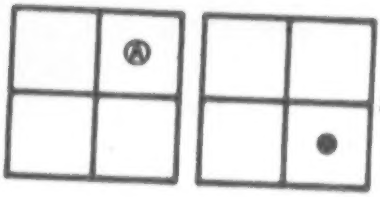
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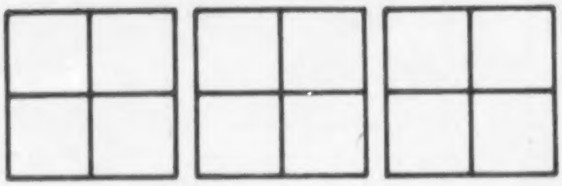
Name ______________________________

W 6 ****

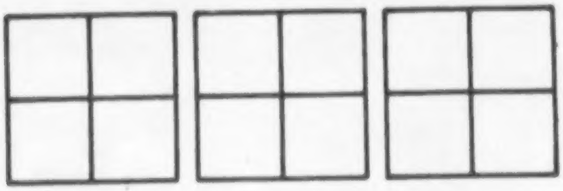
Put these numbers on the Minicomputer.

Use at least one negative checker for each number.

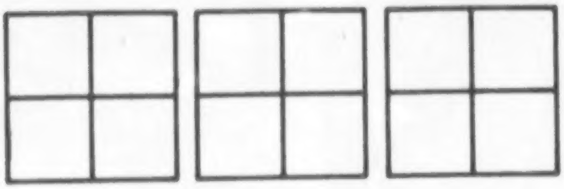
19 =



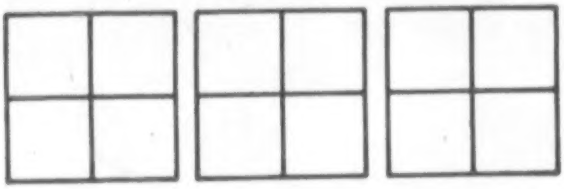
78 =



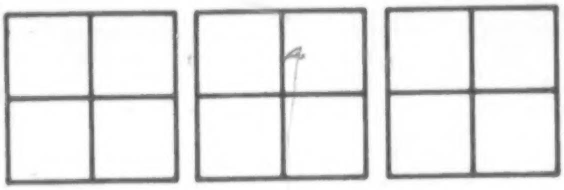
32 =



96 =



199 =



270 =